POSITIVITY BOUNDS ON HIGGS-PORTAL DARK MATTER

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JHEP **06**, 124 (2023) (arXiv: 2302.02879, for WIMP Scalar Dark Matter)
JHEP **11**, 119 (2023) (arXiv: 2308.14629, for Freeze-in Scalar Dark Matter)

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The Future is Flavourful
National Yang Ming Chiao Tung University, Hsinchu, Taiwan

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WIMP Dark Matter

Positivity Bounds (1/12)

- EFT

  heavy degrees of freedom decouple

  for large-distance phenomena
  or small momentum scale

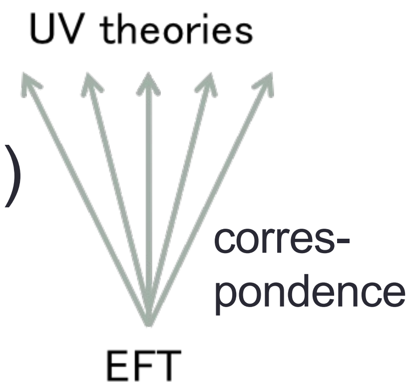
- EFT interaction terms:

$$\mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \cdots$$

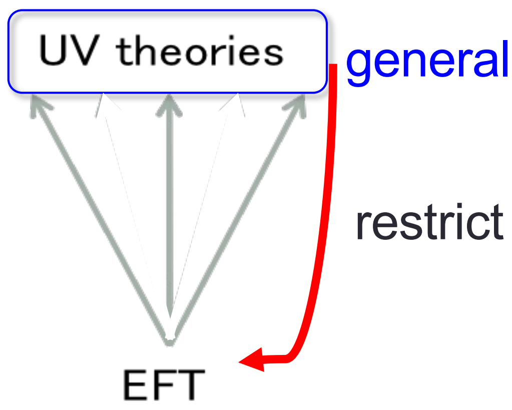
Wilson coefficients

$$\mathcal{L} = \sum_{i=1}^{n_d} \frac{c_i^5}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i^6}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{c_i^7}{\Lambda^3} \mathcal{O}_i^{(7)} + \frac{c_i^8}{\Lambda^4} \mathcal{O}_i^{(8)} + \cdots$$

Positivity Bounds (2/12)



- EFT is for the energy scale
  E << Λ (typical energy scale of the UV physics)
- Many UV models correspond with EFT
- From the general feature of UV theory,
  can we bound on Wilson coefficients of EFT?



If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

1. Special relativity ⟶ Lorentz invariance
2. Conservation of probability ⟶ Unitarity
3. Causality - - -> Analyticity

Positivity Bounds (3/12)

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP **0610**, 014(2006)

- One of the way to do this is Positivity bounds
- Positivity bounds: the signs of certain combinations of Wilson coefficients in EFT have to be positive, e.g. W^4 operators:

$$\frac{F_{T,0}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\hat{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\hat{\mathrm{W}}^{\alpha\beta}] \qquad \frac{F_{T,1}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\nu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\beta}\hat{\mathrm{W}}^{\alpha\nu}]$$

$$\frac{F_{T,2}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\mu}\hat{\mathrm{W}}^{\mu\beta}]\mathrm{Tr}[\hat{\mathrm{W}}_{\beta\nu}\hat{\mathrm{W}}^{\nu\alpha}] \qquad \frac{F_{T,10}}{\Lambda^4}\mathrm{Tr}[\hat{\mathrm{W}}_{\mu\nu}\tilde{\mathrm{W}}^{\mu\nu}]\mathrm{Tr}[\hat{\mathrm{W}}_{\alpha\beta}\tilde{\mathrm{W}}^{\alpha\beta}]$$

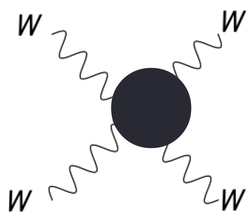
$$\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}W^{I,\mu\nu} \qquad \tilde{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}\left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{I,\rho\sigma}\right)$$

One of the positivity bounds:

$$2F_{T,0} + 2F_{T,1} + F_{T,2} \geq 0$$

KY, C. Zhang, S. Y. Zhou, JHEP **01**, 095 (2021)

Positivity Bounds (4/12)

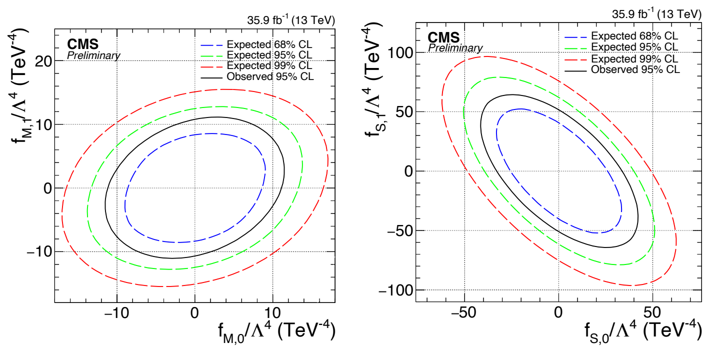


[FFFF]=Dim 8

- Positivity bounds can apply for dim-8 operators in tree-level ← Froissart Bound (⇦Analyticity), etc.

- Dim-8 operators are more suppressed by Λ than lower dimensional ones, however, for dim-8 aQGC operators, LHC experimentalists have been and currently working on constraining them

CMS-PAS-SMP-18-001



- In the future, more dim-8 effects may become accessible

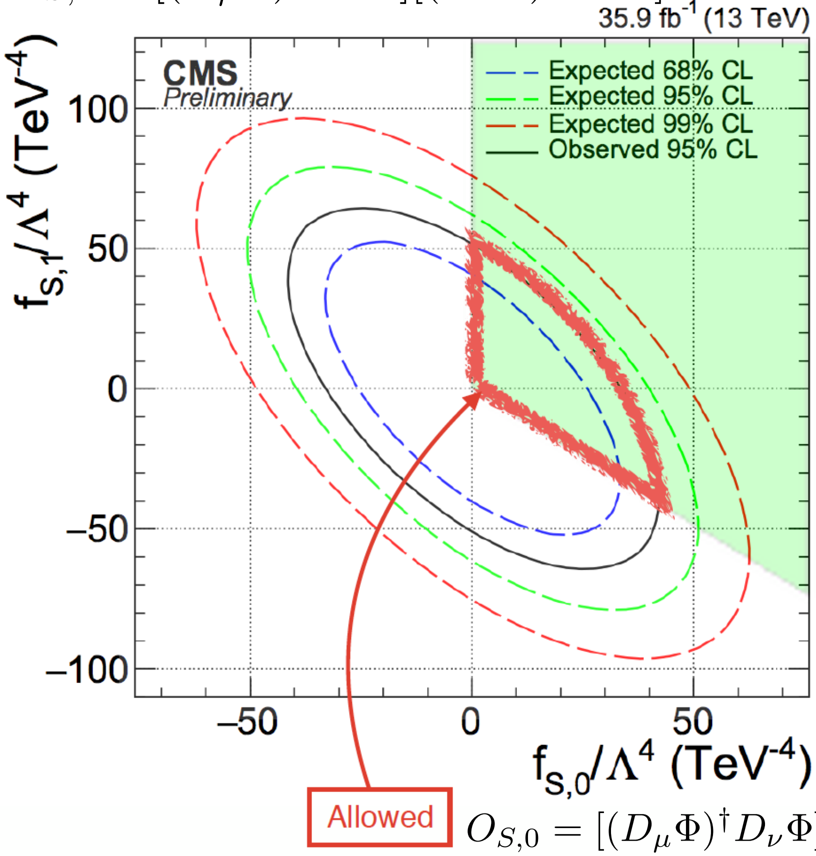
(e.g. new observable proposed for DY process:
Alioli, Boughezal, Mereghetti, Petriello, Phys. Lett. B **809**, 135703 (2020),
X. Li, K. Mimasu, KY, C. Yang, C. Zhang, S. Y. Zhou, JHEP**10**(2022)107)

Positivity Bounds (5/12)

Positivity bounds are important as they offer complementary bounds to the experiments Q. Bi, C. Zhang, S.-Y. Zhou JHEP **1906** (2019) 137

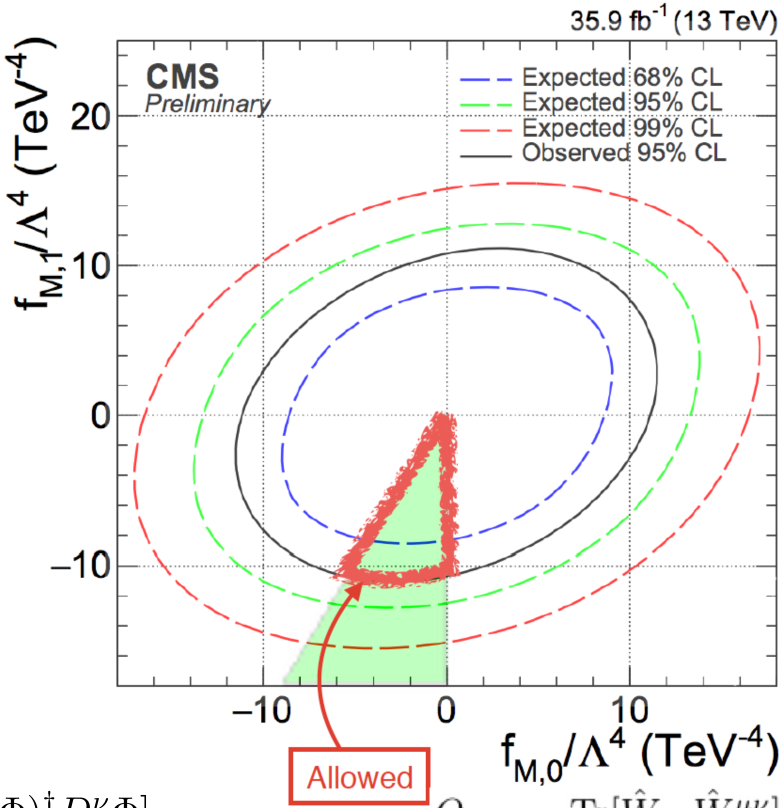
E.g. WZjj (CMS-PAS-SMP-18-001)

$O_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi][(D_\nu\Phi)^\dagger D^\nu\Phi]$



$O_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\mu\Phi)^\dagger D^\nu\Phi]$

$O_{M,1} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}][(D_\beta\Phi)^\dagger D^\mu\Phi]$



$O_{M,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}][(D_\beta\Phi)^\dagger D^\beta\Phi]$

Positivity restricts the directions in which SM deviation is possible

T. N. Pham, T. N. Truong, Phys. Rev. D **31**, 3027 (1985)
B. Ananthanarayan, D. Toublan, G. Wanders, Phys. Rev. D **51**, 1093-1100 (1995)
A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP **0610**, 014 (2006)

Positivity Bounds (6/12)

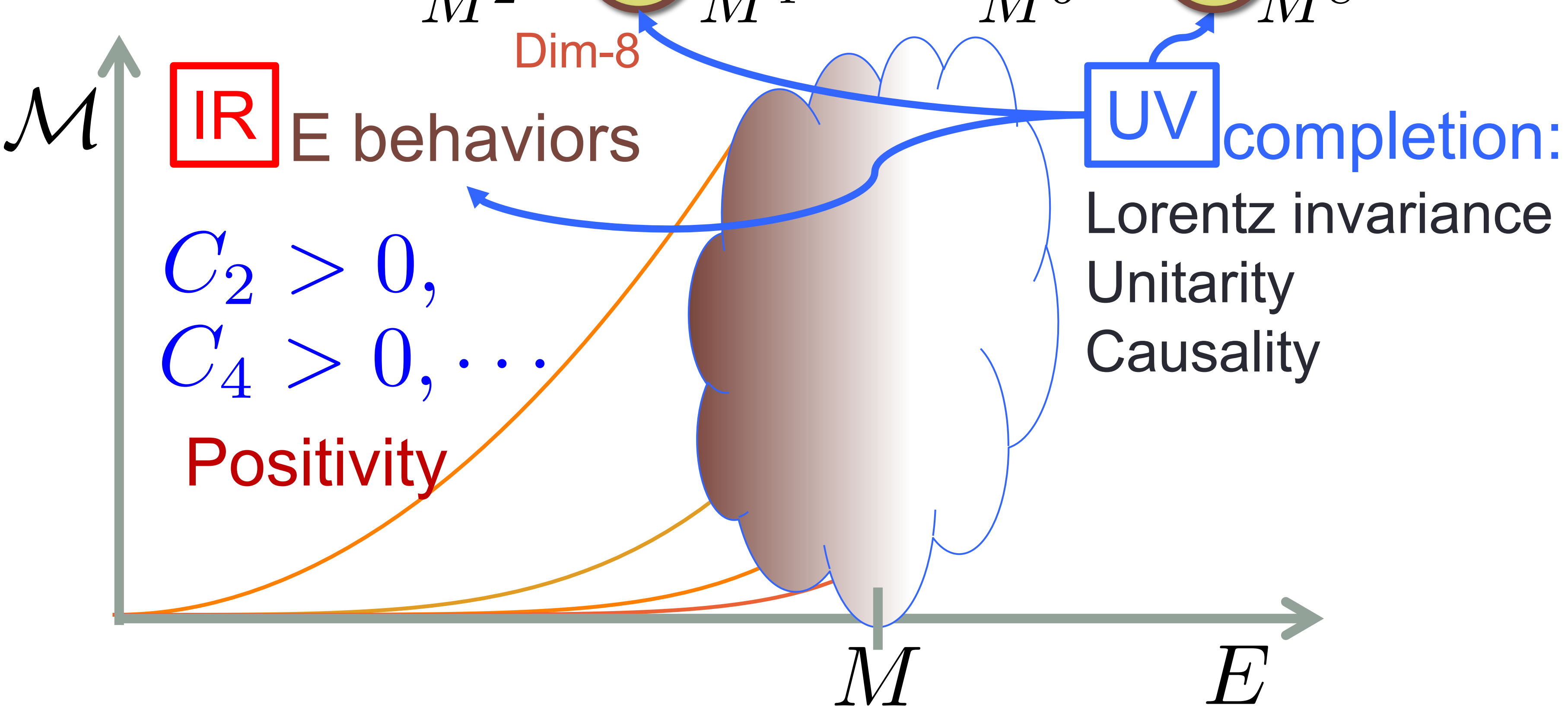
Ref: Slides by Francesco Riva

- Effective Theory Forward Amplitude (IR):
  For D>=8 Wilson Coefficients

Dim-12

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$

Dim-8



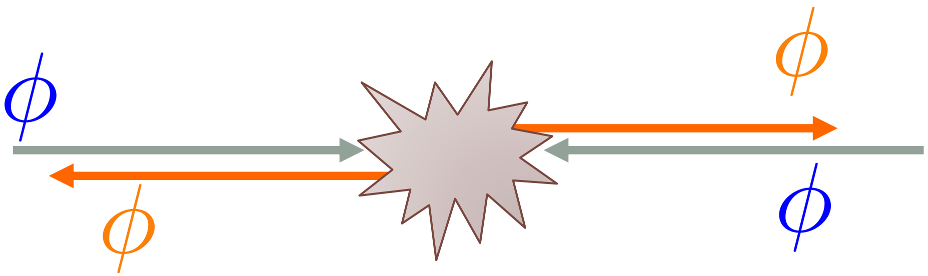
Positivity Bounds (7/12)

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$

$C_2 > 0$

massless scalar 2-2 forward elastic scattering:

forward: $t=0$



I+II $\rightarrow$ I+II

elastic

Let us consider the amplitude of this: $\dfrac{\mathcal{M}(s,0)}{s^3}$

Positivity Bounds (8/12)

Forward limit positivity bounds are from:

1. Lorentz Invariance

2. Unitarity ⇨ Optical theorem:
   e.g., elastic case,

$$\mathrm{Im}\mathcal{M}(k_1, k_2 \to k_1, k_2) = \underline{\underline{s\sigma_{\mathrm{tot}}(k_1, k_2 \to \mathrm{anything})}}$$

Positive

1. Analyticity* ⇨ Froissart Bound:

$$\left|\mathcal{M}(s, \underline{\underline{\cos\theta = 1}})\right| < \mathrm{Const.}\ s(\ln s)^2$$

forward

Froissart, Martin 1960's

(for real s→∞)

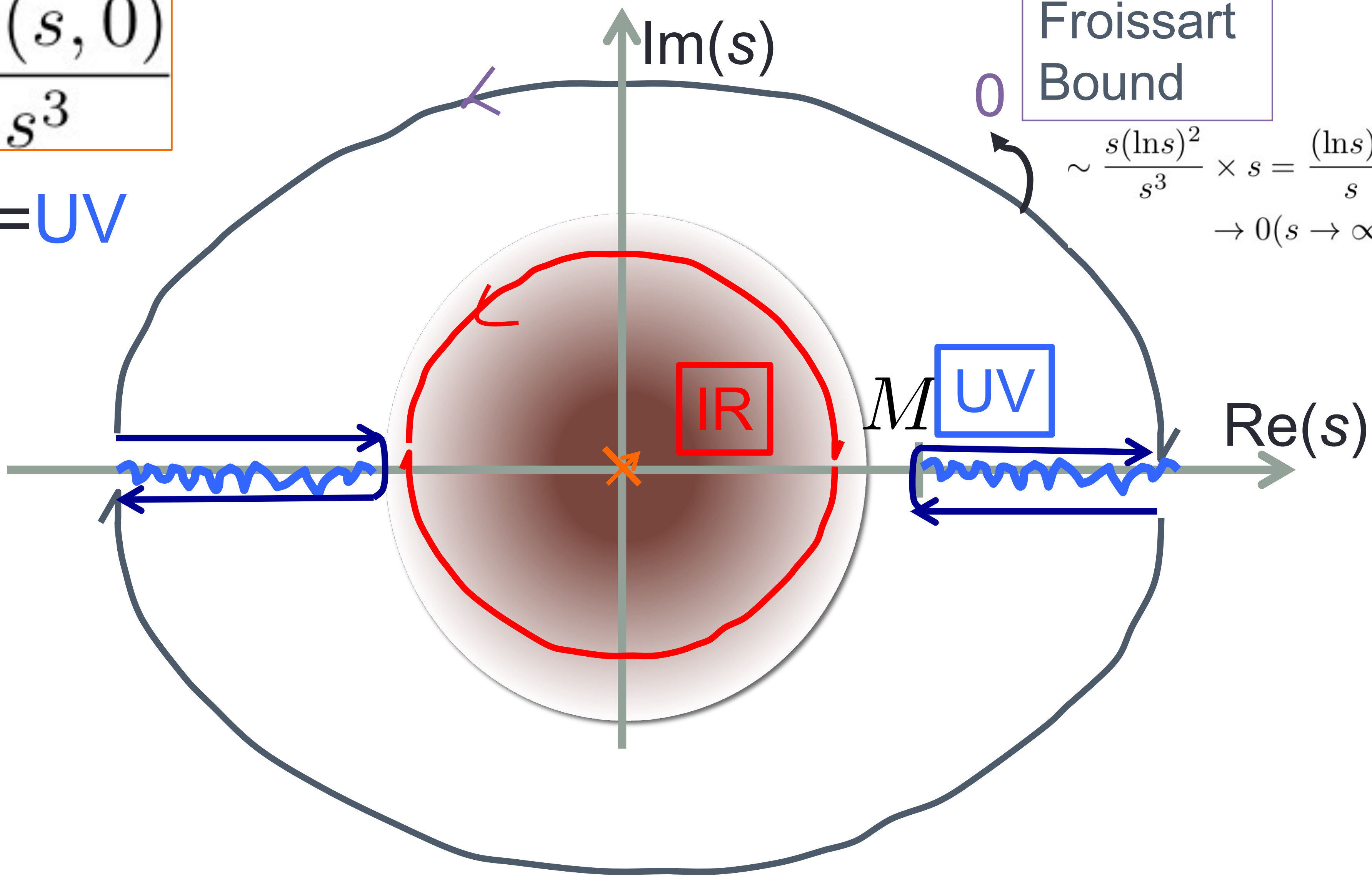
*Analyticity of the amplitude besides poles and branch cuts on real axis

Positivity Bounds (9/12)

massless scalar 2-2 forward elastic scattering amplitude:

$$\frac{\mathcal{M}(s,0)}{s^3}$$

IR=UV

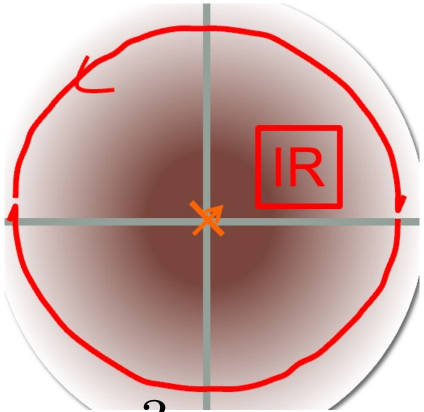


$$\sim \frac{s(\ln s)^2}{s^3} \times s = \frac{(\ln s)^2}{s}$$
$$\to 0 (s \to \infty)$$

Positivity Bounds (10/12)

IR

$$\frac{1}{2\pi i}\oint ds\,\frac{\mathcal{M}(s,0)}{s^3} = \frac{C_2}{M^4}$$



$$\mathcal{M} = C_0 + C_1\frac{s}{M^2} + C_2\frac{s^2}{M^4} + C_3\frac{s^3}{M^6} + C_4\frac{s^4}{M^8} + \cdots$$

Positivity Bounds (11/12)

UV

$$\frac{1}{2\pi i}\int_M^\infty ds\,\underline{\underline{\left(M(s+i\epsilon,0)-M(s-i\epsilon,0)\right)}}/s^3$$

2)&3)
= (2i)Im M(s,0)
= (2i)s $\sigma_{tot}(s)$

|| 1)

$$+\frac{1}{2\pi i}\int_M^\infty ds\,\underline{\underline{\left(M(-s-i\epsilon,0)-M(-s+i\epsilon,0)\right)}}/s^3$$

crossing sym. ↓ ↓ s+t+u = 0 & t=0

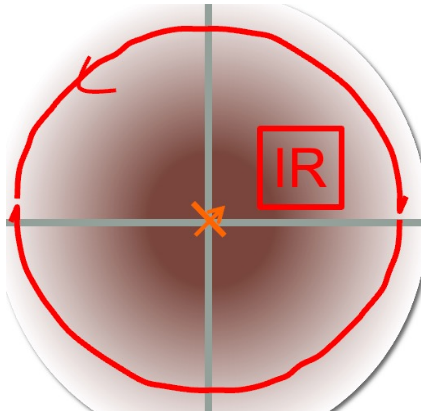
1. Crossing Symmetry: M(s,0)=M(u,0) =M(-s,0),
2. Schwarz reflection principle: M(s*,0)=M(s,0)*
3. Optical theorem: Im M(s,0) = s σ_{tot}(s)

$$= \frac{2}{\pi}\int_M^\infty ds\,\frac{s\sigma_{tot}(s)}{s^3} > 0$$

Positivity Bounds (12/12)

IR

$$\frac{1}{2\pi i}\oint ds \frac{\mathcal{M}(s,0)}{s^3} = C_2$$



$$\mathcal{M} = C_0 + C_1\frac{s}{M^2} + C_2\frac{s^2}{M^4} + C_3\frac{s^3}{M^6} + C_4\frac{s^4}{M^8} + \cdots$$

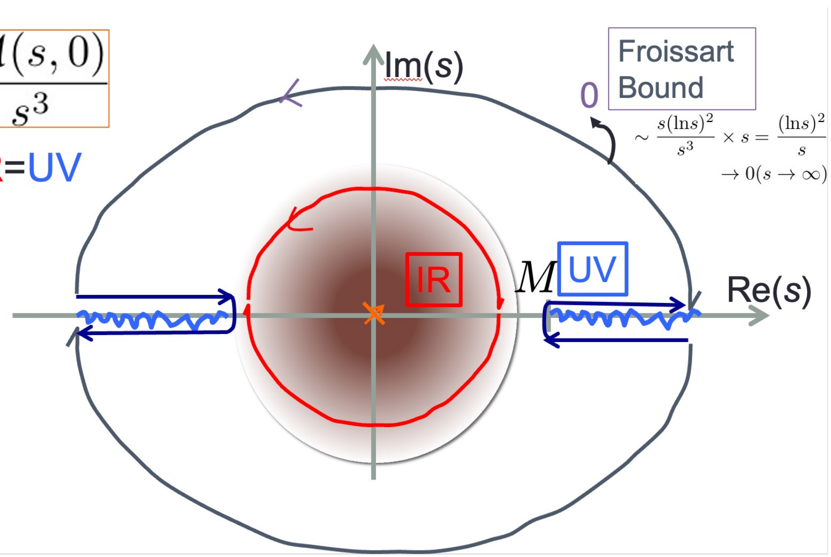
$1/(2\pi i)\int M_{IR}/s^3$ $(=C_2/M^4)$…IR

$=1/(2\pi i)\int M_{UV}/s^3 > 0$ …UV

$$\frac{\mathcal{M}(s,0)}{s^3}$$

IR=UV

→ $C_2 > 0$ …IR



Higgs Portal DM operators (1/4) -positivity side-

- Derivative Coupling for Higgs and Dark Matter Fields

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

- Subject to satisfying positivity bounds
- Spin-2 massive graviton and/or spin-0 radion mediated DM model is one of the candidates of this scenario as the partial UV completion
- Sensitive to high-energy prosses

Higgs Portal DM operators (2/4) -positivity side-

- Positivity bounds from the superposed states:

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

$$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$$

$$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$$
$$O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$
$$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$

Higgs Portal DM operators (3/4) -positivity side-

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

- Results:

Bounds	Channels $(\lvert 1\rangle + \lvert 2\rangle \to \lvert 1\rangle + \lvert 2\rangle)$
$C^{(1)}_{H^4} + C^{(2)}_{H^4} \geq 0$	$\lvert 1\rangle = \lvert \phi_1\rangle\,,\ \lvert 2\rangle = \lvert \phi_3\rangle$
$C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4} \geq 0$	$\lvert 1\rangle = \lvert \phi_1\rangle\,,\ \lvert 2\rangle = \lvert \phi_1\rangle$
$C^{(2)}_{H^4} \geq 0$	$\lvert 1\rangle = \lvert \phi_1\rangle\,,\ \lvert 2\rangle = \lvert \phi_2\rangle$
$C^{(1)}_{H^2\varphi^2} \geq 0$	$\lvert 1\rangle = \lvert \phi_1\rangle\,,\ \lvert 2\rangle = \lvert \varphi\rangle$
$C_{\varphi^4} \geq 0$	$\lvert 1\rangle = \lvert \varphi\rangle\,,\ \lvert 2\rangle = \lvert \varphi\rangle$
$2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq -\left(C^{(1)}_{H^2\varphi^2} + C^{(2)}_{H^2\varphi^2}\right)$	$\lvert 1\rangle = 2\sqrt{C_{\varphi^4}}\,\lvert \phi_1\rangle + \sqrt{-(C^{(1)}_{H^2\varphi^2} + C^{(2)}_{H^2\varphi^2})}\,\lvert \varphi\rangle,$ $\lvert 2\rangle = \lvert 1\rangle$ Superposition
$2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq C^{(2)}_{H^2\varphi^2}$	$\lvert 1\rangle = 2\sqrt{C_{\varphi^4}}\,\lvert \phi_1\rangle + \sqrt{C^{(2)}_{H^2\varphi^2}}\,\lvert \varphi\rangle,$ $\lvert 2\rangle = -2\sqrt{C_{\varphi^4}}\,\lvert \phi_1\rangle + \sqrt{C^{(2)}_{H^2\varphi^2}}\,\lvert \varphi\rangle$ Superposition

Higgs portal DM

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

Higgs Portal DM operators (4/4)
- dim4 and dim6 -

- Dim-4 and Dim-6 Higgs Portal DM operators relevant to the phenomenology (relic density, direct and indirect detections):

$$-\frac{1}{6\Lambda^4}\Big(c_1 m_\varphi^4\varphi^4 + 4c_2 m_H^4|H|^4 + 8c_2'\lambda_H m_H^2|H|^6 + 4c_2''\lambda_H^2|H|^8 \\ \boxed{+4c_3 m_\varphi^2 m_H^2\varphi^2|H|^2 + 4c_3'\lambda_H m_\varphi^2\varphi^2|H|^4}\Big)$$

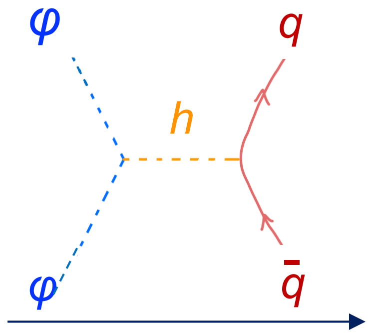
$$+\frac{1}{6\Lambda^4}\Big(d_1 m_\varphi^2\varphi^2(\partial_\mu\varphi)^2 + 4d_2 m_H^2|H|^2|D_\mu H|^2 + 4d_2'\lambda_H|H|^4|D_\mu H|^2 \\ +\boxed{2d_3 m_\varphi^2\varphi^2|D_\mu H|^2 + 2d_4 m_H^2|H|^2(\partial_\mu\varphi)^2 + 2d_4'\lambda_H|H|^4(\partial_\mu\varphi)^2}\Big)$$

WIMP case

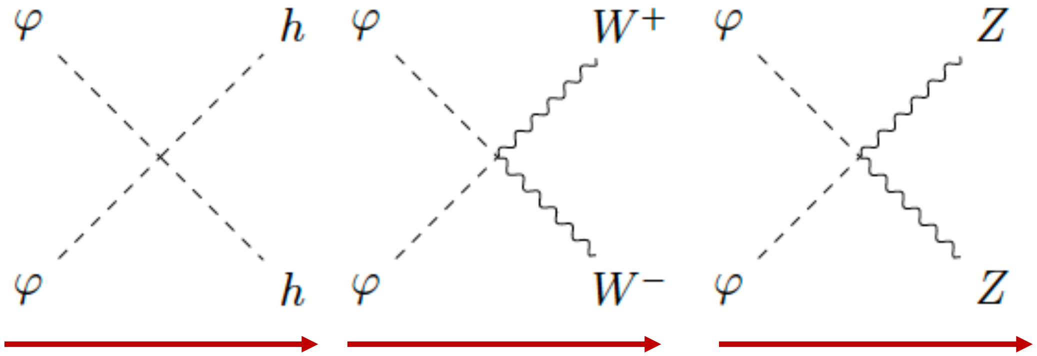
Relic Density (1/2)

- Higgs-portal interactions linear in the Higgs boson *h*

$$\mathcal{L}_{h,\mathrm{linear}} = \frac{1}{3\Lambda^4} h \Big[2(c_3 - c_3')\lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu \varphi)^2 \Big]$$

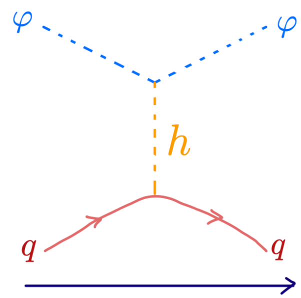


- Feynman diagrams for DM annihilation processes when $c'_3 = c_3$ and $d'_4 = d_4$ ($\varphi\varphi \to h \to ff$ are absent):



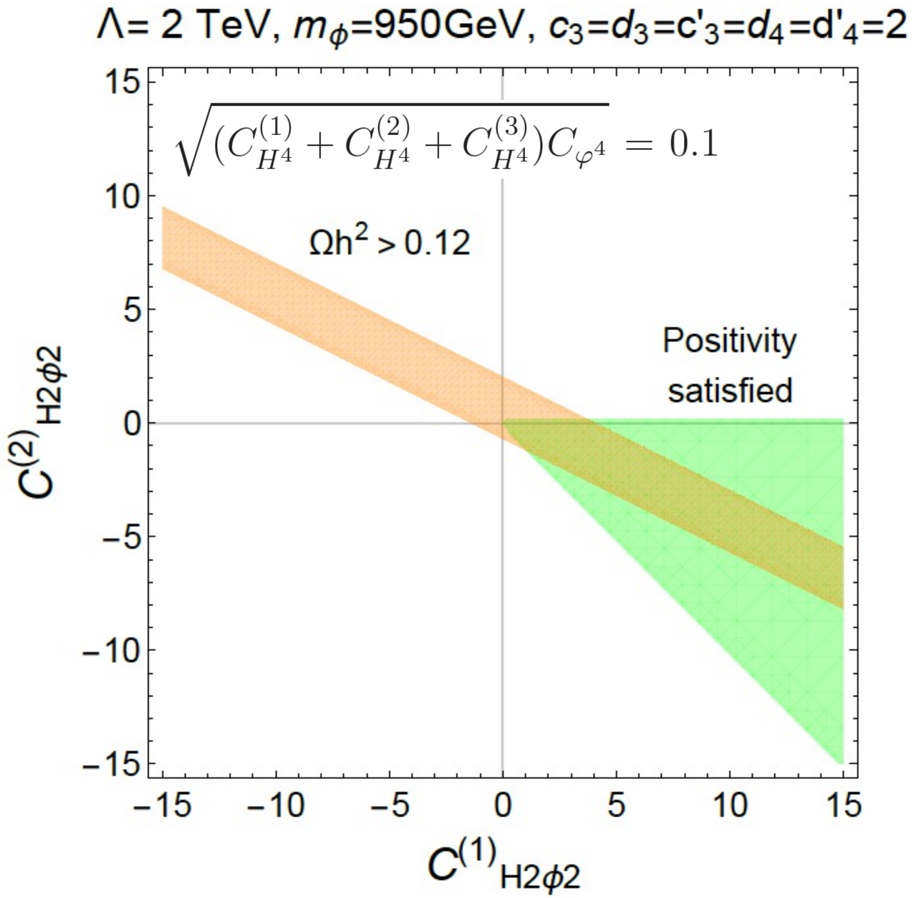
$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

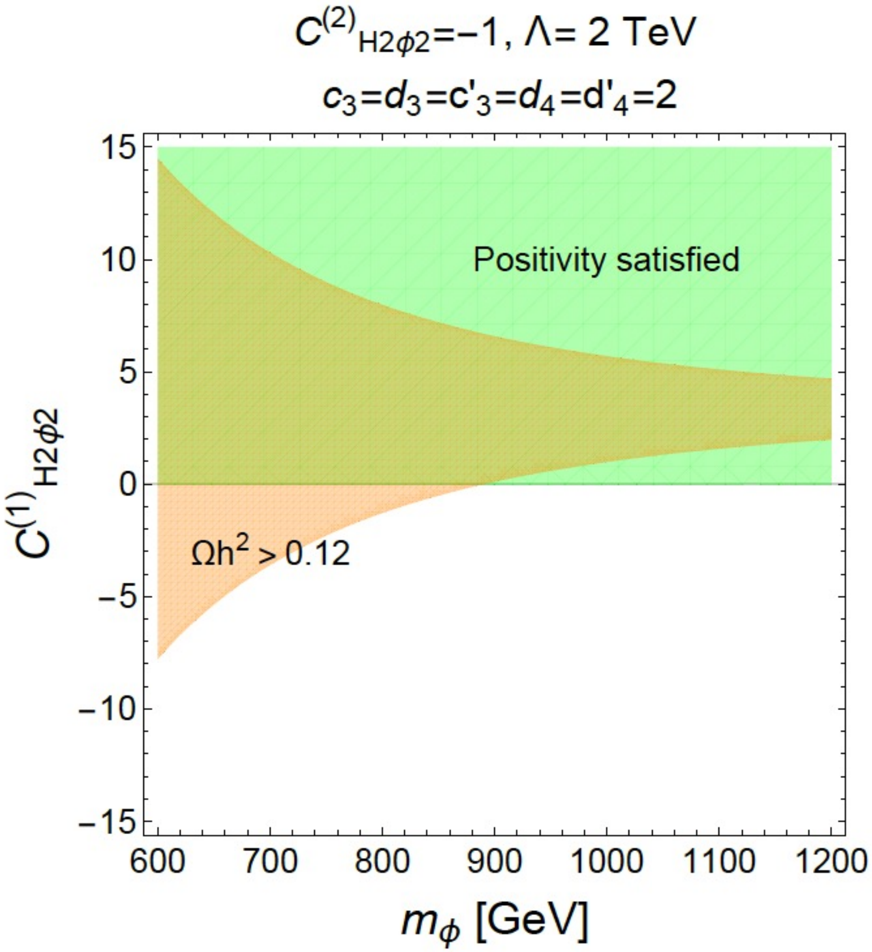
Note that the tree-level direct detection bounds are absent in this case



WIMP case

Relic Density (2/2)





$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi) \quad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

WIMP case

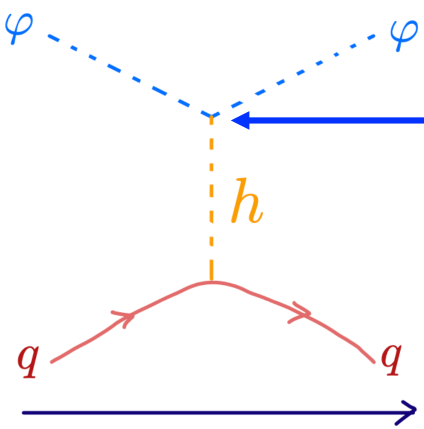
Direct Detection

- Higgs-portal interactions linear in the Higgs boson h

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c_3')\lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

$$\mathcal{L} \supset -\frac{1}{6\Lambda^4}\Big(c_1 m_\varphi^4 \varphi^4 + 4c_2 m_H^4 |H|^4 + 8c_2' \lambda_H m_H^2 |H|^6 + 4c_2'' \lambda_H^2 |H|^8 + 4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c_3' \lambda_H m_\varphi^2 \varphi^2 |H|^4\Big) + \frac{1}{6\Lambda^4}\Big(d_1 m_\varphi^2 \varphi^2 (\partial_\mu \varphi)^2 + 4d_2 m_H^2 |H|^2 |D_\mu H|^2 + 4d_2' \lambda_H |H|^4 |D_\mu H|^2 + 2d_3 m_\varphi^2 \varphi^2 |D_\mu H|^2 + 2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d_4' \lambda_H |H|^4 (\partial_\mu \varphi)^2\Big),$$

- Tree-level direct detection bounds are absent when $c'_3 = c_3$ and $d'_4 = d_4$
e.g., Massive Graviton/Radion cases



WIMP case

Indirect Detection

$$\mathcal{L} \supset -\frac{1}{6\Lambda^4}\left(4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c_3' \lambda_H m_\varphi^2 \varphi^2 |H|^4\right) + \frac{1}{6\Lambda^4}\left(2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d_4' \lambda_H |H|^4 (\partial_\mu \varphi)^2\right)$$

- When $c'_3 = c_3$ and $d'_4 = d_4$, $\varphi\varphi \to h \to ff$ are absent:

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c_3')\lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu \varphi)^2\right]$$

- In this case $\varphi\varphi \to hh$, *WW, and ZZ* can be constrained by indirect detection

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

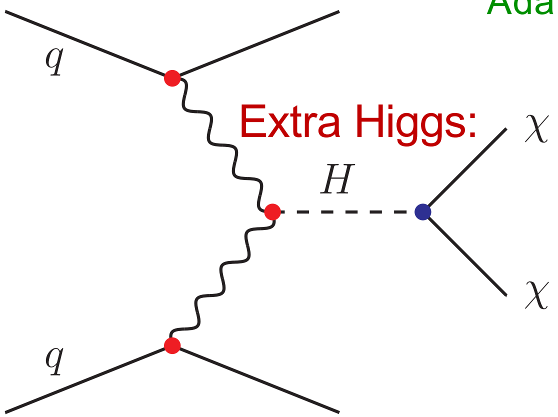
- If we assume that only massive graviton is involved, $\varphi\varphi \to hh$ also vahish at *s*-wave, but $\varphi\varphi \to WW/ZZ$ are *s*-wave dominant

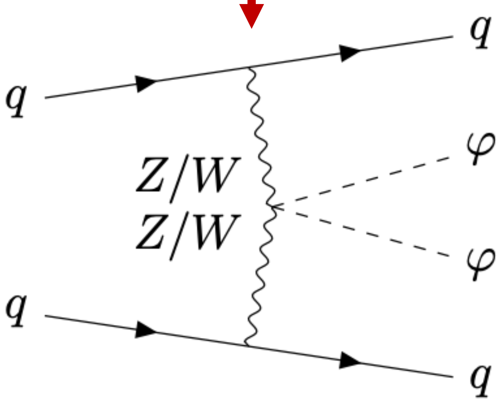
WIMP case

LHC Search (1/3)

- ATLAS measurement with 139/fb at the 13 TeV LHC

Adapted from Fig. 1 in G. Aad *et al.* [ATLAS], JHEP **08**, 104 (2022)





- For our dim-8 operators, H in Fig. is integrated out $\chi \to \varphi$

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

- Higgs takes vev
- Covariant Derivative contains vector bosons

WIMP case

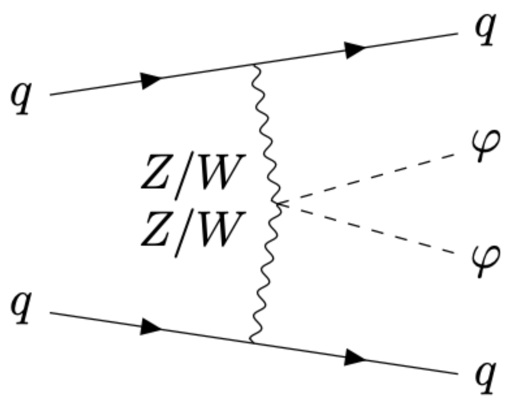
LHC Search (2/3)

ATLAS measurement with 139/fb at the 13 TeV LHC

- 95% upper limits: 0.11 pb

G. Aad *et al.* [ATLAS], JHEP **08**, 104 (2022)

$\sqrt{s} = 13$ TeV LHC, $L_{\rm int} = 139\ {\rm fb}^{-1}$	$\sigma^{\rm VBF} \times B_{\rm inv} = 0.11$ pb ($m_H = 1$ TeV)
$\Lambda = 1$ TeV, $m_\varphi = 375$ GeV	cross section from EFT operators
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (40, 40)$	0.28 pb Excluded
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (32, 32)$	0.11 pb Excluded
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (40, 0)$	0.012 pb
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (0, 40)$	0.097 pb

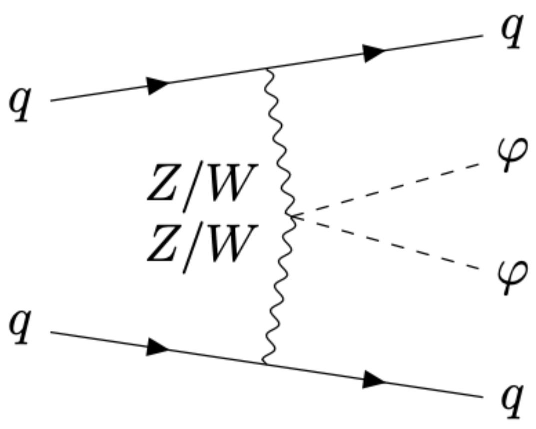


$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

WIMP case

LHC Search (3/3)

- High Luminosity LHC (HL-LHC) Search



Amplitude for $W^+W^-/ZZ \to \varphi\varphi$

- $O^{(2)}_{H^2\varphi^2}$ shows only Mandelstam *s* and mass dependencies
- $O^{(1)}_{H^2\varphi^2}$ causes *t* dependency also

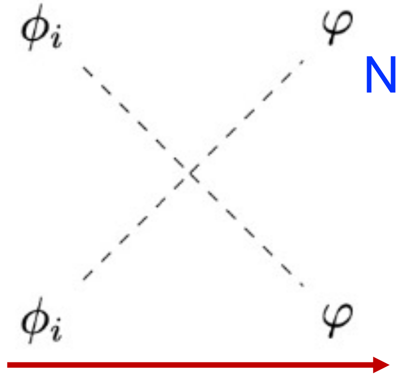
Checking angular distributions may help to distinguish between $O^{(1)}_{H^2\varphi^2}$ and $O^{(2)}_{H^2\varphi^2}$

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$
$$O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

X. Li, K. Mimasu, KY, C. Yang, C. Zhang, S. Y. Zhou, JHEP**10**(2022)107

Freeze-in Dark Matter (1/3)

- We assume that the electroweak symmetry is unbroken during the freeze-in production of dark matter
- Feynman diagrams for
  DM production due to effective Higgs-portal actions:



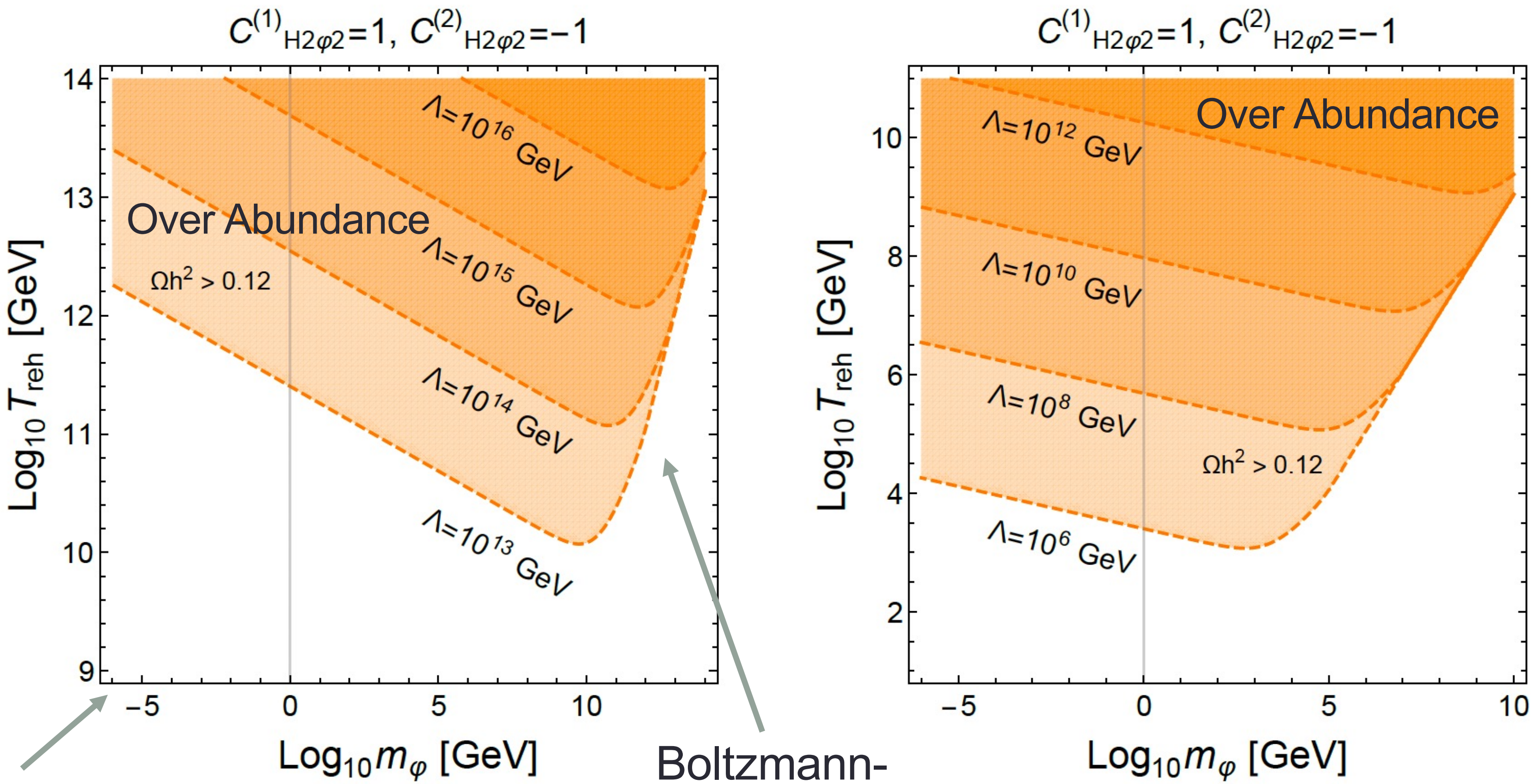
Non-thermal DM production

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

- Taking $s, t \gg m_\varphi^2, m_H^2,$

$$|\mathcal{M}_{\phi_i\phi_i \to \varphi\varphi}|^2 \simeq \frac{1}{576\Lambda^8} \left[3(C^{(1)}_{H^2\varphi^2} + 2C^{(2)}_{H^2\varphi^2}) s^2 + 6C^{(1)}_{H^2\varphi^2}\, t(t+s) \right]^2$$

Freeze-in Dark Matter (2/3)



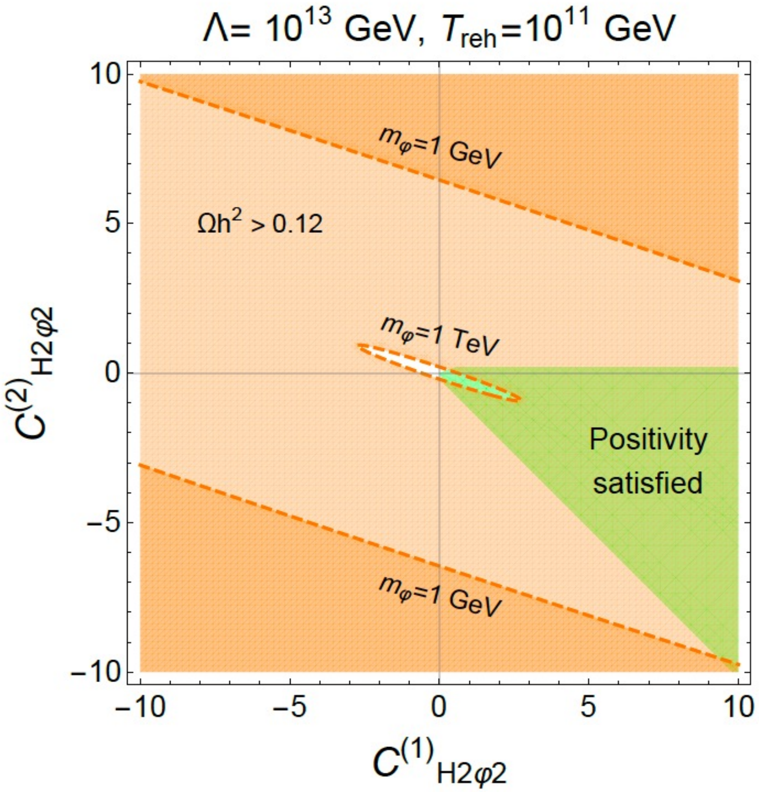
Boltzmann-suppressed

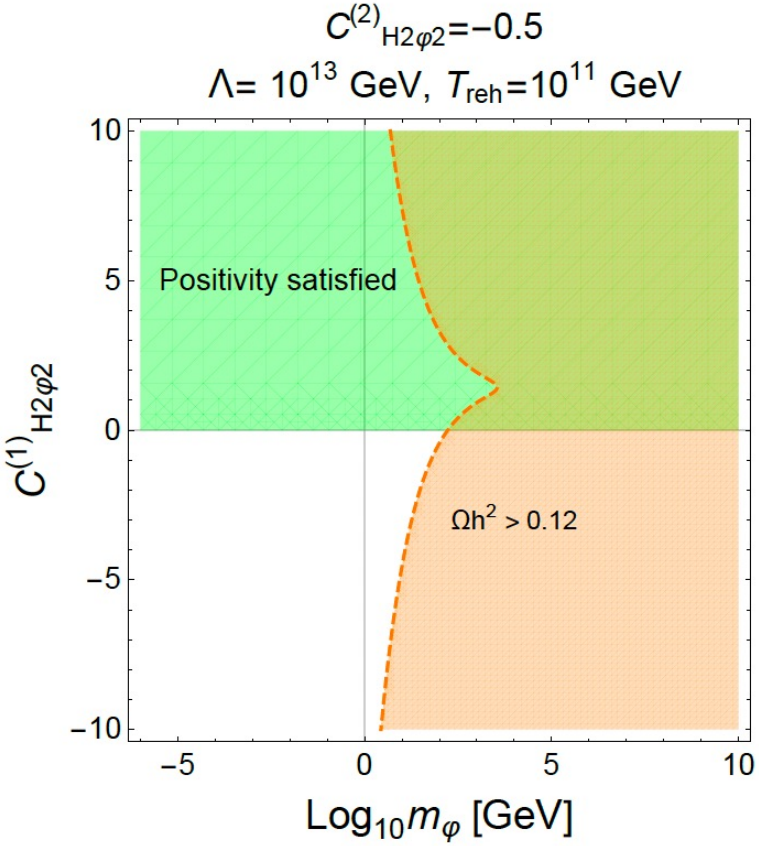
$m_\varphi > 1$ keV
Safe from Lyman-alpha forest bound

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu\varphi\partial^\nu\varphi) \quad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu\varphi\partial^\nu\varphi)$$

Freeze-in Dark Matter (3/3)

$$\sqrt{(C^{(1)}_{H^4}+C^{(2)}_{H^4}+C^{(3)}_{H^4})C_{\varphi^4}}=0.1$$





$$O^{(1)}_{H^2\varphi^2}=(D_\mu H^\dagger D_\nu H)(\partial^\mu\varphi\partial^\nu\varphi)\quad O^{(2)}_{H^2\varphi^2}=(D_\mu H^\dagger D^\mu H)(\partial_\nu\varphi\partial^\nu\varphi)$$

Summary

- We consider Higgs portal dark matter derivative coupled dim-8 interactions and apply the positivity conditions to them
- We also included dim-4 and dim-6 Higgs portal interactions
- We see constraints from relic density, direct and indirect detections
- For HL-LHC search, utilizing the kinematical distributions may be useful
- We also see the interplay between the positivity and relic density for the freeze-in dark matter case

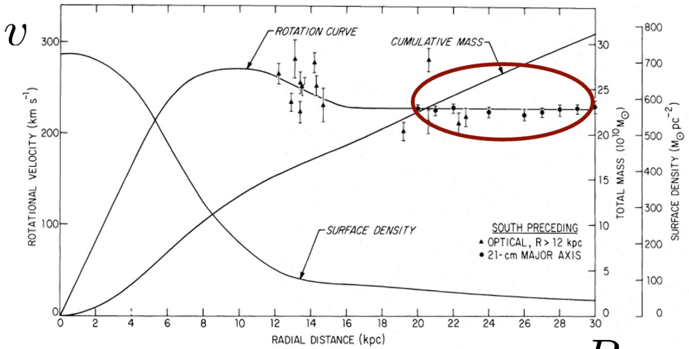
Backup

Dark Matter (1/4)

Galaxy Rotation Curve

V. C. Rubin W. K. Ford (1970)
M. S. Roberts R. N. Whitehurst (1975)

v Speed of the star and gass



Distance from the Center of Galaxy R

$$G\frac{mM(R)}{R^2} = m\frac{v^2}{R} \longrightarrow v = \sqrt{\frac{GM(R)}{R}}$$

Gravitational Force | Centrifugal Force

Gravitational Lensing

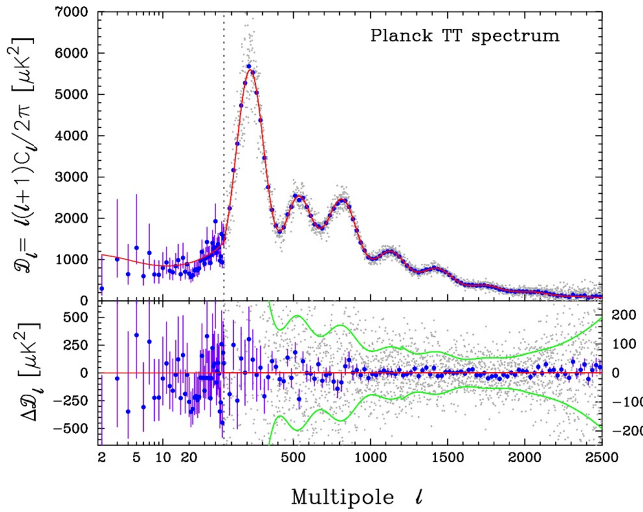
Gravitational lensing system called SDSS J0928+2031 observed by Hubble telescope <https://esahubble.org/images/potw1903a/>

Bullet Cluster

Bullet Cluster photo in X-ray (red) with the gravitational lensing (blue) <https://chandra.harvard.edu/photo/2006/1e0657/>

Cosmic Microwave Background (CMB)

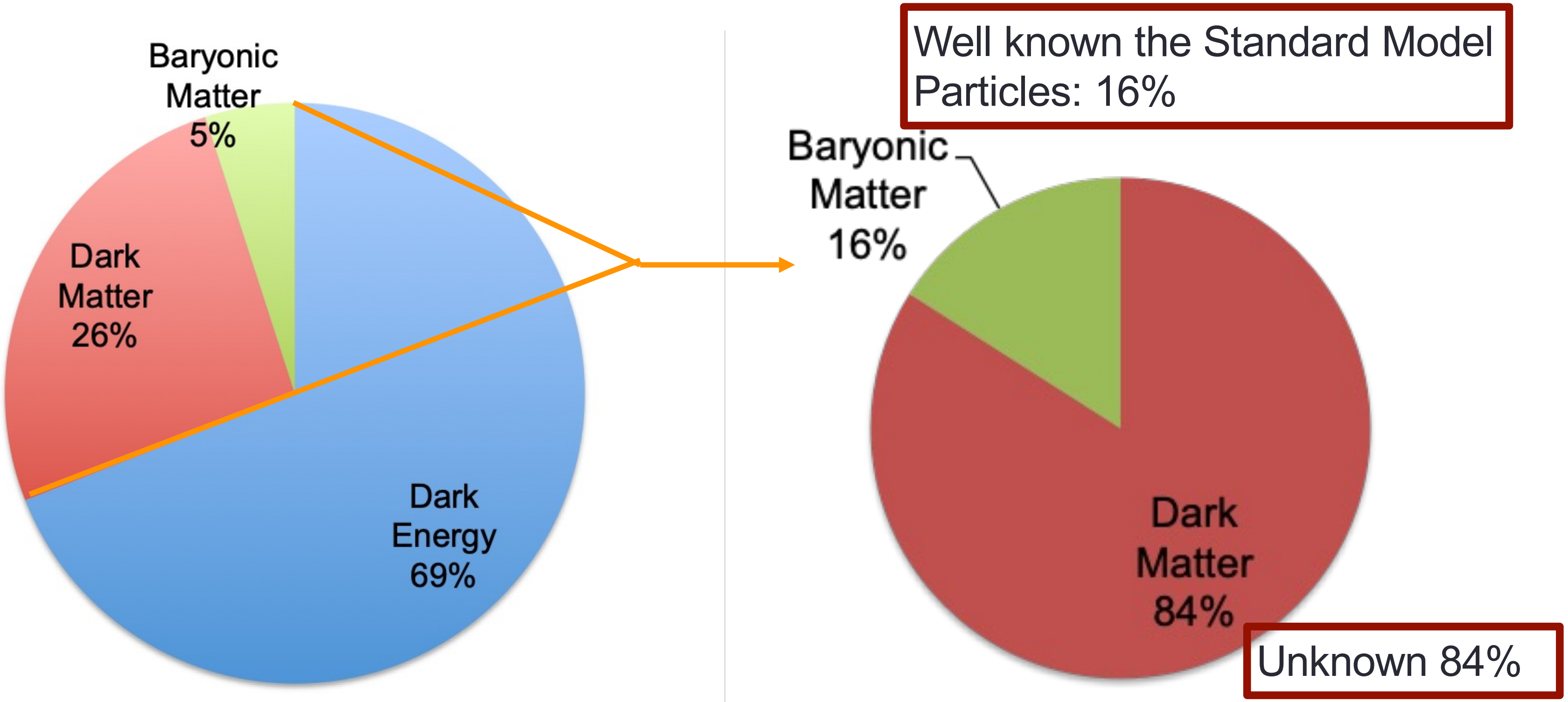
P. A. R. Ade *et al.* [Planck], Astron. Astrophys. **571**, A16 (2014)



Dark Matter (2/4)

Particle Data Group, Prog. Theor. Exp. Phys. 2022, 083C01 (2022) and 2023 update

Energy density ratios in the Universe



Energy Density Ratio

Energy Density Ratio of the Matters

Dark Matter (3/4) –DM Models-

Particle Data Group, Prog. Theor. Exp. Phys. 2022, 083C01 (2022) and 2023 update

- Framework approach
  - Hierarchy Problem: Supersymmetry, Extra-dimension
  - Strong CP problem: Axions
  - Neutrino masses and mixing: Sterile neutrino

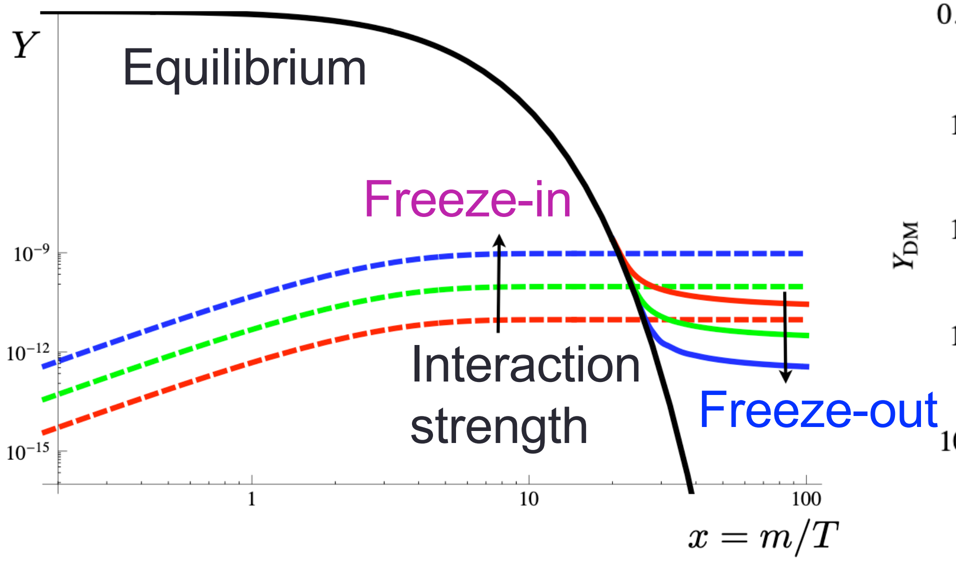
- Anomaly/signal approach
  - Extension of gauge sectors:
    Extra U(1)/Dark SU(2) symmetry,
    e.g., dark photon/Z' boson, SU(2) gauge boson

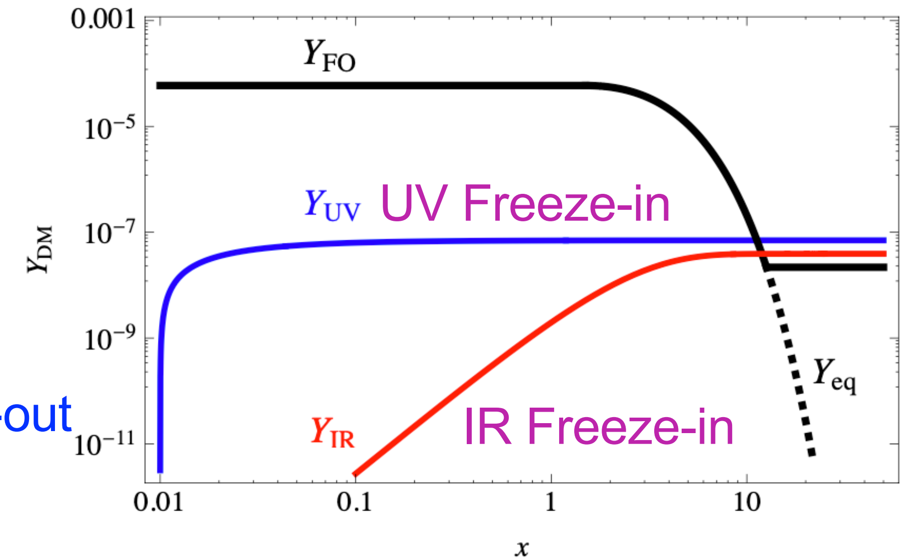
- Renormalizable "portals" approach
  Higgs-portal, Hypercharge field strength, Neutrino-portal

- Effective Field Theory approach

Dark Matter (4/4) -Production Mechanism-

- WIMPs (Weakly Interacting Massive Particles)
  Freeze-out mechanism: Thermal equilibrium → Away from it
- FIMPs (Feebly Interacting Massive Particles)
  Freeze-in mechanism: Out-of-equilibrium DM Production





L. J. Hall, K. Jedamzik, J. March-Russell, S. M. West
JHEP **03**, 080 (2010)

F. Elahi, C. Kolda and J. Unwin,
JHEP **03**, 048 (2015)

Positivity Bounds (13/16)

Example of Positivity

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Heisenberg-Euler Lagrangian:

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3}\exp(-m^2 s) \times\left[(es)^2\mathfrak{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2\mathfrak{F}\right]$$

$$X = \sqrt{2(\mathcal{F} + i\mathcal{G})},$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{H}^2 - \vec{E}^2\right)$$

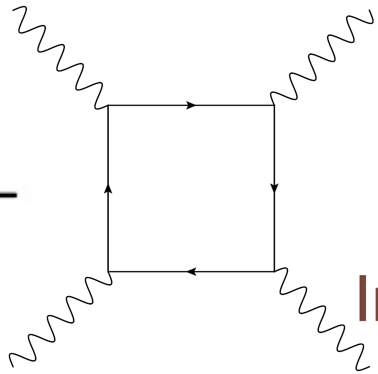
$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{H}$$

$$= \tfrac{1}{2}(\mathbf{E}^2 - \mathbf{H}^2) + \frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2}\times[(\mathbf{E}^2-\mathbf{H}^2)^2 + 7(\mathbf{E}\cdot\mathbf{H})^2] + \cdots$$

>0

from J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}^2 +$$

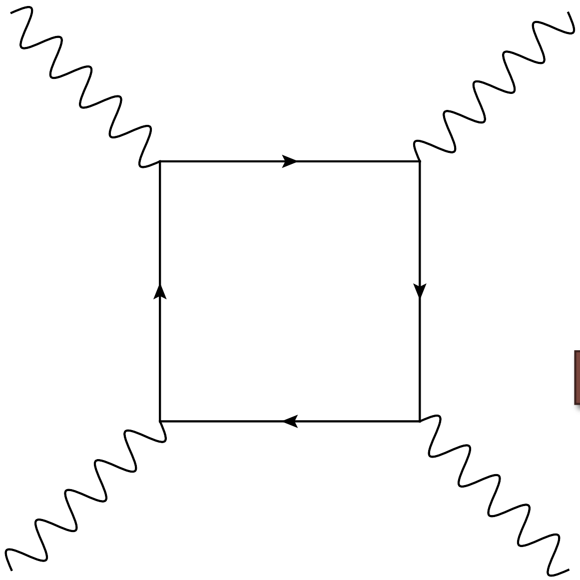


$$+ \quad \cdots$$

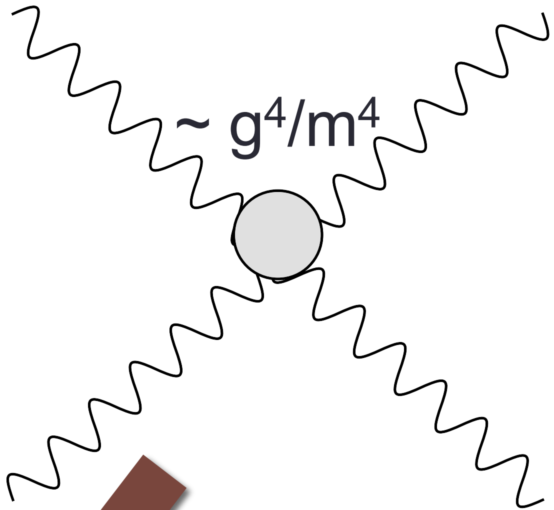
Including this

Positivity Bounds (14/16)

Example of Positivity



Photon Energy
^
^
Fermion Mass



$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2}$$

CP even case

Consistent with QED

Positivity bounds: a>0, b>0

Dispersion Relation (for Positivity Bounds) (15/16)

Forward scattering amp, at low energy (EFT)

(Amp by Dim.8) $\propto (F/\Lambda^4)\, s^2$

$$M^2 = m_i^2 + m_j^2 + m_k^2 + m_l^2$$

$$M_{ijkl} = \frac{1}{2}\frac{d^2}{ds^2} M_{ij\to kl}\left(s = \frac{1}{2}M^2, t = 0\right) + c.c.$$

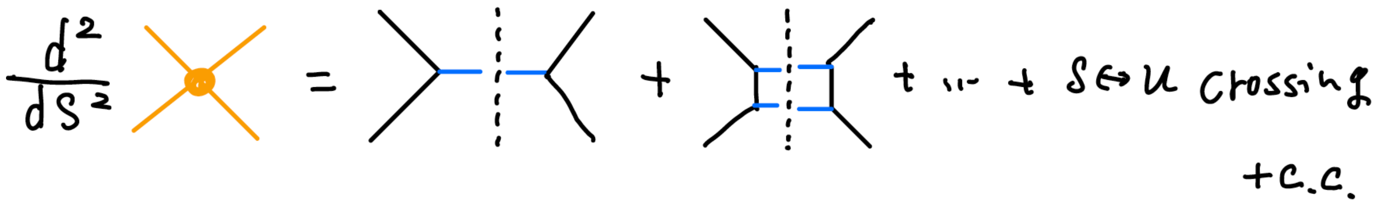
$$= \sum_X \int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds\, M_{ij\to X} M^*_{kl\to X}}{2\pi s^3} + (j \leftrightarrow l) + c.c$$

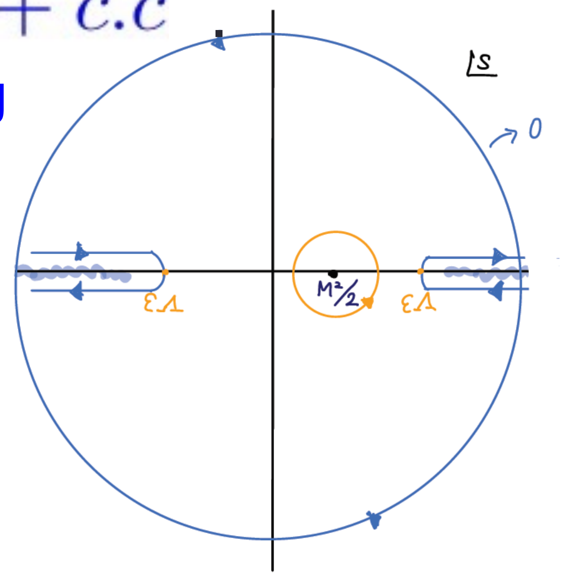
$\epsilon \lesssim 1$

Amplitude of SM →X

Σ_X: BSM states, X summation& LIPS integration

s⇆u crossing





Dispersion Relation (for Positivity Bounds) (16/16)

- Useful to rewrite Dispersion Relation for Positivity Bounds

(Amp by Dim.8)
∝ (F/Λ^4) s^2

$$M_{ijkl} = \frac{F_\alpha M_\alpha^{ijkl}}{\Lambda^4}$$

$$M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu\, m_{K}{}_X^{ij} m_{K}{}_X^{kl}}{\pi\mu^3} + (j \leftrightarrow l)$$

where $M(ij \to X) \equiv m_{R_X}^{ij} + i m_{I_X}^{ij}$

- When i=k, j=l, RHS complete squares >=0

$M^{ijij} \geq 0$ because $m_{K}{}_X^{ij} m_{K}{}_X^{ij} \geq 0$

- More generally,
  Elastic Forward Scattering between Superposed States :

$M(ab \to ab)$ with $|a\rangle = u^i |i\rangle\,, \quad |b\rangle = v^i |i\rangle$

$$\| $$

$$u^i v^j u^{*k} v^{*l} M^{ijkl} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_X{}' \sum_{K=R,I} \frac{d\mu}{\pi\mu^3} \left[|u \cdot m_{K_X} \cdot v|^2 + |u \cdot m_{K_X} \cdot v^*|^2 \right] \geq 0$$

(generalized) Elastic Positivity Bounds

Higgs Portal DM operators (5/5)
- Massive Graviton and Radion case-

- Higgs/DM and Graviton Interaction:

$$-\frac{c_H}{M} G^{\mu\nu} T^H_{\mu\nu} - \frac{c_\varphi}{M} G^{\mu\nu} T^\varphi_{\mu\nu}$$

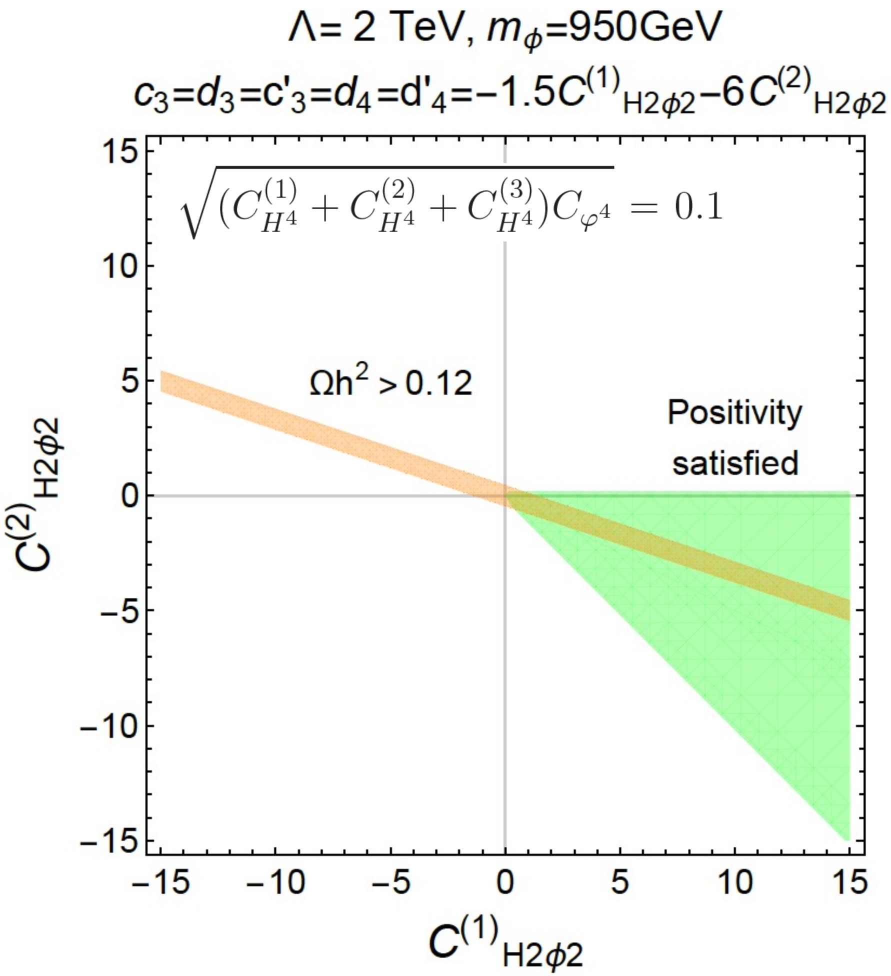
- Higgs/DM and Radion Interaction:

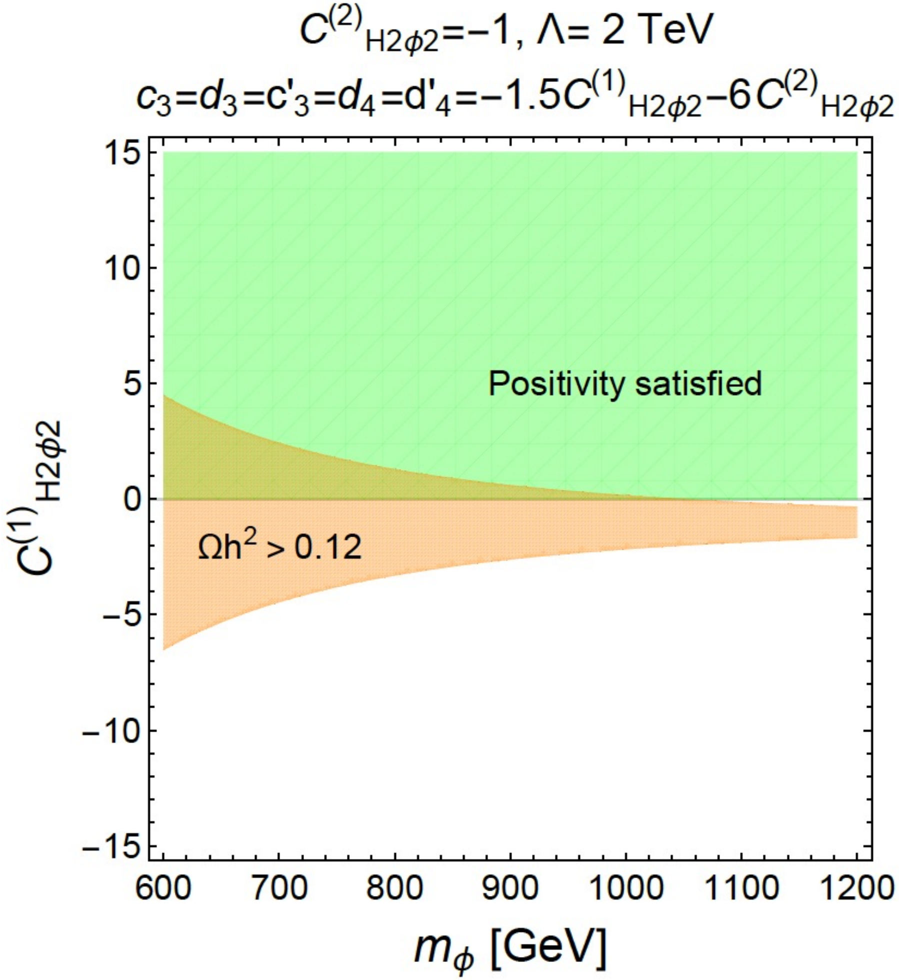
$$\mathcal{L}_r = \frac{c^r_H}{\sqrt{6}M} r\, T^H + \frac{c^r_\varphi}{\sqrt{6}M} r\, T^\varphi$$

- After Integrating out Massive Graviton/Radion, we can identify coefficients of dim-4, 6, and 8 operators as an example
- We found that they satisfied the positivity conditions as far as $c_H c_\varphi \geq 0$ (attractive force for the graviton)

WIMP case

Relic Density (3/3) -Graviton and Radion case-





$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu\varphi\partial^\nu\varphi) \quad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu\varphi\partial^\nu\varphi)$$