Lattice investigations of the chimera baryon spectrum in the Sp(4) gauge theory

2024.06.06 @ Future is Flavourful

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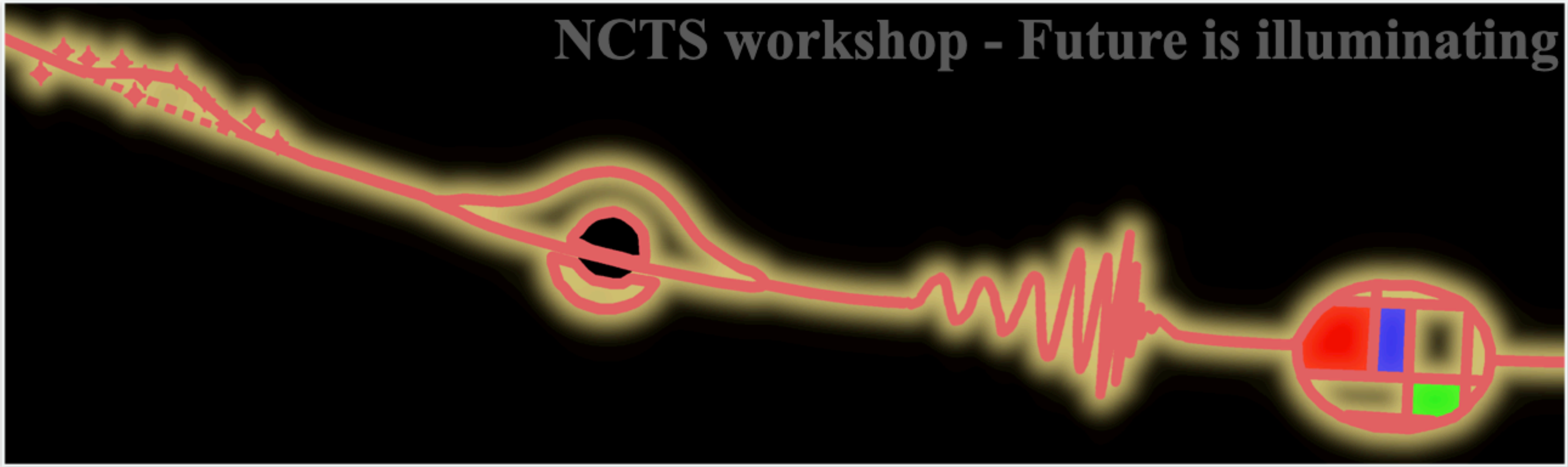
SU(4)/Sp(4) composite Higgs model and lattice simulations



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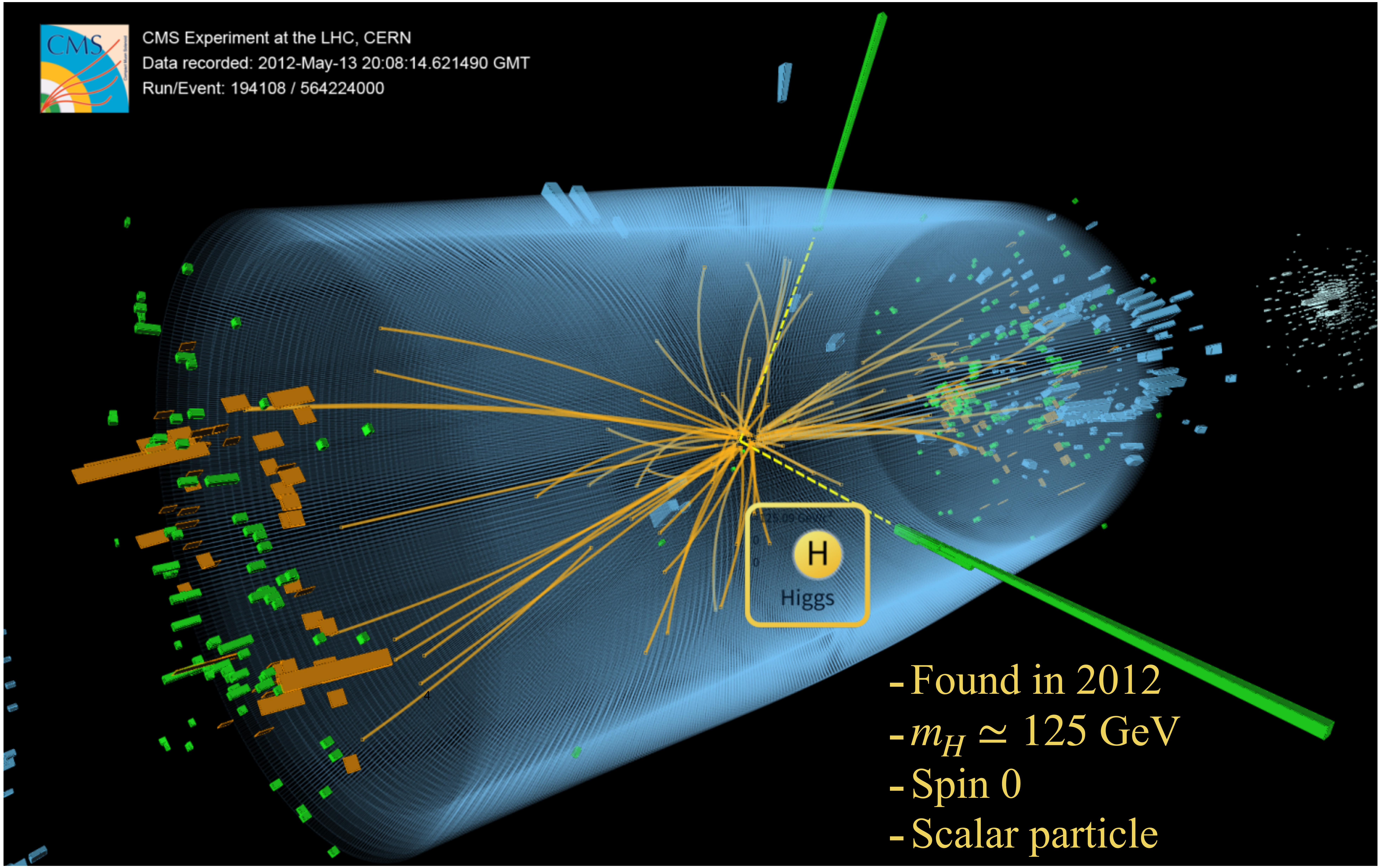
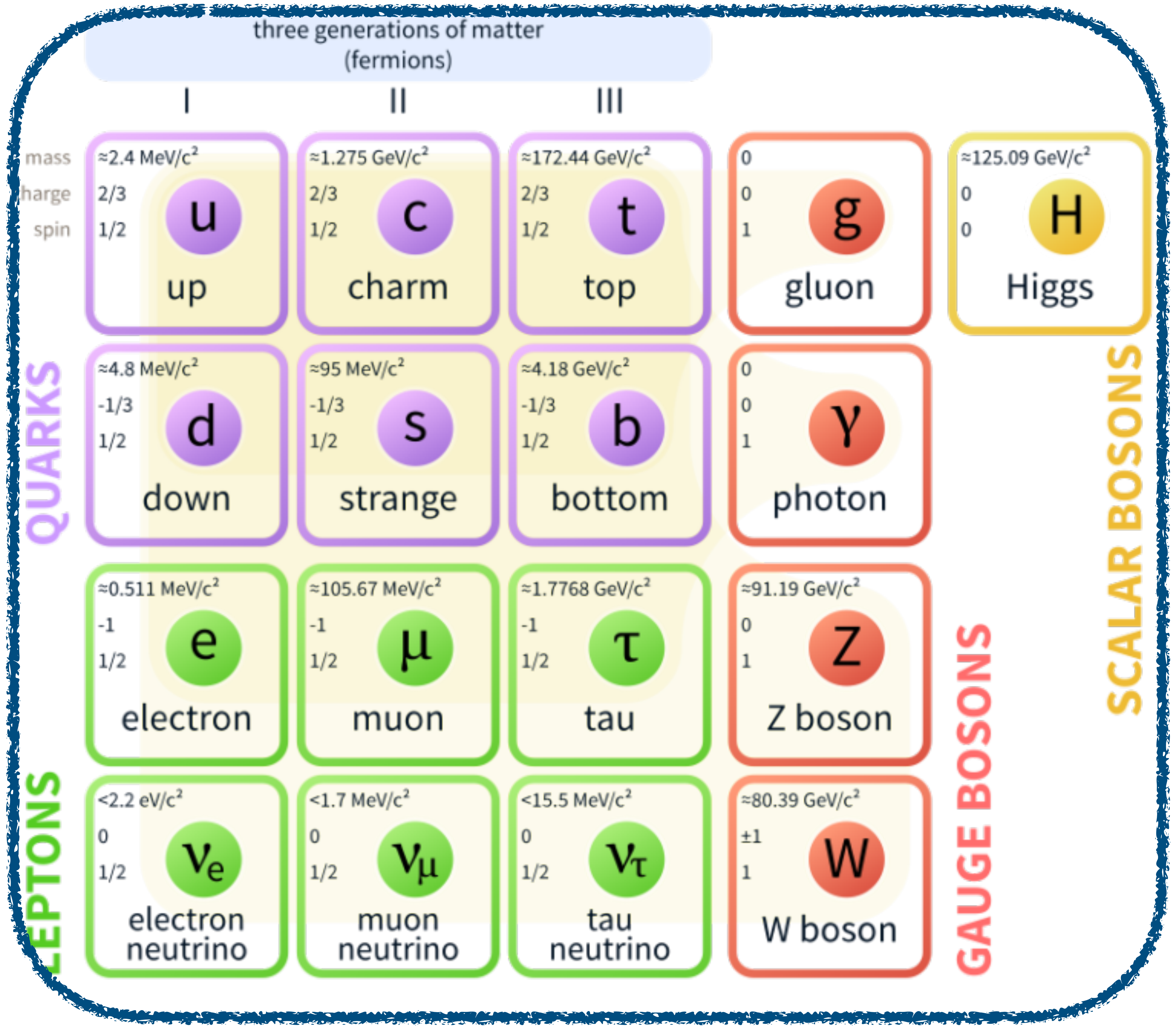
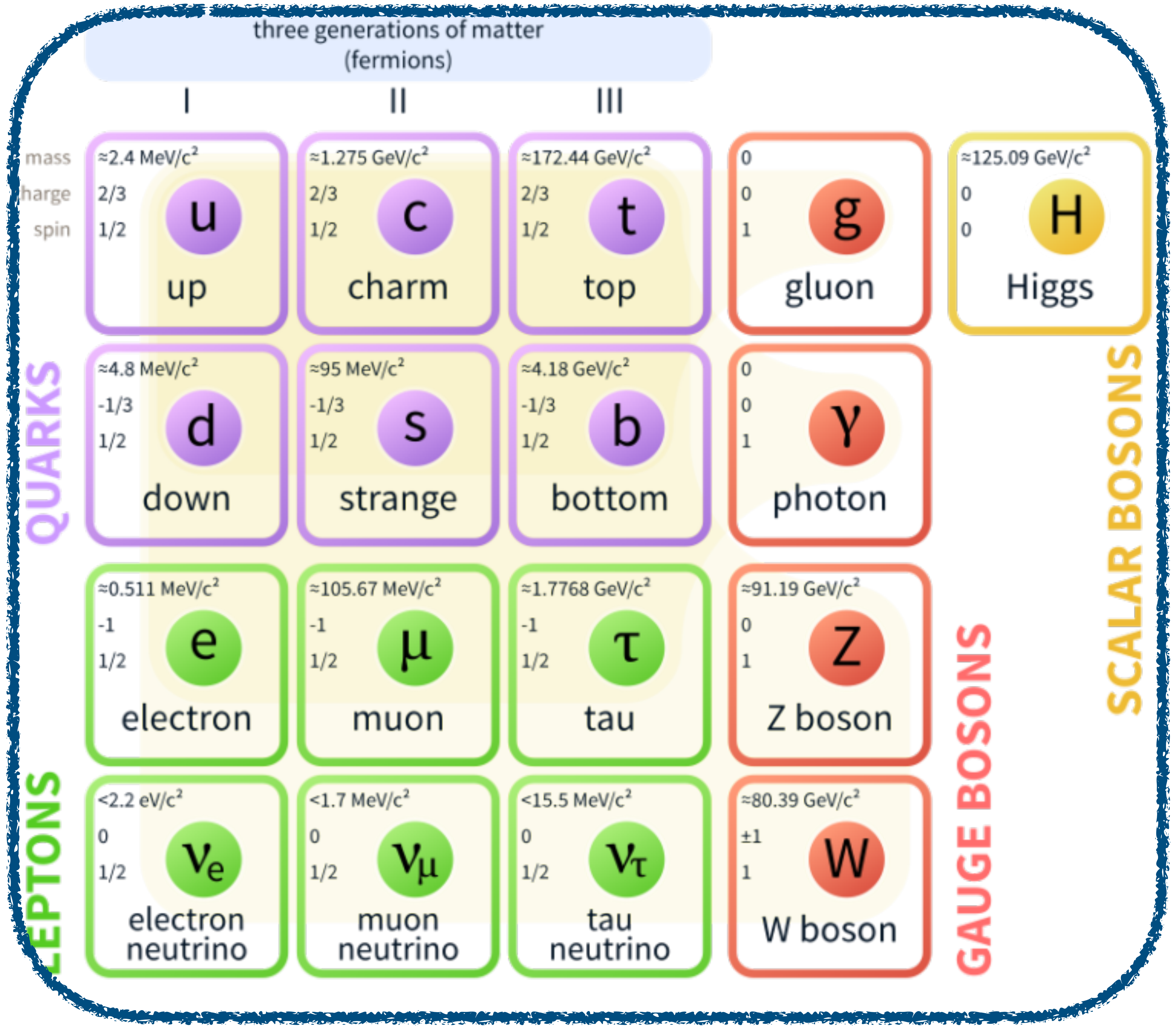
Outline

- Introduction:
  - Sp(4) gauge theory: A Composite Higgs model
  - Chimera baryon
- Results
  - Projections
  - Mass hierarchy of chimera baryons
  - Chiral EFT and AIC
- Summary and Outlook



- Found in 2012
- $m_H \simeq 125$ GeV
- Spin 0
- Scalar particle


three generations of matter
(fermions)
I
II
III
mass
harge
spin
≈2.4 MeV/c²
2/3
1/2
u
up
≈1.275 GeV/c²
2/3
1/2
c
charm
≈172.44 GeV/c²
2/3
1/2
t
top
0
0
1
g
gluon
≈125.09 GeV/c²
0
0
H
Higgs
≈4.8 MeV/c²
-1/3
1/2
d
down
≈95 MeV/c²
-1/3
1/2
s
strange
≈4.18 GeV/c²
-1/3
1/2
b
bottom
0
0
1
γ
photon
≈0.511 MeV/c²
-1
1/2
e
electron
≈105.67 MeV/c²
-1
1/2
μ
muon
≈1.7768 GeV/c²
-1
1/2
τ
tau
≈91.19 GeV/c²
0
1
Z
Z boson
<2.2 eV/c²
0
1/2
νe
electron neutrino
<1.7 MeV/c²
0
1/2
νμ
muon neutrino
<15.5 MeV/c²
0
1/2
ντ
tau neutrino
≈80.39 GeV/c²
±1
1
W
W boson
QUARKS
EPTONS
GAUGE BOSONS
SCALAR BOSONS



triviality of the scalar sector

→SM is an EFT

Composite Higgs Model

D. B. Kaplan and H. Georgi, Phys.Lett.B 136 (1984)

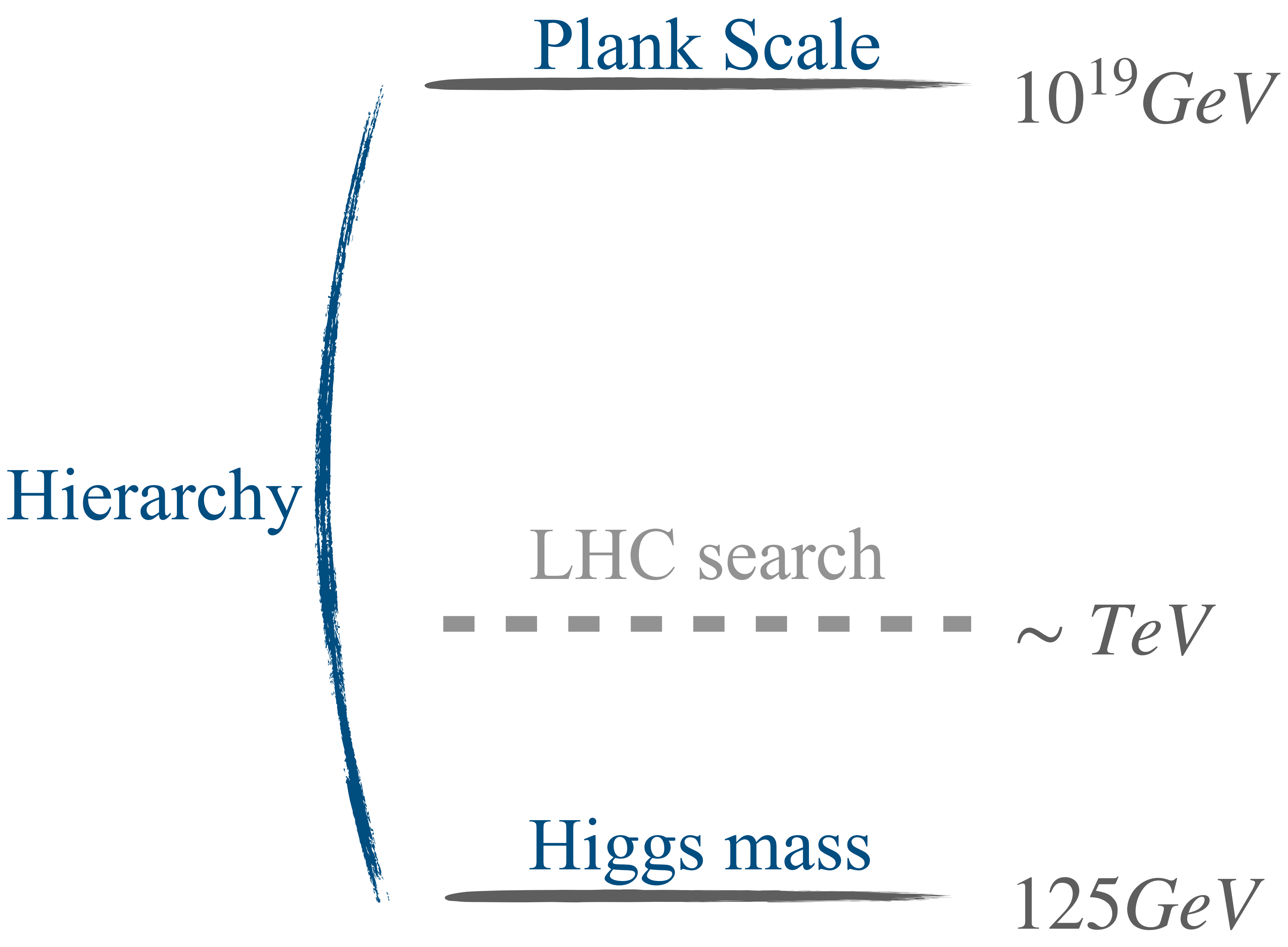
Plank Scale $10^{19} GeV$

Hierarchy

Higgs mass $125 GeV$

Composite Higgs Model

D. B. Kaplan and H. Georgi, Phys.Lett.B 136 (1984)



Plank Scale $10^{19} GeV$

Hierarchy

LHC search $\sim TeV$

Higgs mass $125 GeV$

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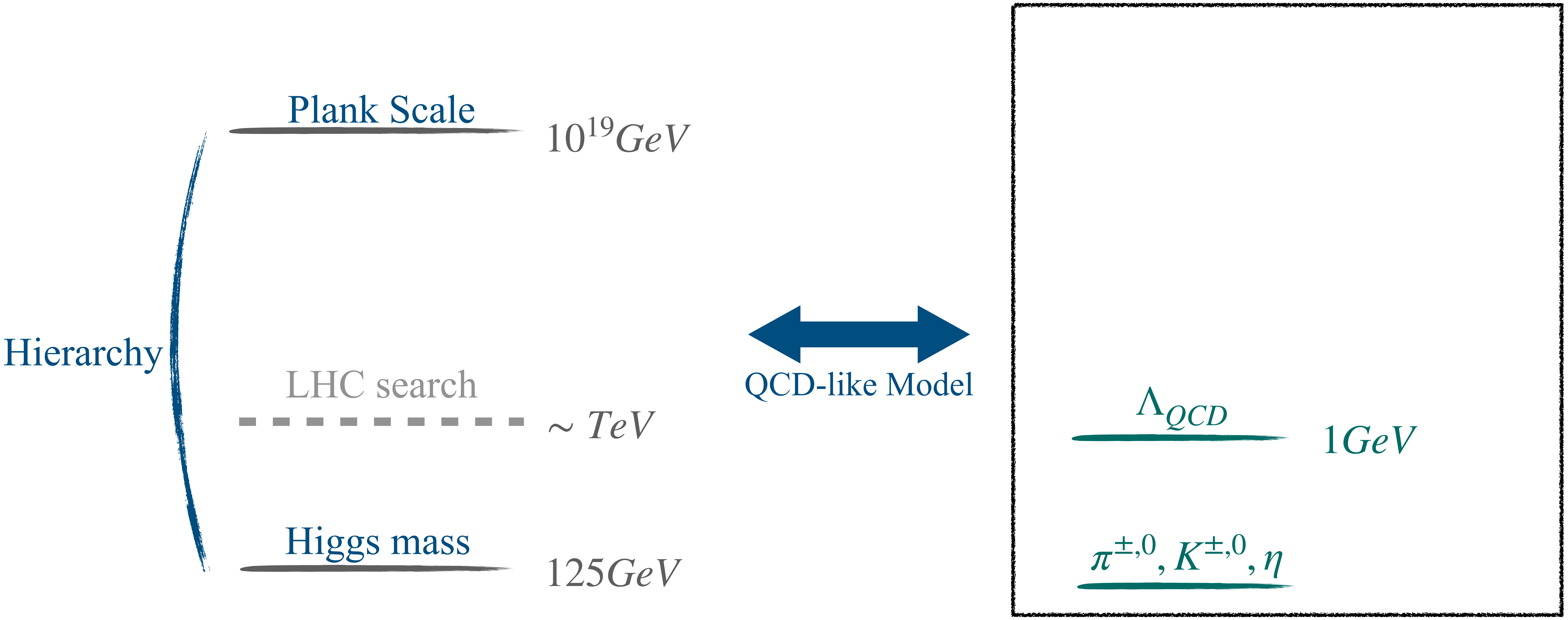

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QCD-like Model

Λ_{QCD} $1GeV$

$\pi^{\pm,0}, K^{\pm,0}, \eta$

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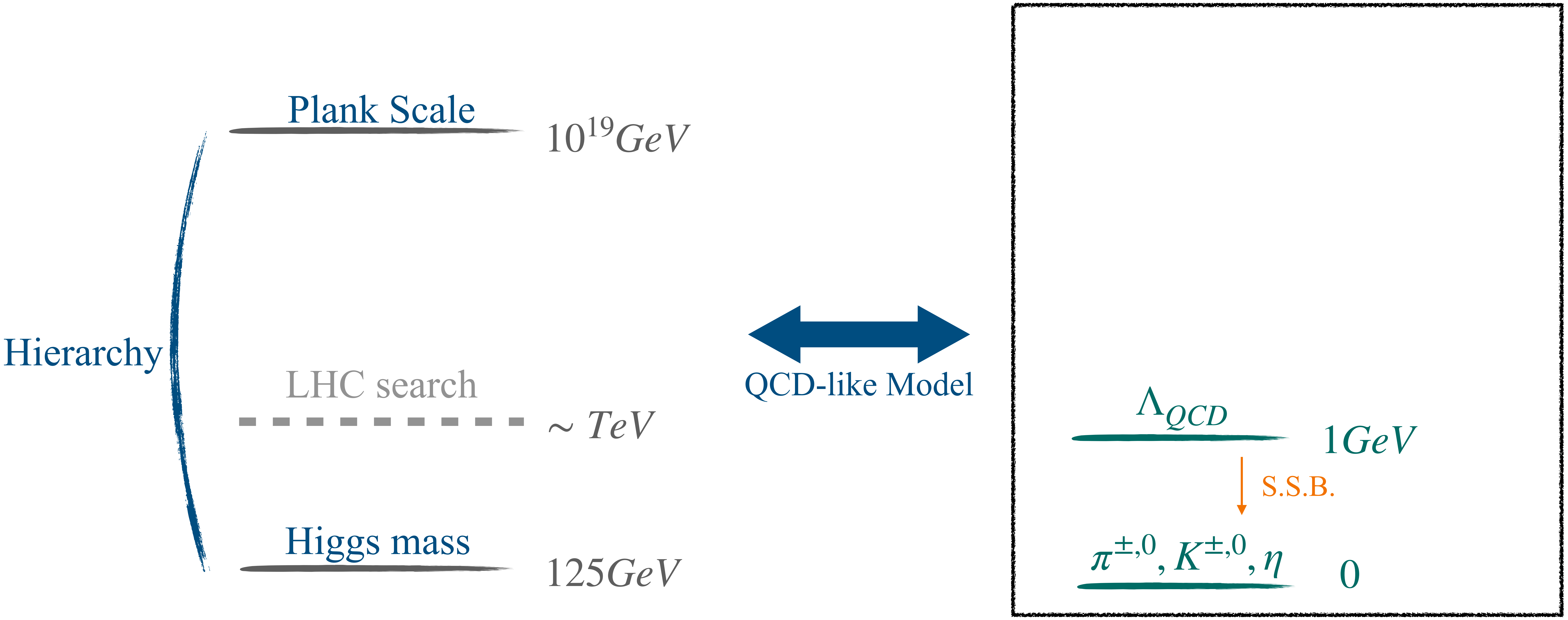
D. B. Kaplan and H. Georgi, Phys.Lett.B 136 (1984)



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Λ_{QCD} $1GeV$

S.S.B.

$\pi^{\pm,0}, K^{\pm,0}, \eta$ 0

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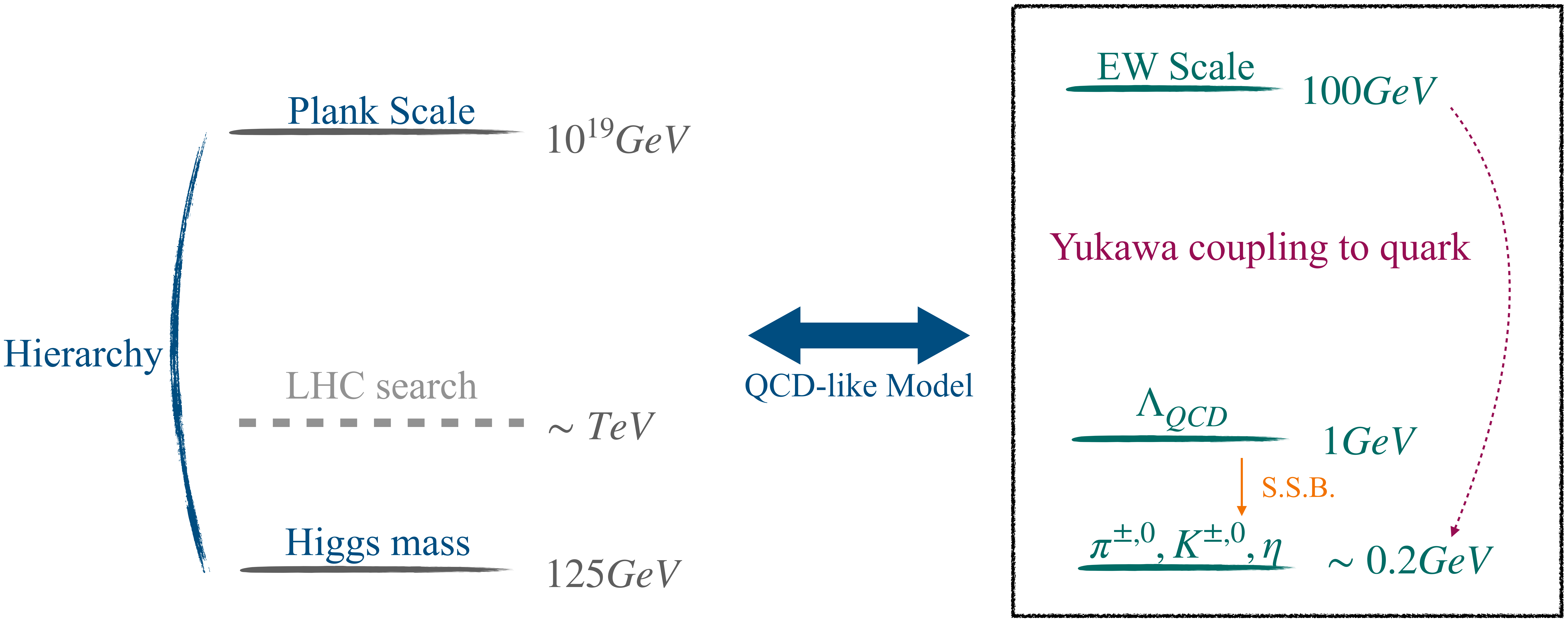

Hierarchy

Plank Scale $10^{19}GeV$

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QCD-like Model

EW Scale $100GeV$

Yukawa coupling to quark

Λ_{QCD} $1GeV$

S.S.B.

$\pi^{\pm,0}, K^{\pm,0}, \eta$ $\sim 0.2GeV$

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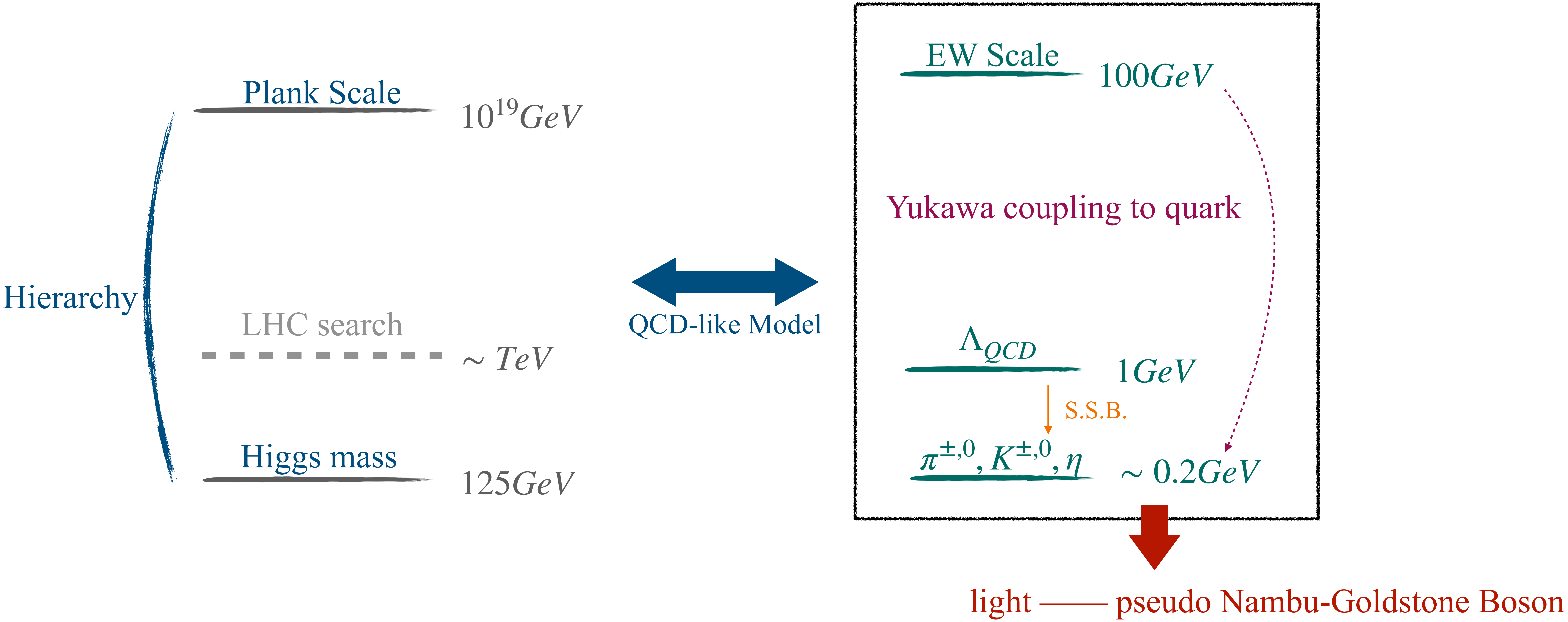

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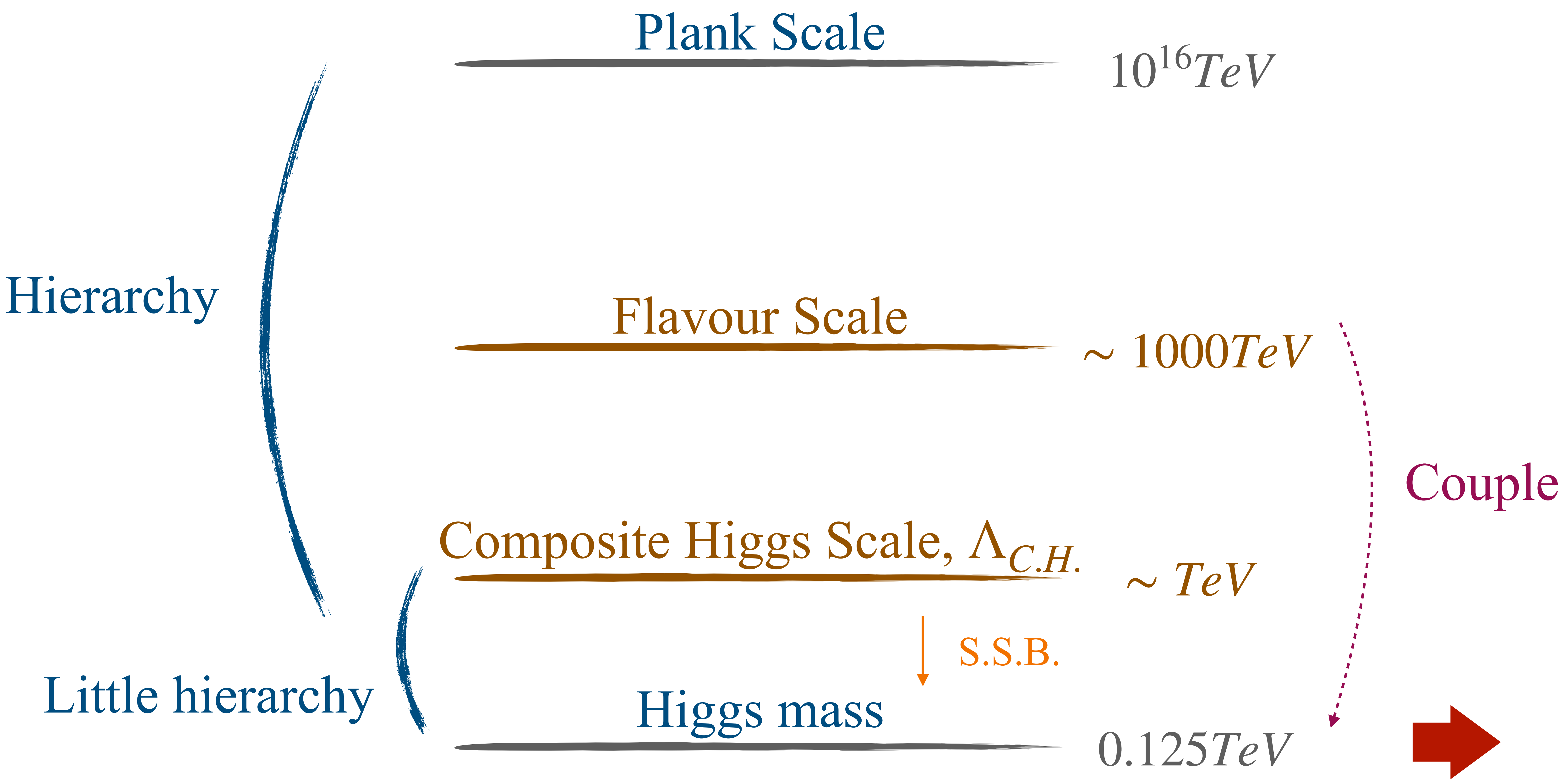
light —— pseudo Nambu-Goldstone Boson

Composite Higgs Model



Plank Scale $10^{16} TeV$

Flavour Scale $\sim 1000 TeV$

Composite Higgs Scale, $\Lambda_{C.H.}$ $\sim TeV$

S.S.B.

Higgs mass $0.125 TeV$

Hierarchy

Little hierarchy

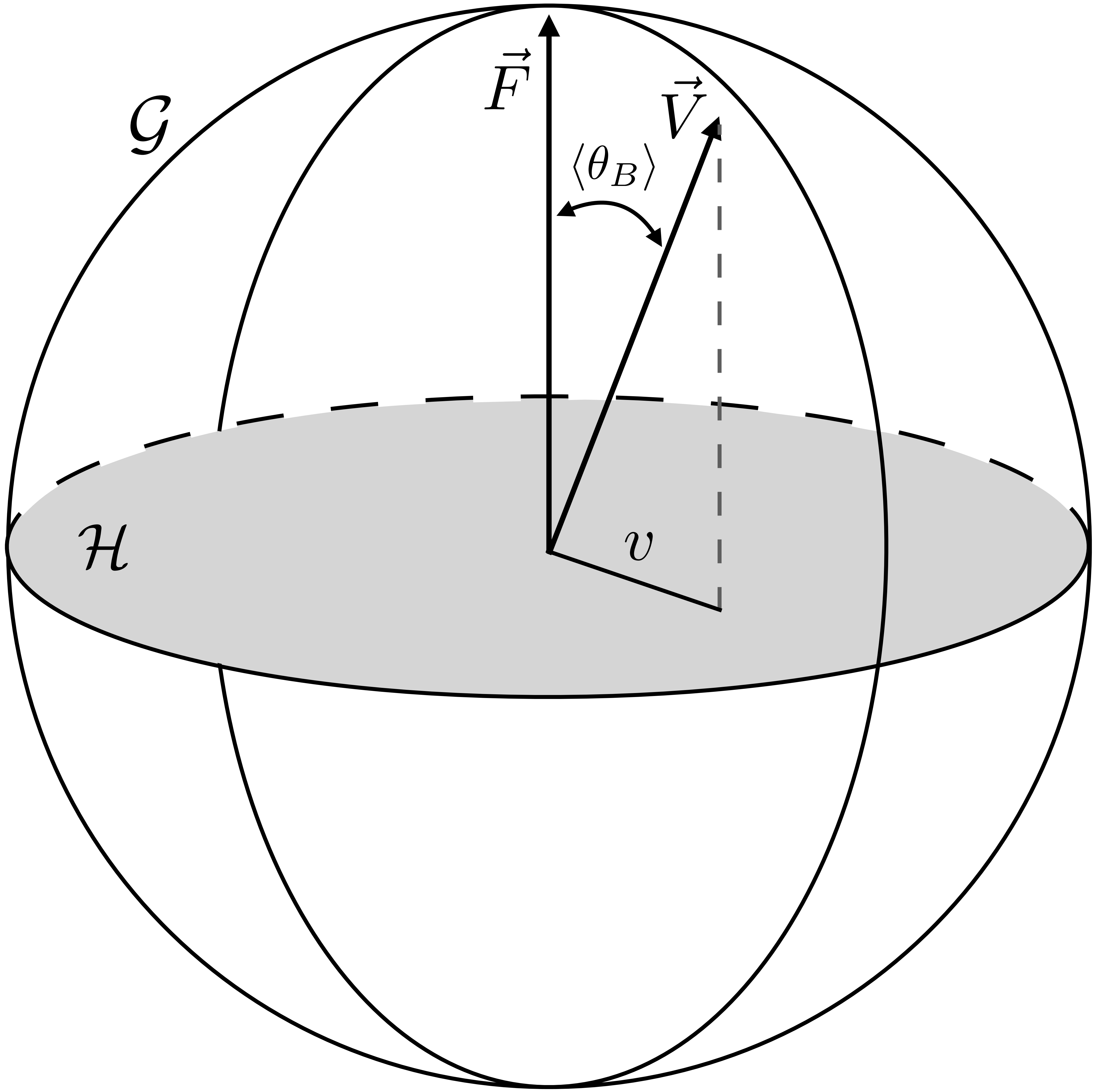
Couple

Higgs boson as a bound state of new strong dynamics, which is lighter because of being a pseudo Nambu-Goldstone Boson.

Composite Higgs Model

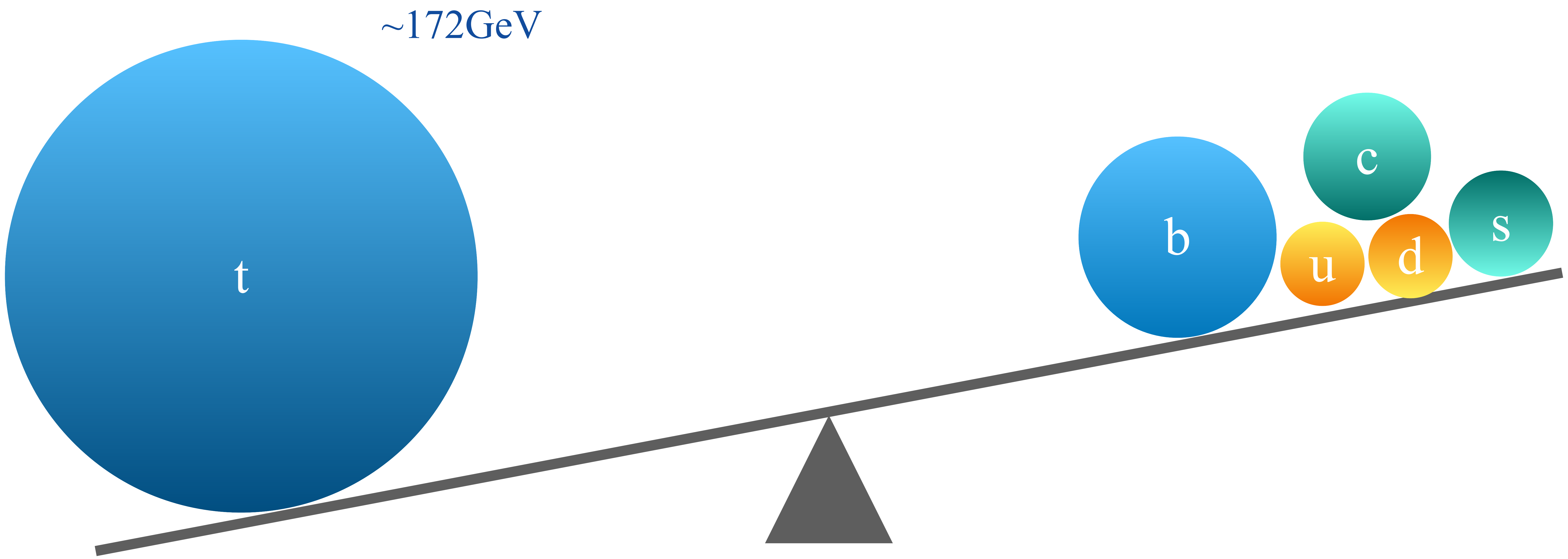
Symmetries

- Global symmetry: $\mathcal{G}$
- Subgroup: $\mathcal{H}$ with $G_{\rm EW} \subset \mathcal{H}$
- Vacuum misalignment angle: θ_B
- Coset $\mathcal{G}/\mathcal{H} \to$ pNGBs
- The scale of the EWSB: $v = f \sin\theta_B \ (f = |\vec{F}|)$



Composite Higgs Model

Top partial compositeness



Composite Higgs Model

Top partial compositeness

Top partners:

- Share the same quantum number as the top
  - Spin-1/2 bound states emerging from the novel strong-interaction sector
  - Carry QCD colour charge

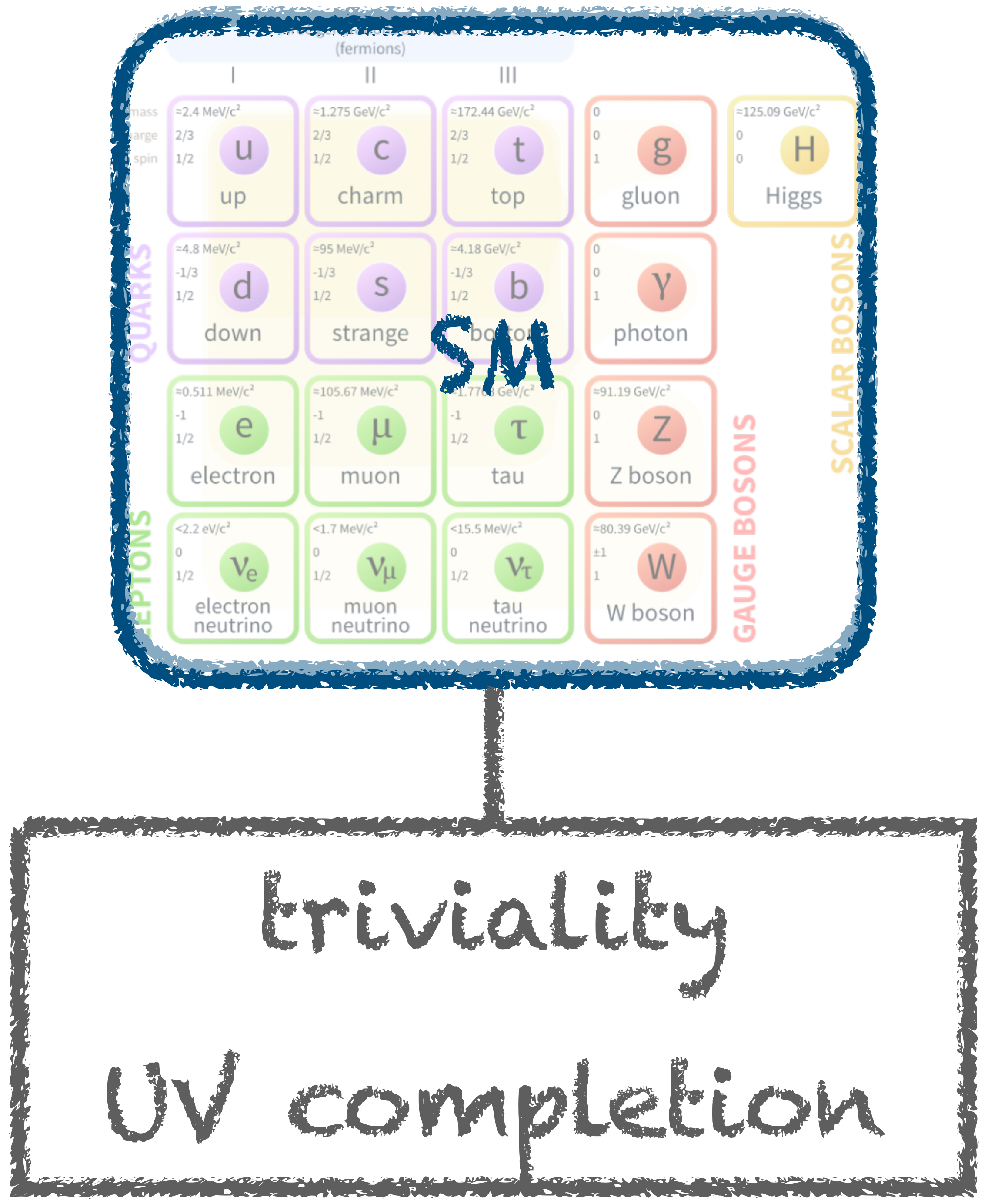
➡ Introducing higher representation

- Hypercolour-neutral
- Give the mass to the top by mixing with it

$$\mathcal{L}_{\text{mass}} = -M\bar{T}_L T_R - y\frac{v}{\sqrt{2}}\bar{t}_L T_R - \lambda f \bar{T}_L t_R + \text{h.c.}\,, \qquad \Rightarrow m_t \simeq \frac{yv}{\sqrt{2}}\frac{\lambda f}{\sqrt{\lambda^2 f^2 + M^2}}\,.$$

$$= (\bar{t}_L \quad \bar{T}_L)\begin{pmatrix} 0 & \frac{yv}{\sqrt{2}} \\ \lambda f & M \end{pmatrix}\begin{pmatrix} t_R \\ T_R \end{pmatrix} + \text{h.c.}\,.$$

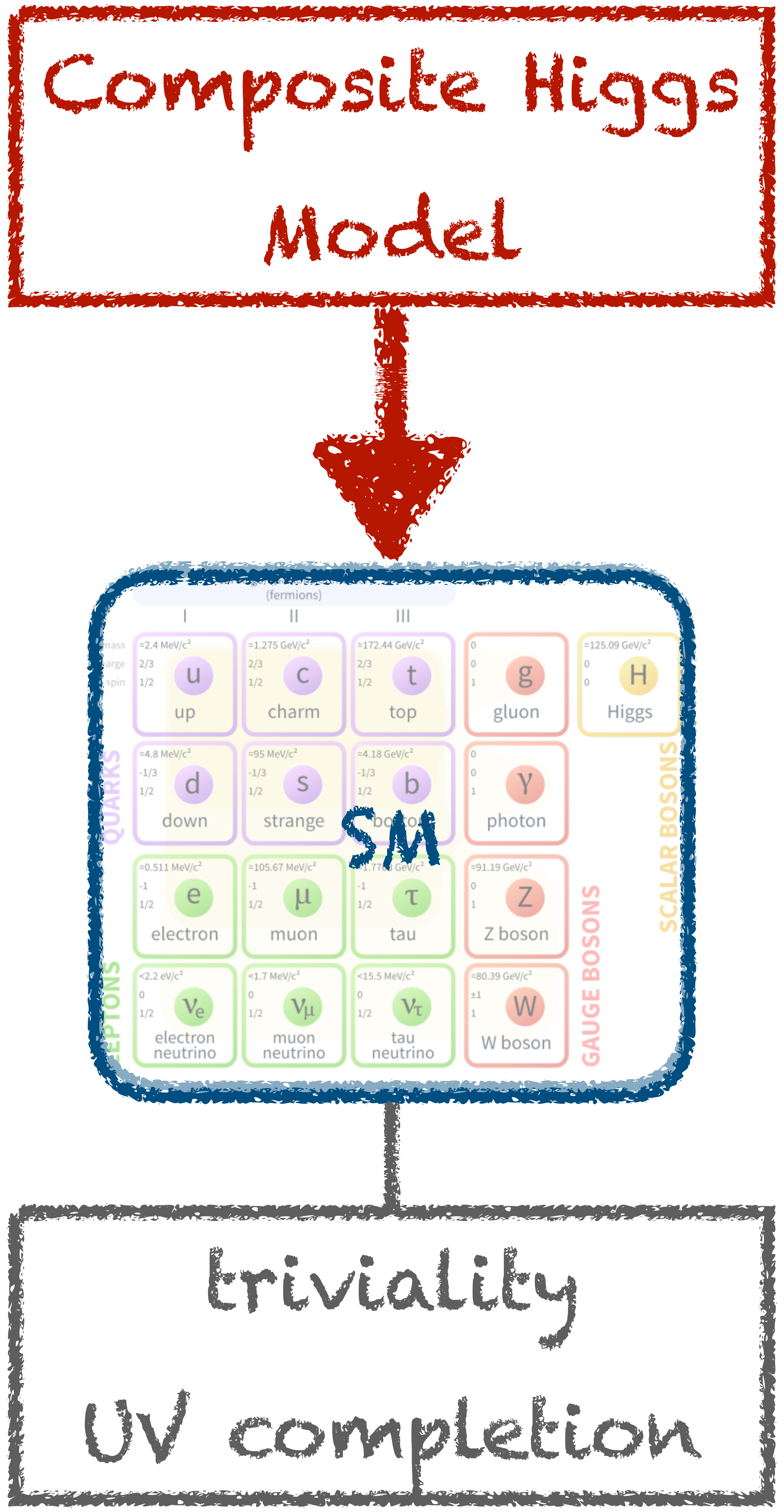
Composite Higgs Model



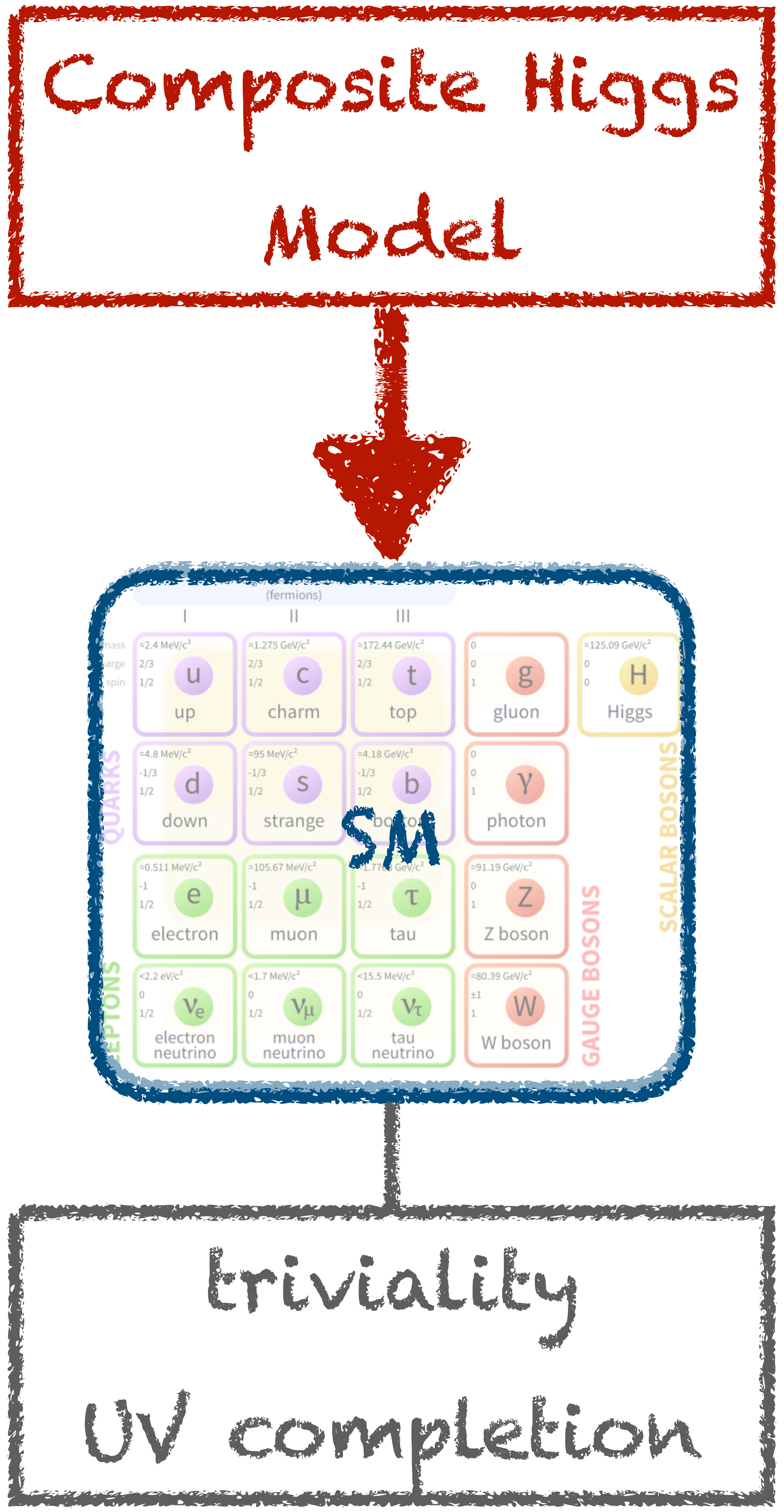
triviality

UV completion

Composite Higgs Model

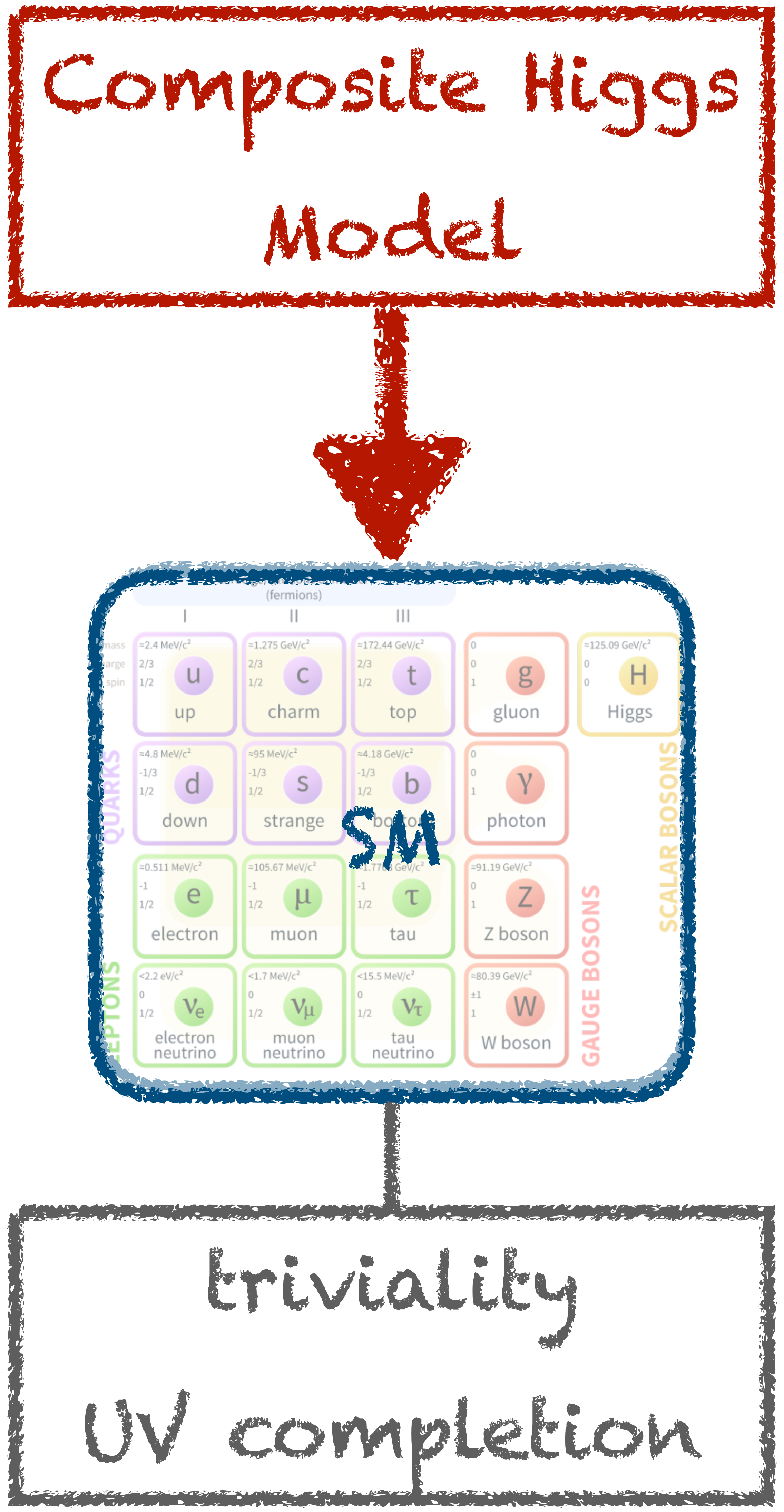


Composite Higgs Model



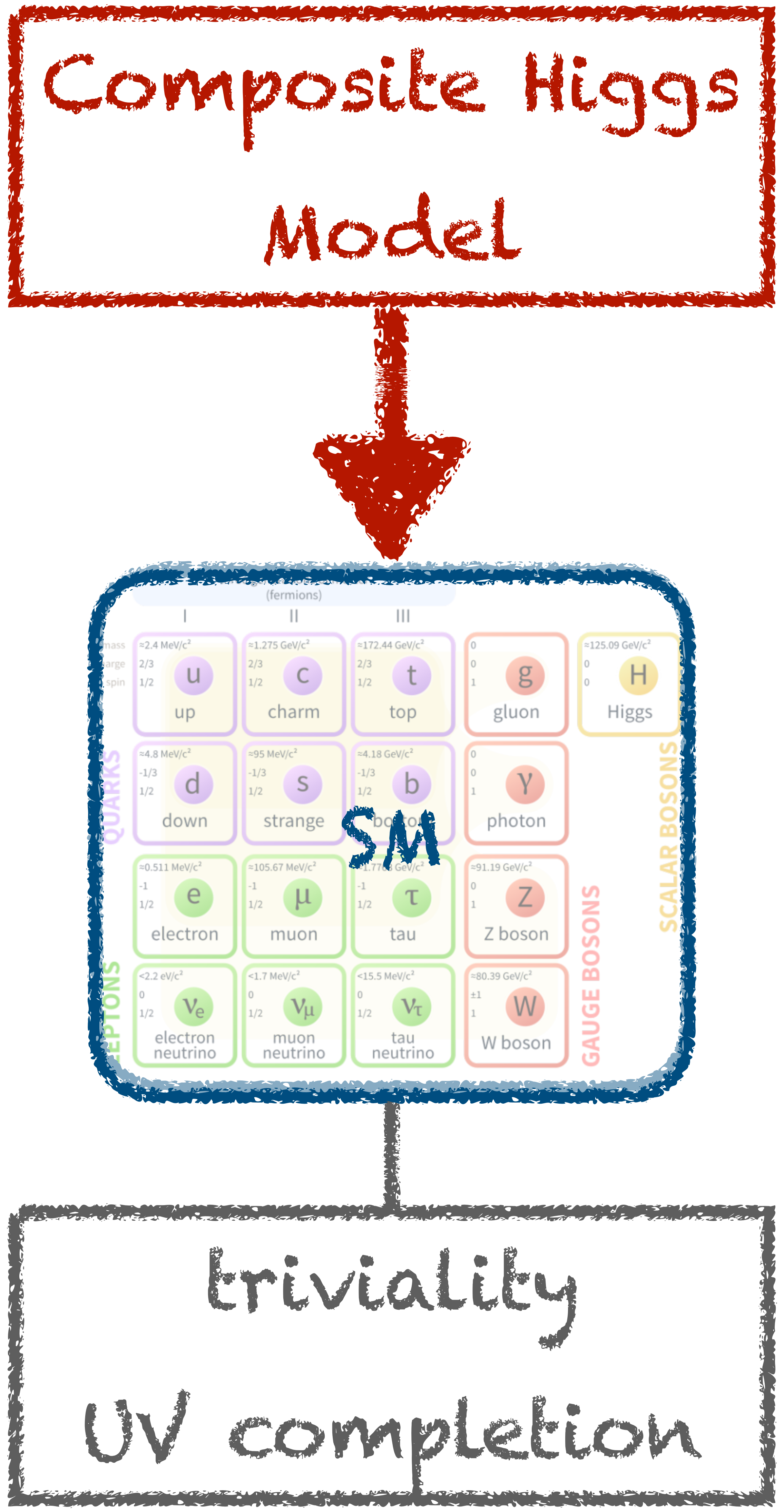
- SM Higgs is a composite object.

Composite Higgs Model



- SM Higgs is a composite object.
- Introduce a novel strong-interaction sector and hyperquarks.

Composite Higgs Model



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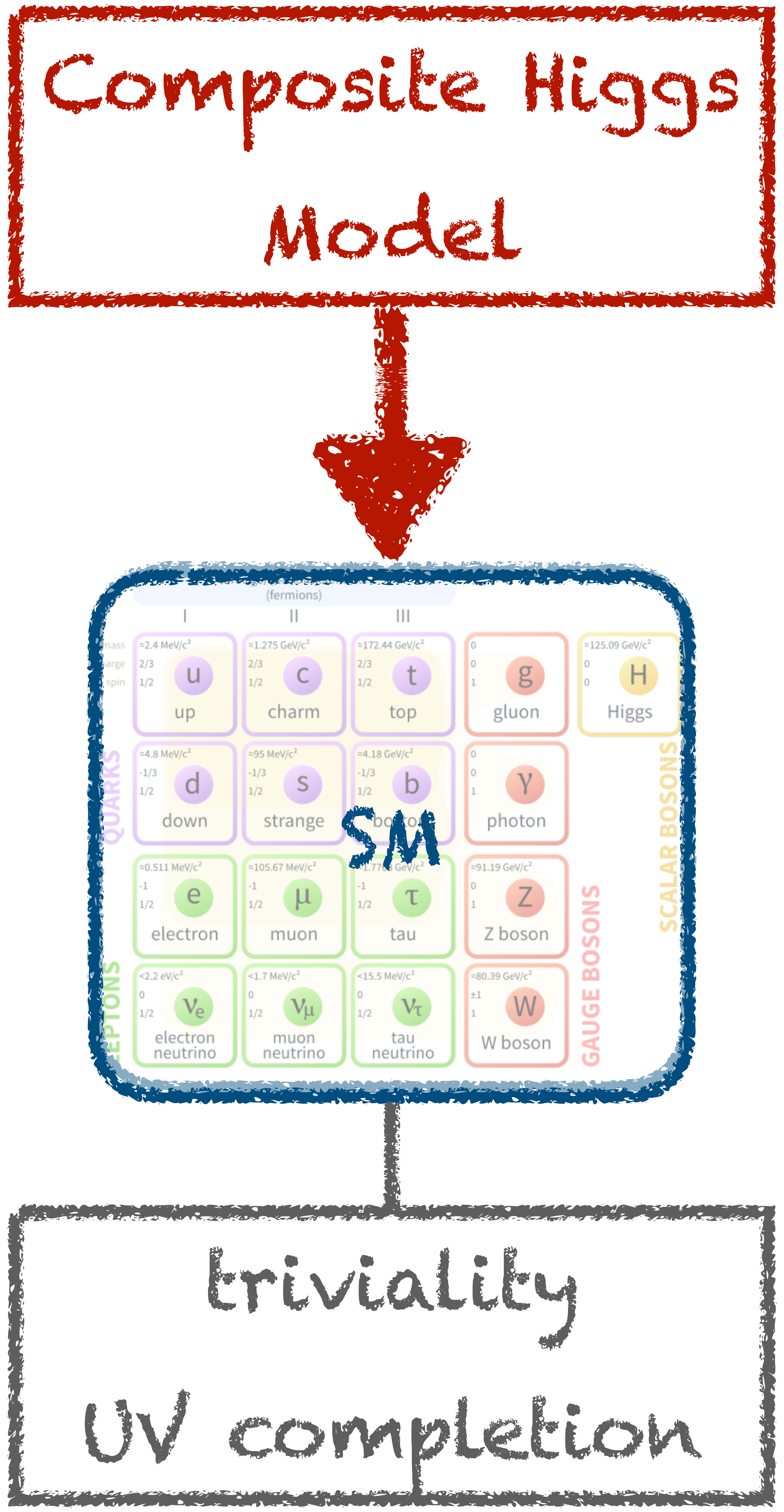
- Accommodate a light Higgs boson: SM Higgs is interpreted as one of the Goldstone modes (in the coset).

Composite Higgs Model



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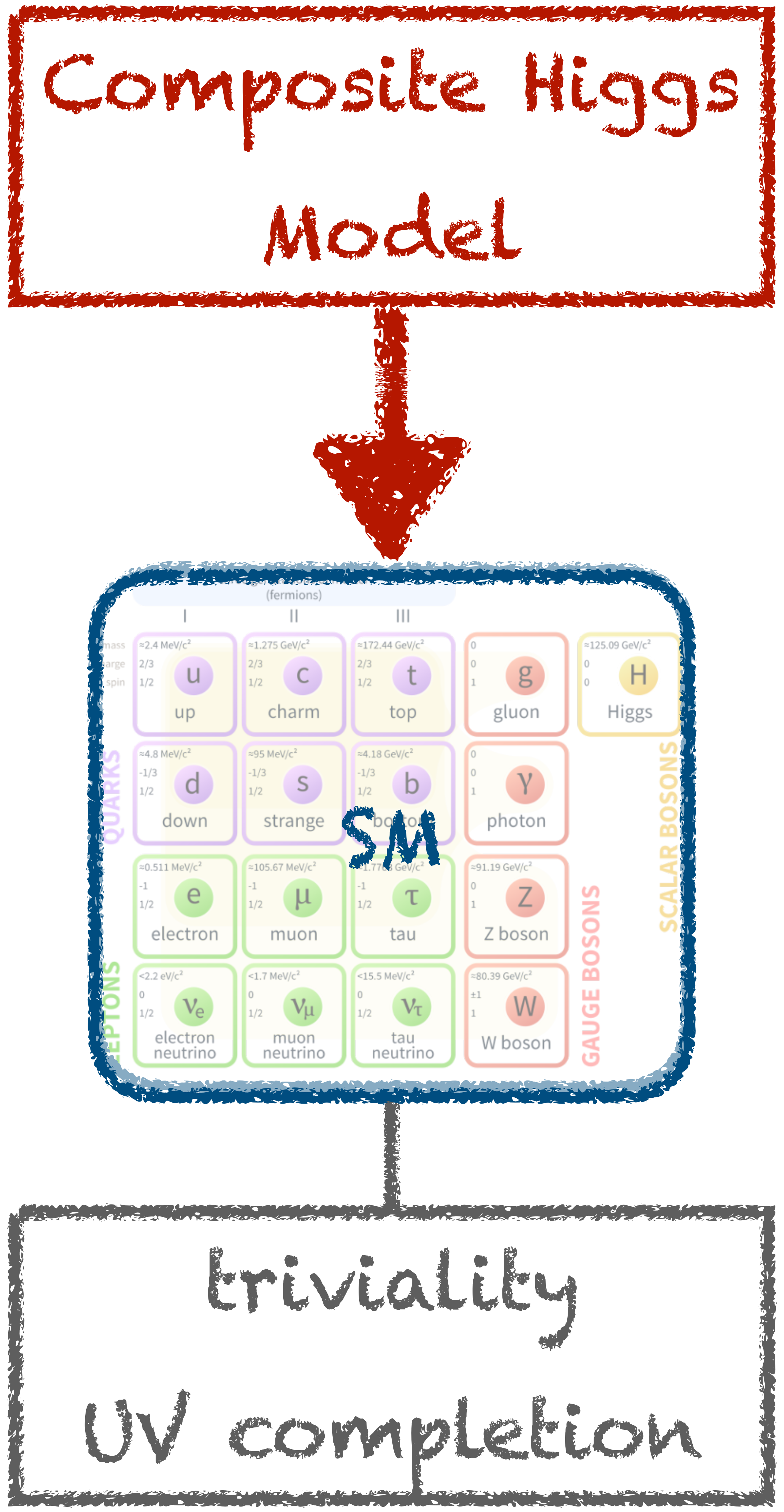

- SM Higgs is a composite object.
- Introduce a novel strong-interaction sector and hyperquarks.
- Accommodate a light Higgs boson: SM Higgs is interpreted as one of the Goldstone modes (in the coset).
- A UV complete theory.
- Can embed top partial compositeness with a higher representation.

Composite Higgs Models

*Weyl fermions

Name	Gauge group	ψ	χ	Baryon type
M1	$SO(7)$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$\psi\chi\chi$
M2	$SO(9)$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$\psi\chi\chi$
M3	$SO(7)$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M4	$SO(9)$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M5	$Sp(4)$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$\psi\chi\chi$
M6	$SU(4)$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$\psi\chi\chi$
M7	$SO(10)$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$\psi\chi\chi$
M8	$Sp(4)$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$\psi\psi\chi$
M9	$SO(11)$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M10	$SO(10)$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M11	$SU(4)$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$\psi\psi\chi$
M12	$SU(5)$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	$\psi\psi\chi$, $\psi\chi\chi$

D. Franzosi and G. Ferretti, arXiv:1905.08273

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The minimal model

Barnard et al, arXiv:1311.6562

D. Franzosi and G. Ferretti, arXiv:1905.08273

Our choice of model

- Sp(4) gauge theory with 2F+3AS Dirac fermions

$$\Big\downarrow \quad \text{(4F+6AS 2-component Weyl fermions)}$$

- Breaking pattern:

$$G/H = \underbrace{SU(4)\times SU(6)}_{\text{Enhanced global symmetry due to the (pseudo-) reality}} \,/\, Sp(4)\times SO(6)$$

● $SU(4)/Sp(4)$ gives 5 goldstone bosons.

- 4: SM Higgs doublet
- 1: made heavy in model building

● SU(3) embedded in antisymmetric representation:

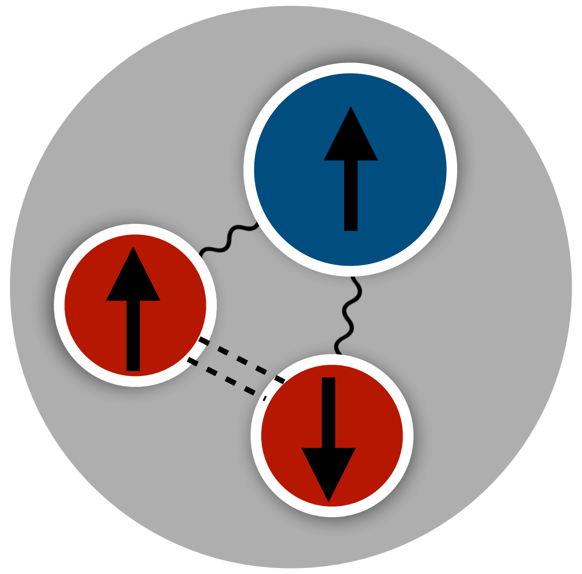
$$SU(6) \to SO(6) \supset SU(3)$$

↳ QCD colour SU(3)

Chimera Baryon

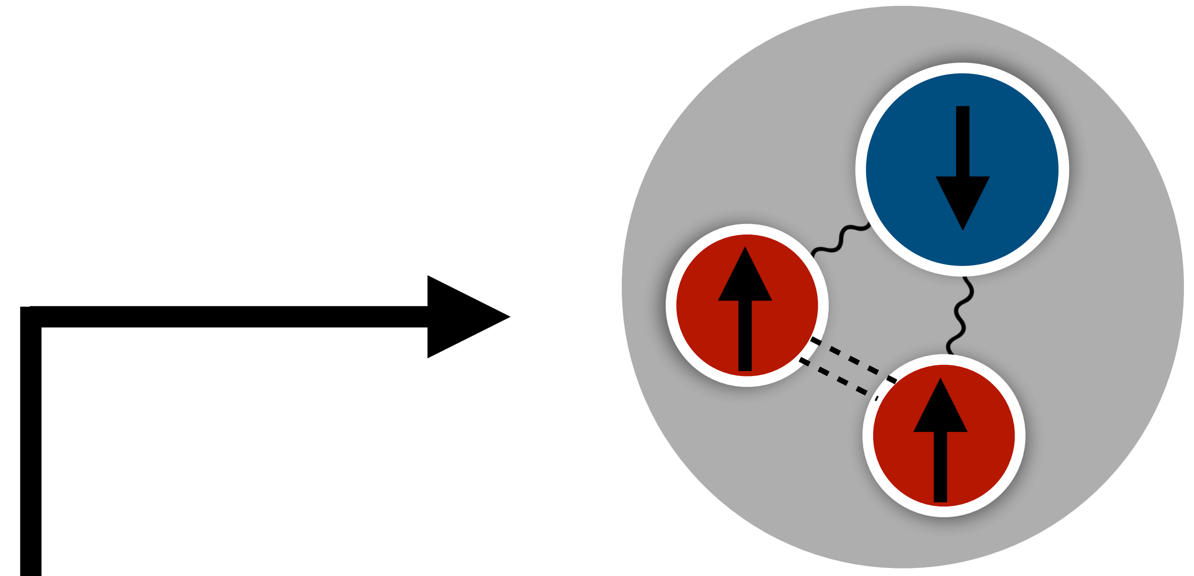
a, b, c: hypercolour
Ω: 4×4 symplectic matrix
J: spin
R: irreducible rep. of the fundamental sector

- Interpolating operators
  - Λ type: $\mathcal{O}_{\mathrm{CB},\gamma^5} = \left(\bar{\psi}^{1\,a}\gamma^5\psi^{2\,b}\right)\Omega_{ad}\Omega_{bc}\chi^{k\,cd}$

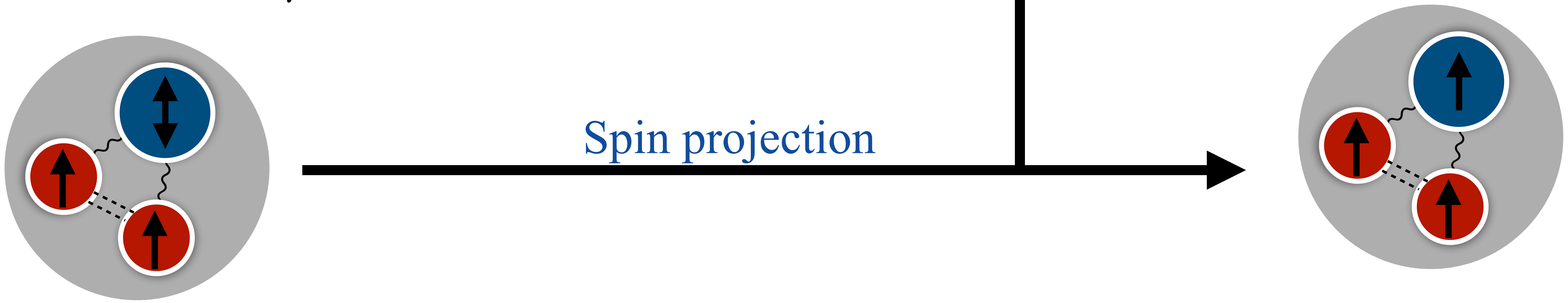


$(J, R) = (1/2, 5)$
*top partner

  - Σ type: $\mathcal{O}_{\mathrm{CB},\gamma^\mu} = \left(\bar{\psi}^{1\,a}\gamma^\mu\psi^{2\,b}\right)\Omega_{ad}\Omega_{bc}\chi^{k\,cd}$



Σ: $(J, R) = (1/2, 10)$
*top partner



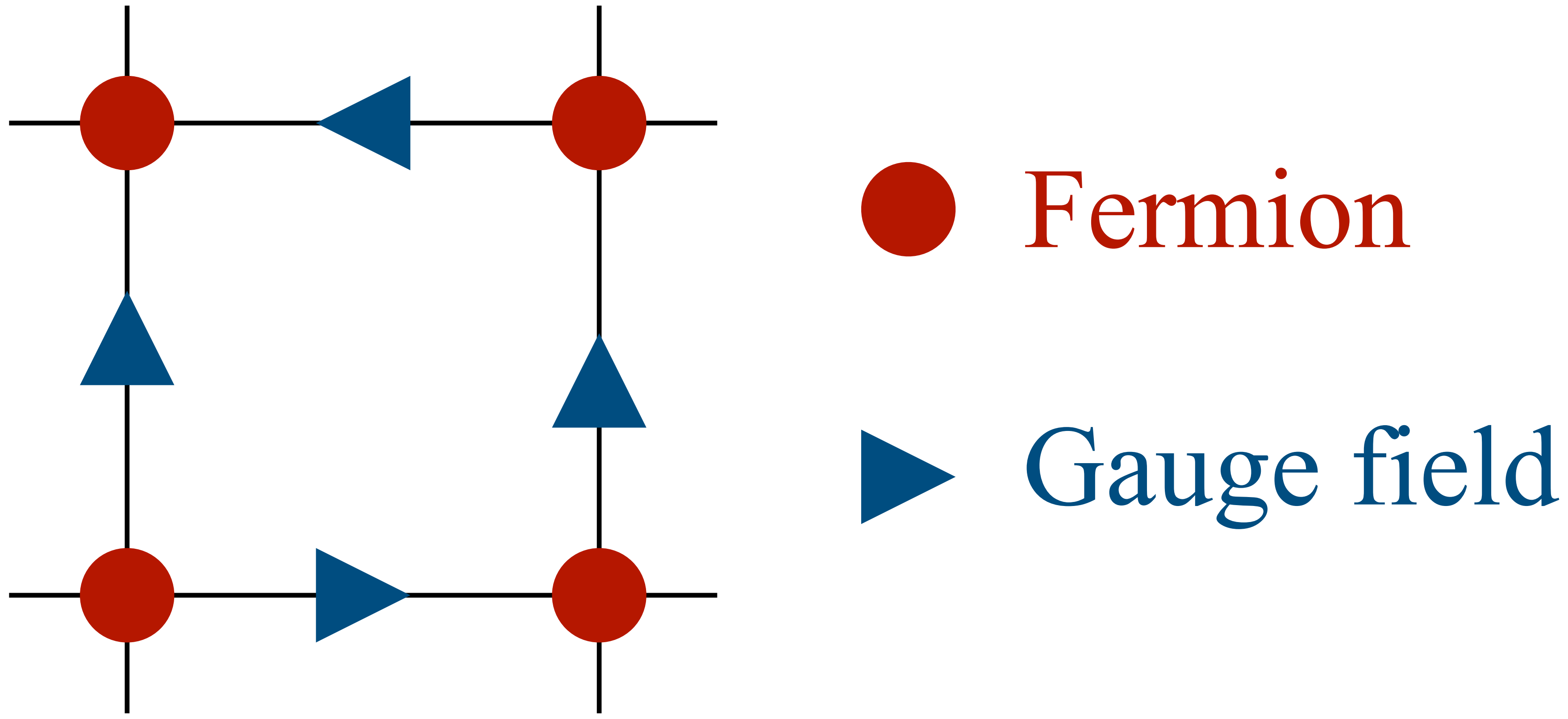
Spin projection

Σ^*: $(J, R) = (3/2, 10)$

Lattice Method

- Strongly coupled theory → lattice field theory
- Fermions on the grids, carrying colours, spin or flavours
- Gauge fields on the links
- Generating functional

$$
\begin{aligned}
Z &= \int DUD\psi D\bar{\psi} e^{-S[U]} e^{-\int d^4 x \bar{\psi}(D[U]+m)\psi} \\
&= \int DU \det(D[U]+m) e^{-S[U]}
\end{aligned}
$$



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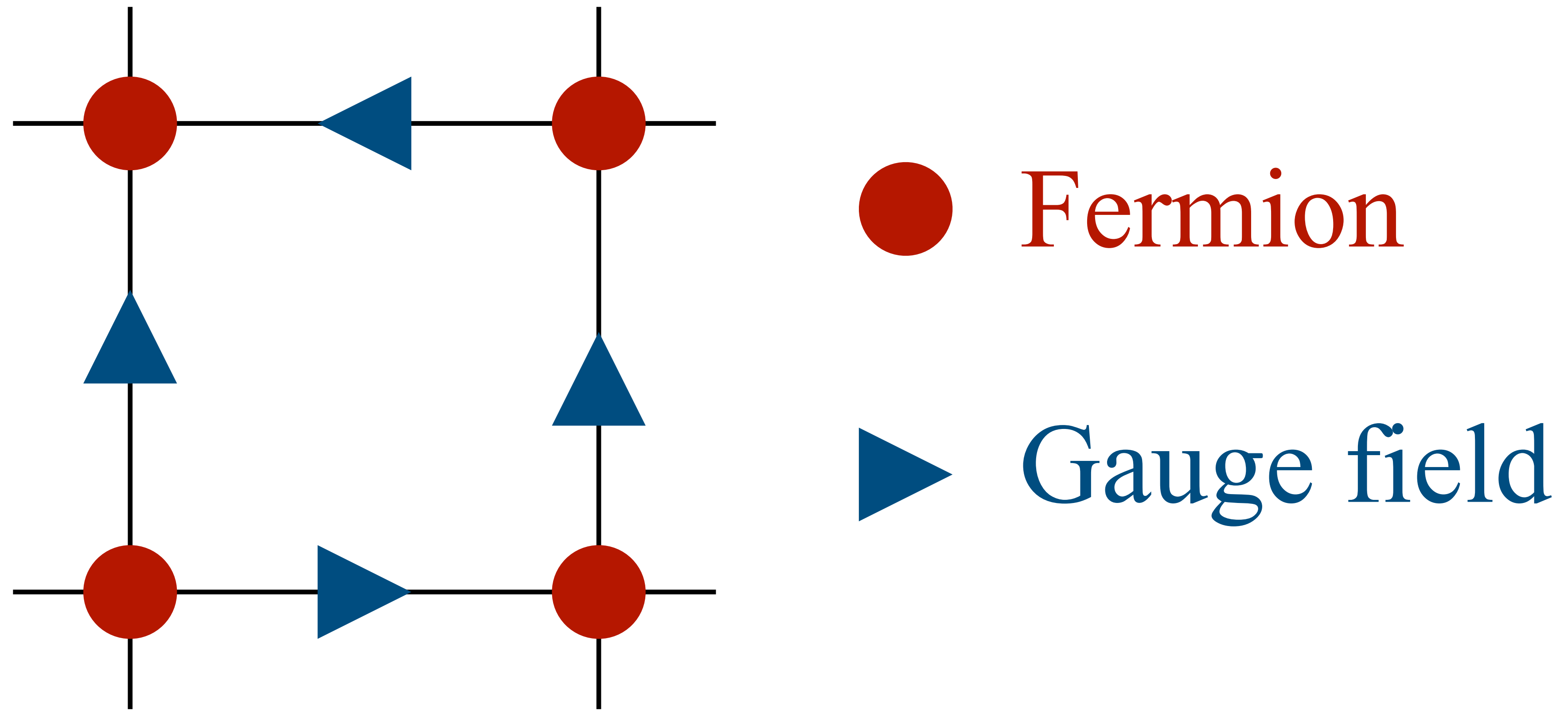

● Fermion

▶ Gauge field

$$Z = \int DUD\psi D\bar{\psi} e^{-S[U]} e^{-\int d^4x \bar{\psi}(D[U]+m)\psi}$$

$$= \int DU \det(D[U]+m) e^{-S[U]}$$

➡ (Hybrid) Monte-Carlo simulation

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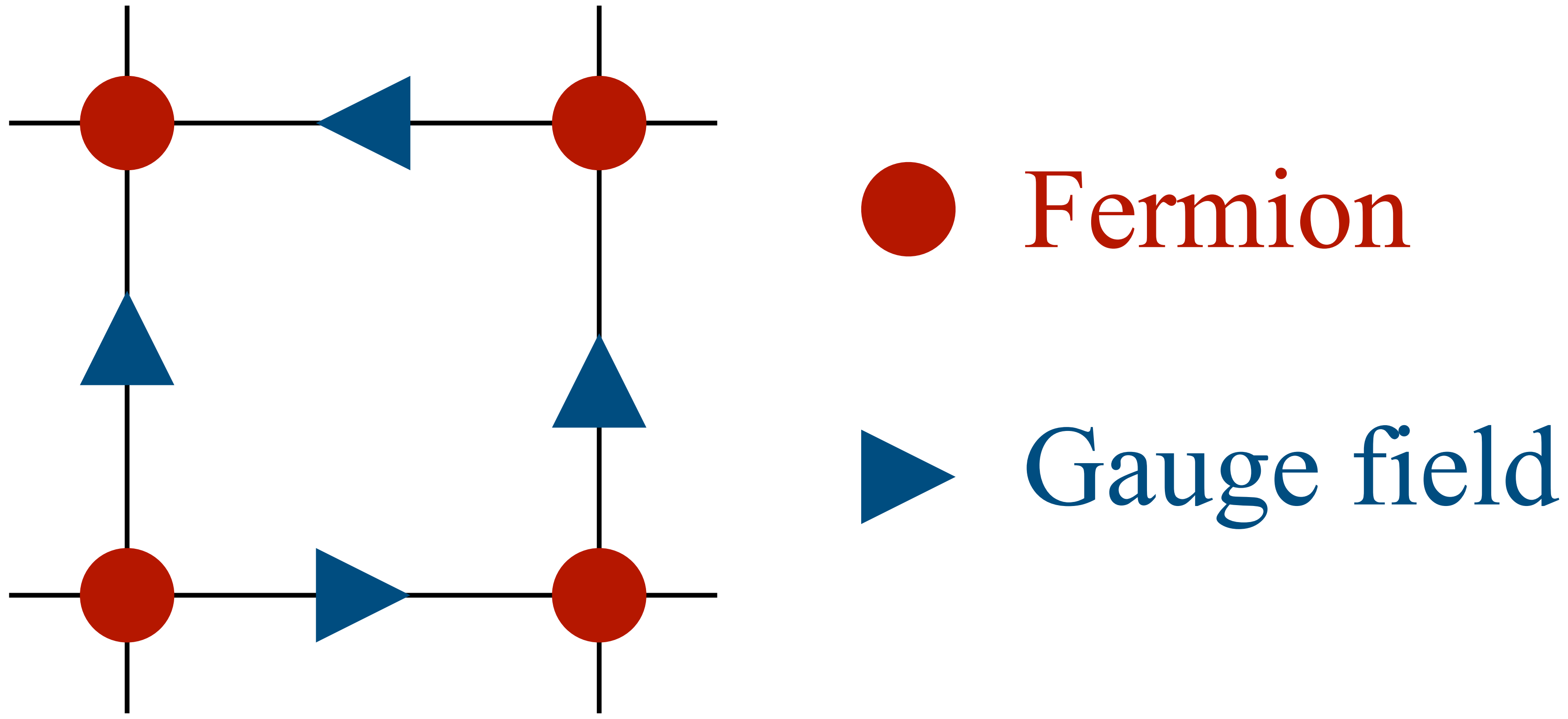

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(Hybrid) Monte-Carlo simulation

Quench calculation: $\det(D[U]+m)=1$ Heat bath algorithm

Lattice Method

Numerical calculations are accomplished by modifying the HiRep code.

repository: https://github.com/sa2c/HiRep Del Debbio et al, arXiv:0805.2058

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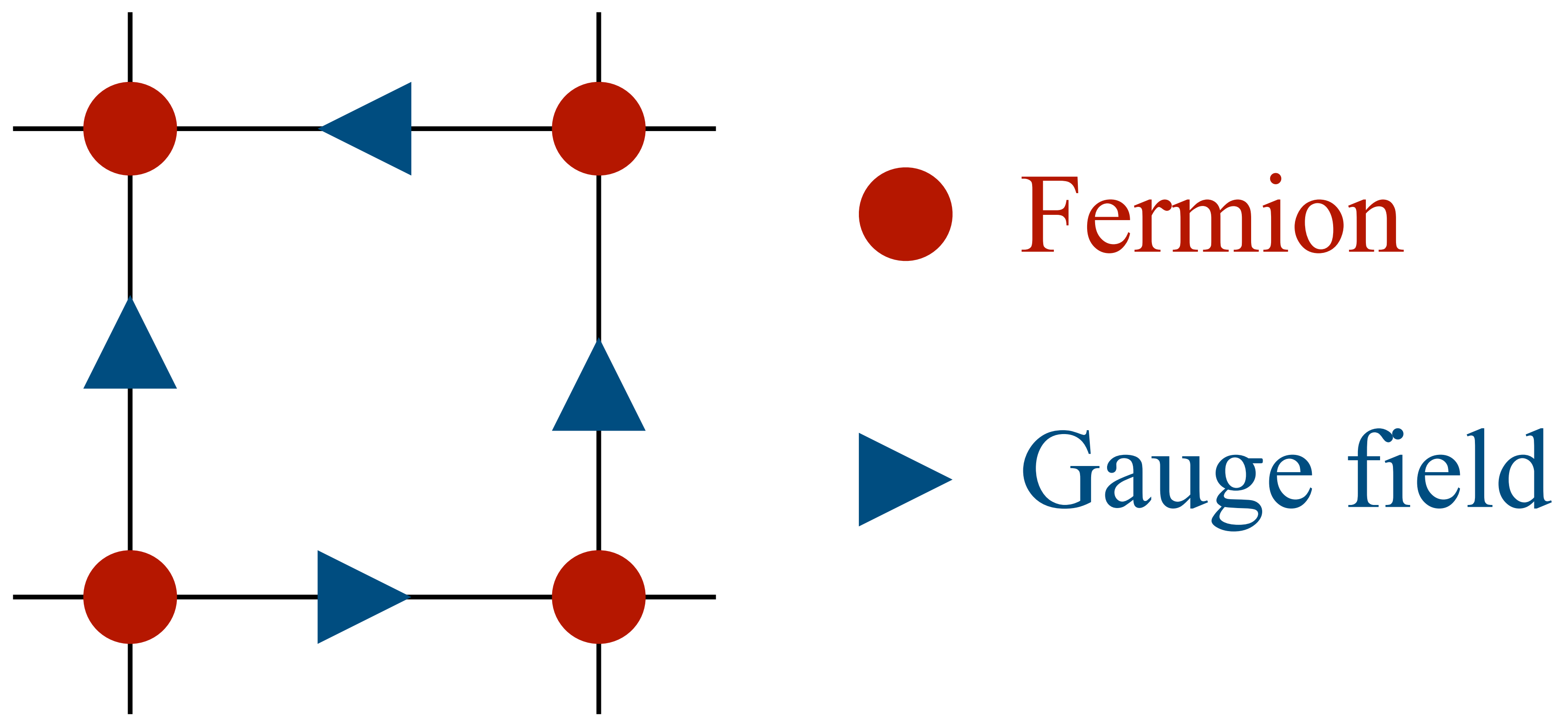

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Lattice Method

Extracting mass

- Mesonic 2-point correlation function

$$C(t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 \Big| T\left[O(\vec{x},t)O^{\dagger}(0,0)\right] \Big| 0 \rangle$$

$$\sum_{n} \frac{\langle 0 \big| O_{\pi} \,|n\rangle \langle n \big| O_{\pi}^{\dagger} \big| 0 \rangle}{2E_n} e^{-E_n t}$$

$$\xrightarrow{t\to\infty} \frac{1}{2m_{\pi}} \left| \langle 0 \big| O_{\pi} \,|\pi\rangle \right|^2 e^{-M_{\pi} t}$$

$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 \Big| \left[\bar{u}\gamma_5 d\right](\vec{x},t)\left[\bar{d}\gamma_5 u\right](0,0) \Big| 0 \rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathrm{Tr}\left[S_u(0,0;\vec{x},t)S_d^{\dagger}(0,0;\vec{x},t)\right]$$

$$S = M^{-1} q$$

M is the Dirac operator calculated on a given background field.

- Effective Mass

$$M_{eff}(t) = -\ln\left[\frac{C(t+1)}{C(t)}\right]$$

Results

quenched approximation

- Projections
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- Projections
- Mass hierarchy of chimera baryons
- Chiral EFT and AIC

Ensemble	β	$N_t \times N_s^3$	$\langle P \rangle$	ω_0/a
QB1	7.62	48×24^3	0.60192	1.448(3)
QB2	7.7	60×48^3	0.608795	1.6070(19)
QB3	7.85	60×48^3	0.620381	1.944(3)
QB4	8.0	60×48^3	0.630740	2.3149(12)
QB5	8.2	60×48^3	0.643228	2.8812(21)

Results

quenched approximation

- Projections
- Mass hierarchy of chimera baryons
- Chiral EFT and AIC

$\hat{m}_{\mathrm{PS}}$: fundamental
$\hat{m}_{\mathrm{ps}}$: Antisymmetric

$\hat{a} \equiv a/\omega_0$ and $\hat{m} \equiv \omega_0 m$

Ensemble	β	$N_t \times N_s^3$	$\langle P \rangle$	ω_0/a
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QB5	8.2	60×48^3	0.643228	2.8812(21)

Results

Projection-CB two-point function

- Interpolating operator

$$\mathcal{O}^{\gamma}_{\mathrm{CB}}(x) \equiv \left(Q^{i\,a}{}_{\alpha}(x) \Gamma^{1\,\alpha\beta} Q^{j\,b}{}_{\beta}(x) \right) \Omega_{ad} \Omega_{bc} \Gamma^{2\,\delta\gamma} \Psi^{k\,cd}{}_{\gamma}(x)$$

- two-point function

$$\begin{aligned} C^{\gamma\gamma'}(t) &\equiv \sum_{\vec{x}} \langle \mathcal{O}^{\gamma}_{\mathrm{CB}}(x) \overline{\mathcal{O}^{\gamma'}_{\mathrm{CB}}}(0) \rangle \\ &= -\sum_{\vec{x}} \left(\Gamma^{2} S_{\Psi}^{k\,cd}{}_{c'd'}(x,0) \overline{\Gamma^{2}} \right)_{\gamma\gamma'} \Omega_{cb} \Omega^{b'c'} \Omega_{ad} \Omega^{d'a'} \\ &\quad \times \mathrm{Tr}\left[\Gamma^{1} S_{Q}^{b}{}_{b'}(x,0) \overline{\Gamma^{1}} S_{Q}^{a}{}_{a'}(x,0) \right] \end{aligned}$$

Results

Projection-CB two-point function

- Interpolating operator

$$\mathcal{O}^{\gamma}_{\mathrm{CB}}(x) \equiv \left(Q^{i\,a}{}_{\alpha}(x) \Gamma^{1\,\alpha\beta} Q^{j\,b}{}_{\beta}(x) \right) \Omega_{ad} \Omega_{bc} \Gamma^{2\,\delta\gamma} \Psi^{k\,cd}{}_{\gamma}(x)$$

- two-point function

$$C^{\gamma\gamma'}(t) \equiv \sum_{\vec{x}} \langle \mathcal{O}^{\gamma}_{\mathrm{CB}}(x) \overline{\mathcal{O}^{\gamma'}_{\mathrm{CB}}}(0) \rangle$$

$$= - \sum_{\vec{x}} \left(\Gamma^2 S_{\Psi}^{k\,cd}{}_{c'd'}(x,0) \overline{\Gamma^2} \right)_{\gamma\gamma'} \Omega_{cb} \Omega^{b'c'} \Omega_{ad} \Omega^{d'a'}$$

$$\times \mathrm{Tr} \left[\Gamma^1 S_Q^{b}{}_{b'}(x,0) \overline{\Gamma^1} S_Q^{a}{}_{a'}(x,0) \right]$$

At large Euclidean time

$$\to P_e \left[c_e e^{-m_e t} + c_o e^{-m_o (T-t)} \right] - P_o \left[c_o e^{-m_o t} + c_e e^{-m_e (T-t)} \right]$$

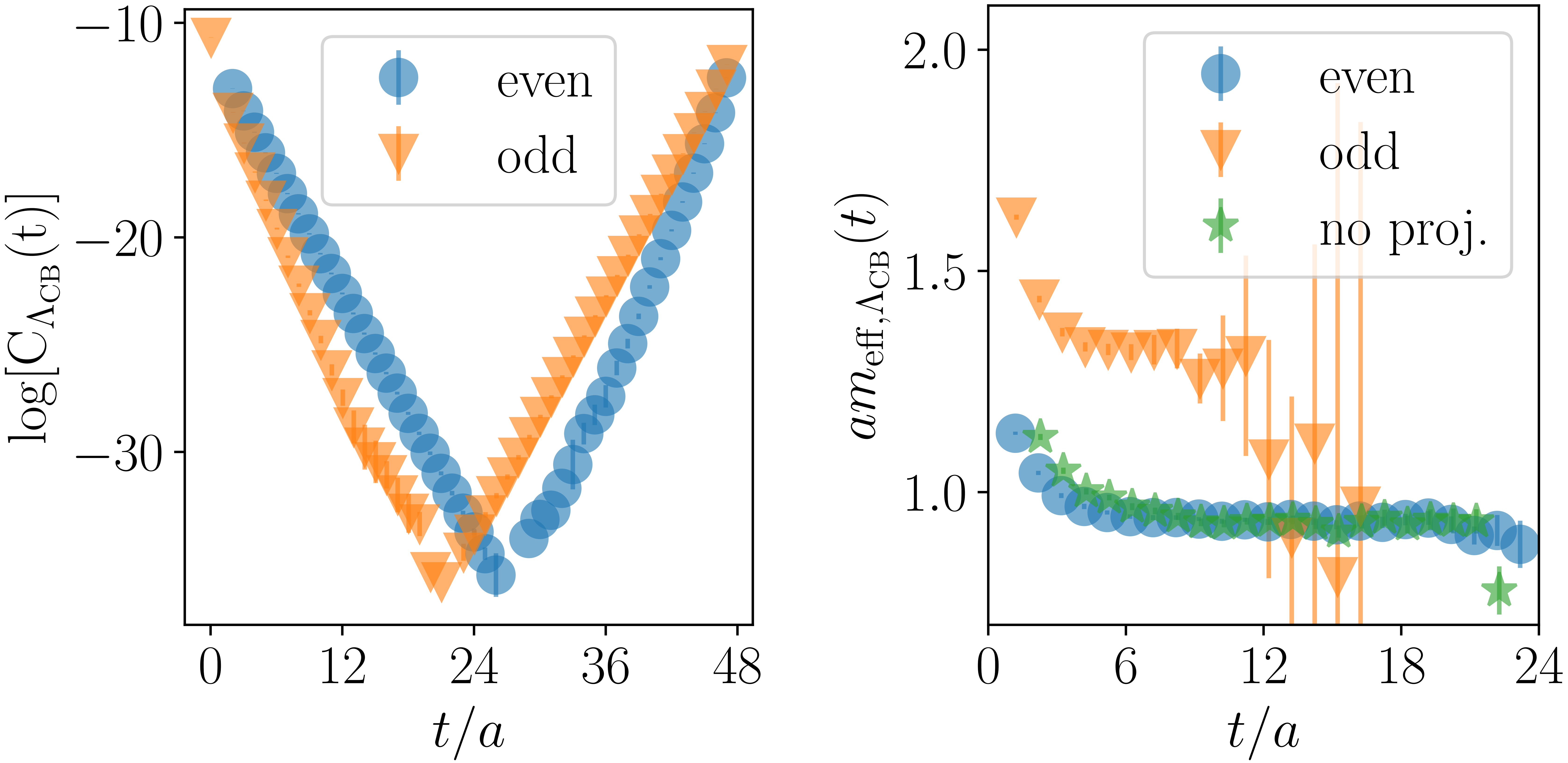
$$P_e \equiv \frac{1}{2}(1 + \gamma^0) \text{ and } P_o \equiv \frac{1}{2}(1 - \gamma^0)$$

Results

Projection-Parity

$$C_{\mathrm{CB}}(t) \to P_e\left[c_e e^{-m_e t} + c_o e^{-m_o(T-t)}\right] - P_o\left[c_o e^{-m_o t} + c_e e^{-m_e(T-t)}\right]$$

- The log plot of the chimera baryon correlators (left) and their effective mass plot (right) with the parity projection.



Chimera Baryon

- Spin projector for Σ-type baryon:

$$\left(P^{3/2}\right)^{ij} = \delta^{ij} - \frac{1}{3}\gamma^i\gamma^j$$

$$\left(P^{1/2}\right)^{ij} = \frac{1}{3}\gamma^i\gamma^j$$

- Two-point function

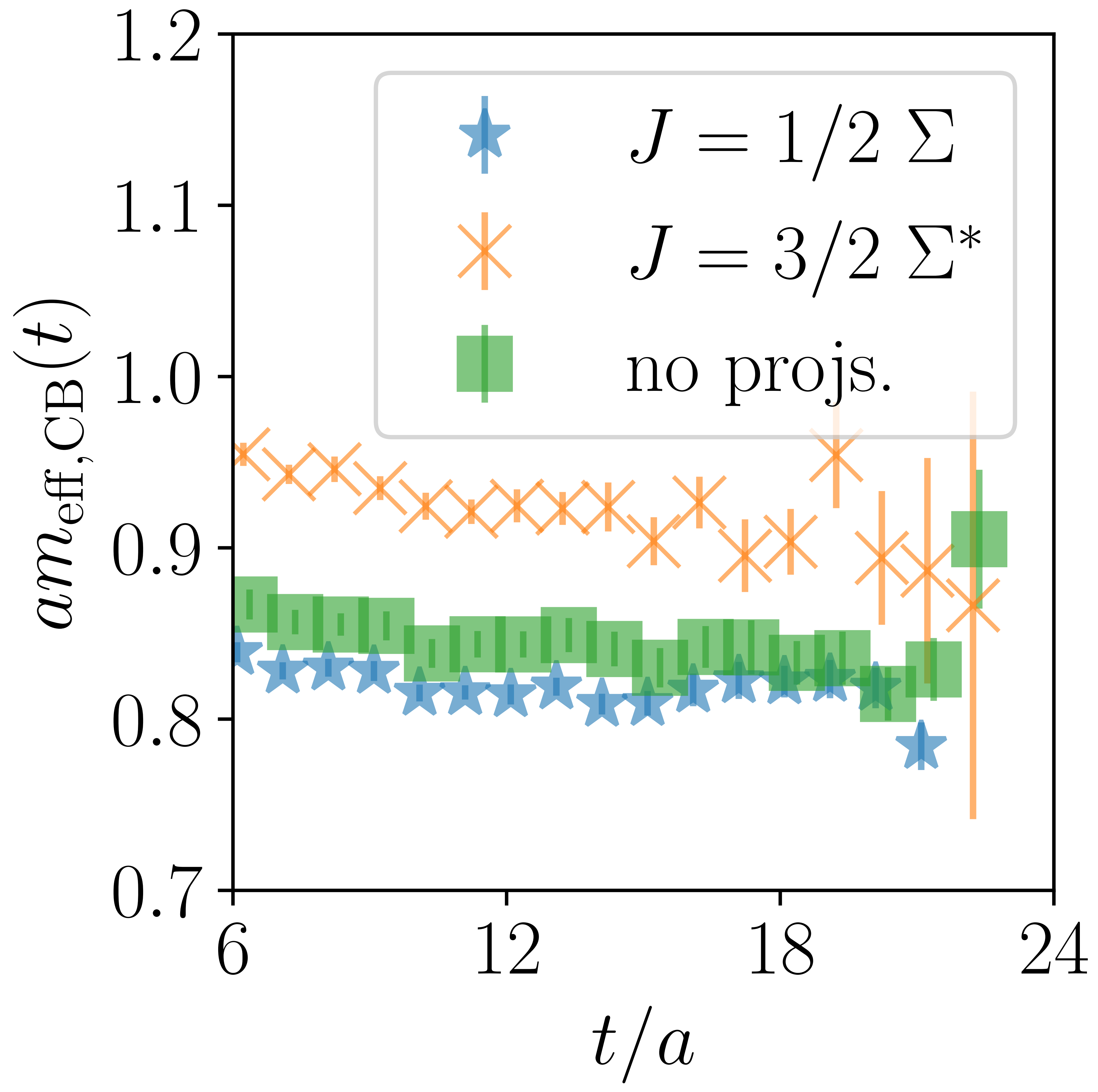
$$C_{ij}(t) = \sum_{\vec{x}} \left\langle \mathcal{O}^i{}_{\mathrm{CB}}(x)\bar{\mathcal{O}}^j{}_{\mathrm{CB}}(0) \right\rangle \text{ with } \mathcal{O}^i_{CB} = \left(\bar{\psi}\gamma^i\psi\right)\chi$$

$$\to C^{1/2}_{\Sigma}(t) = \mathrm{Tr}\left[\left(P^{1/2}\right)^{ij} C_{jk}(t)\right]$$

Results

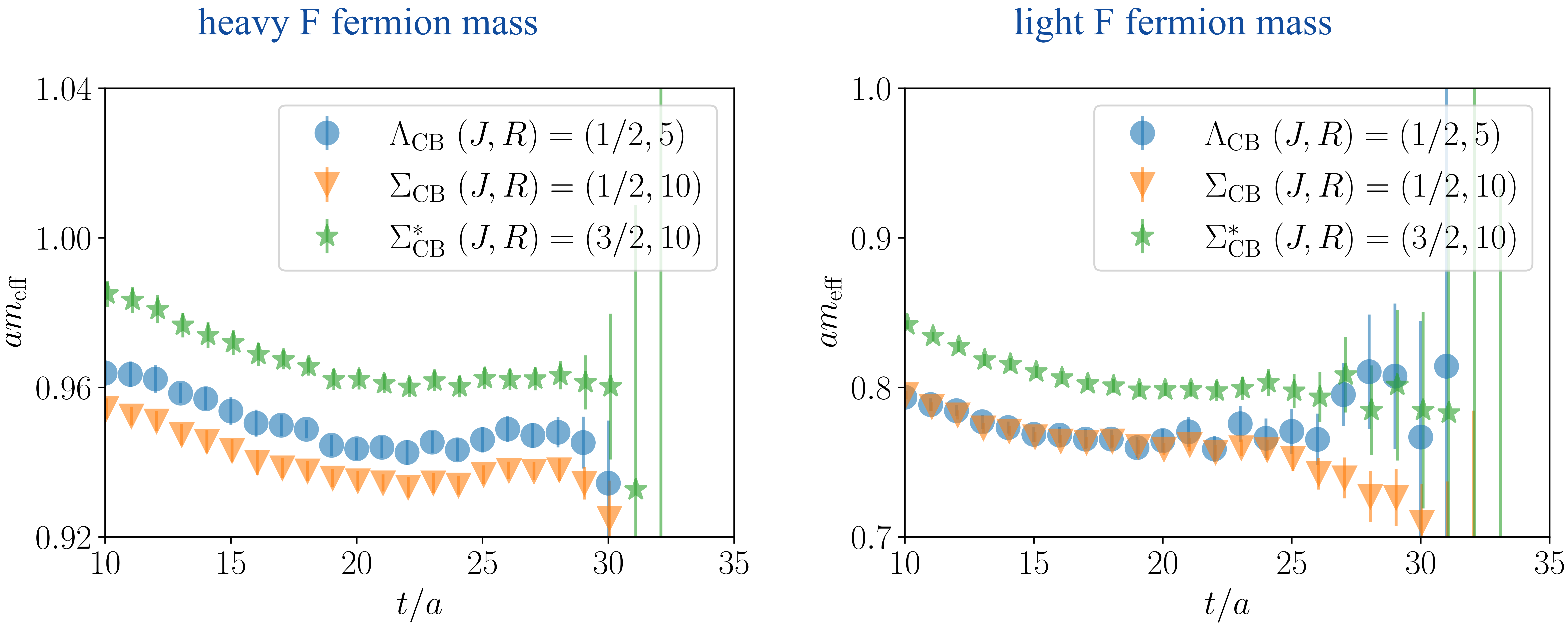
Projection-Spin

- Comparison of effective mass plot between two spin projected states and the state without spin projection.



Results

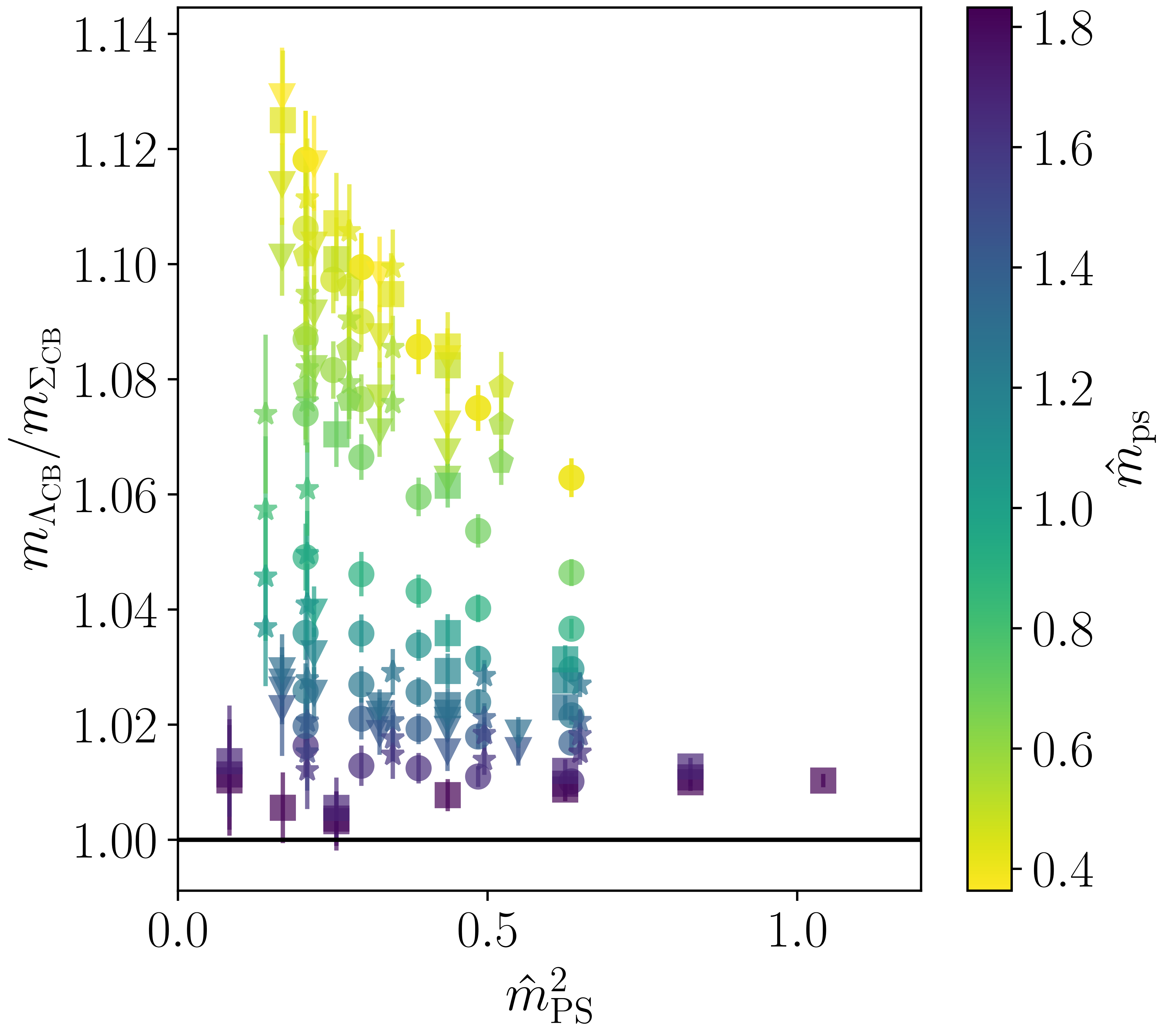
Mass hierarchy

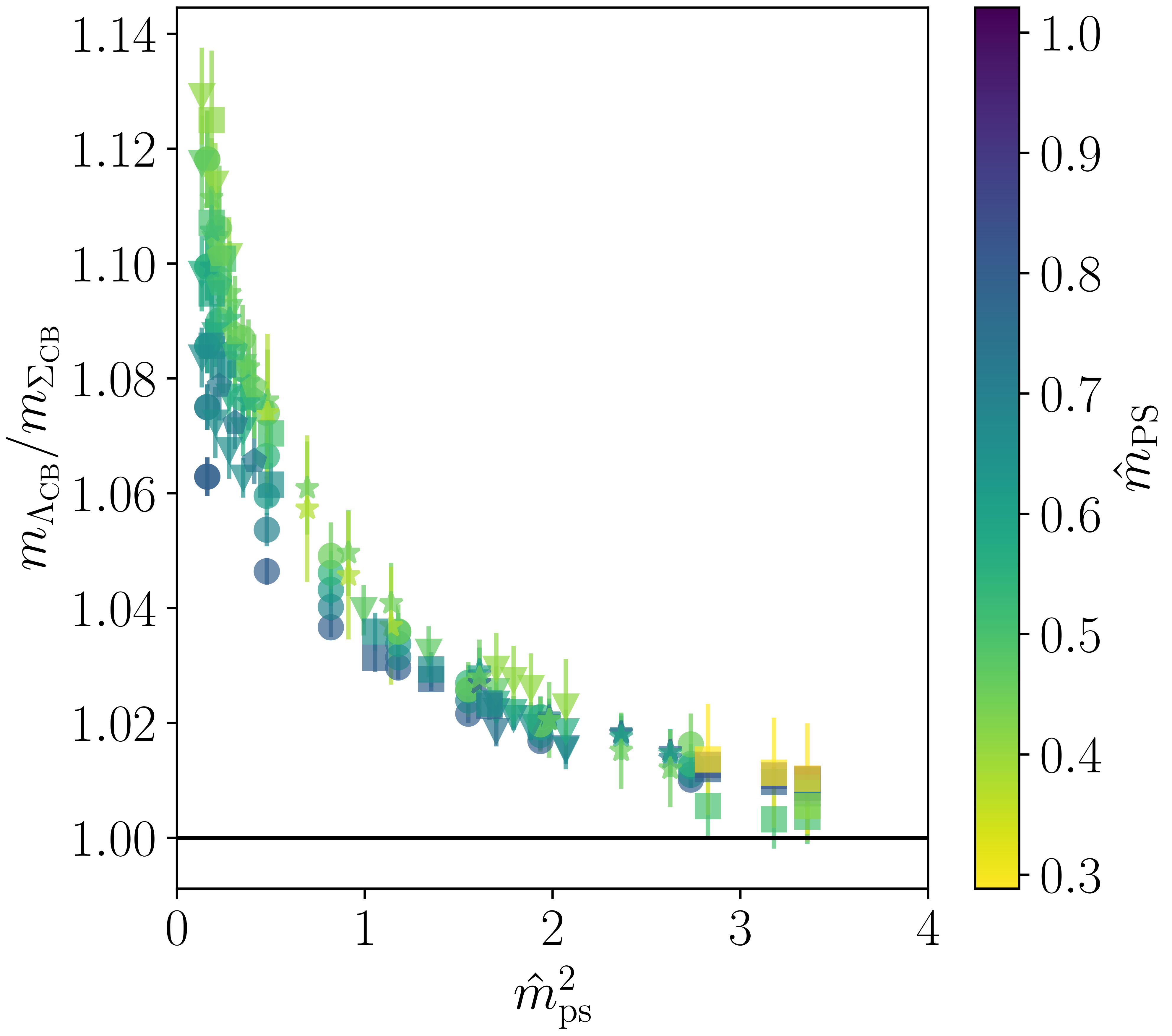


Effective mass plot of chimera baryons calculated with different F fermion masses, at fixed AS fermion mass. The lattice size is 60×48^3 with $\beta = 8.0$.

Results

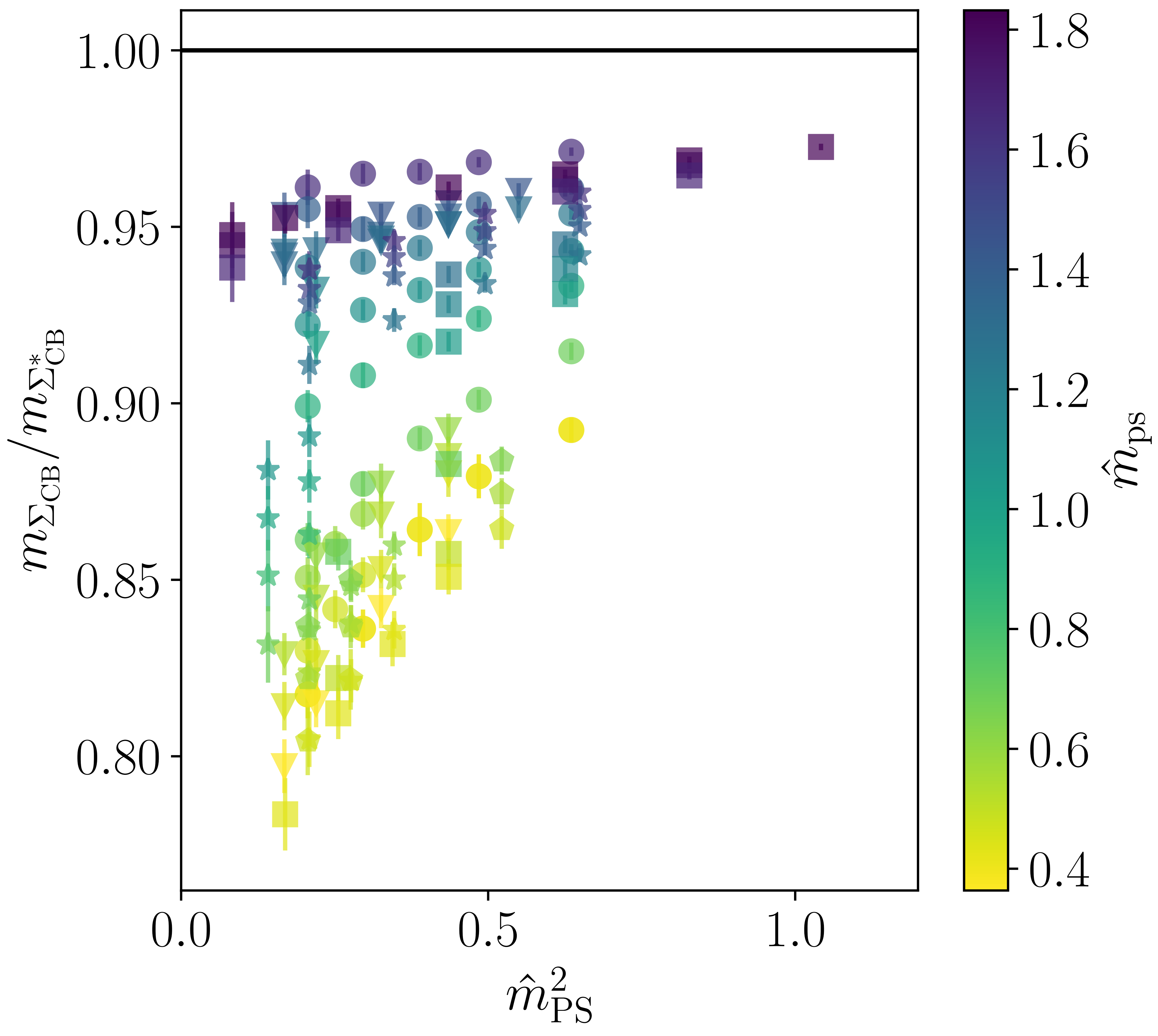
Mass hierarchy

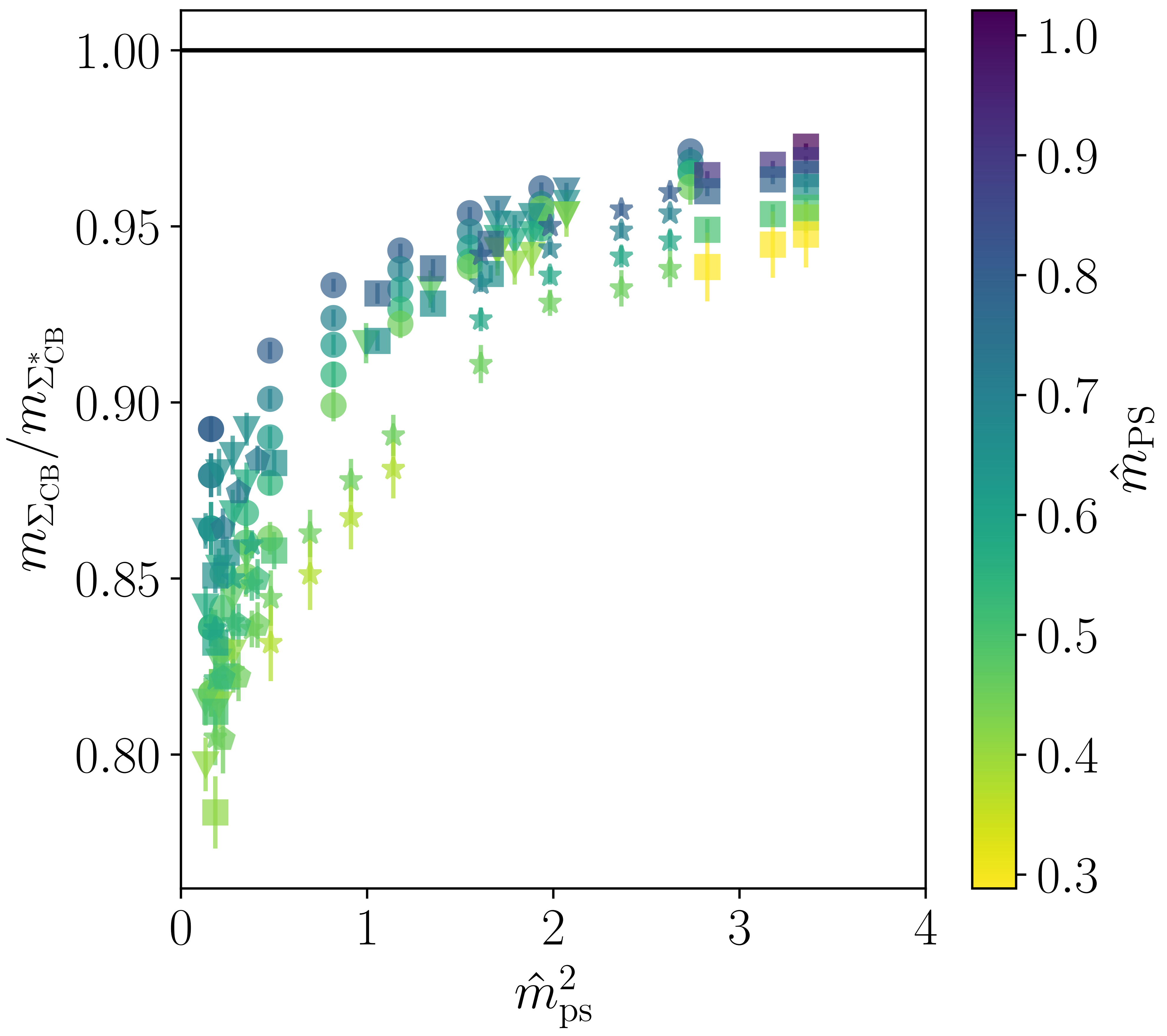




Results

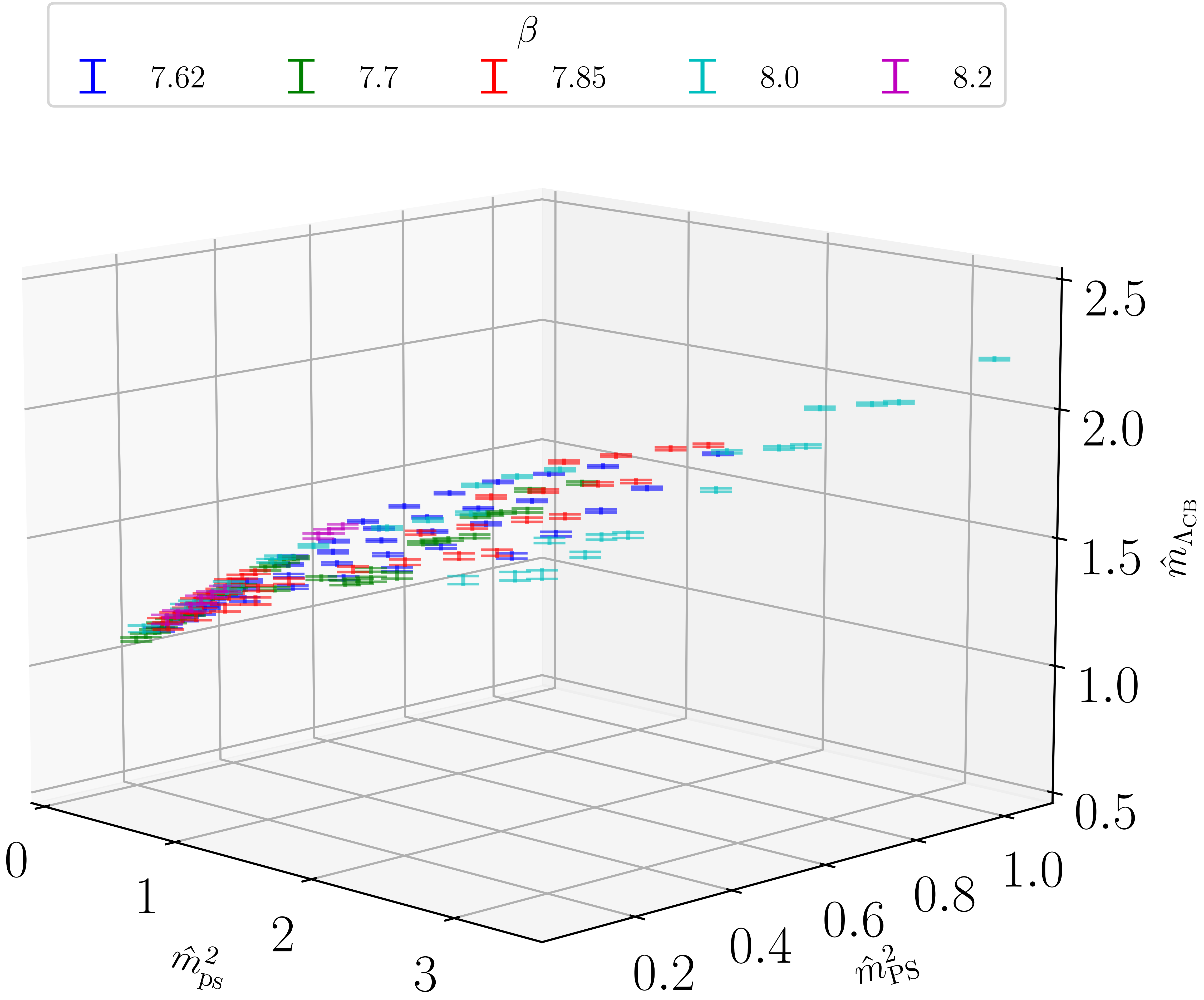
Mass hierarchy





Results

Fitting

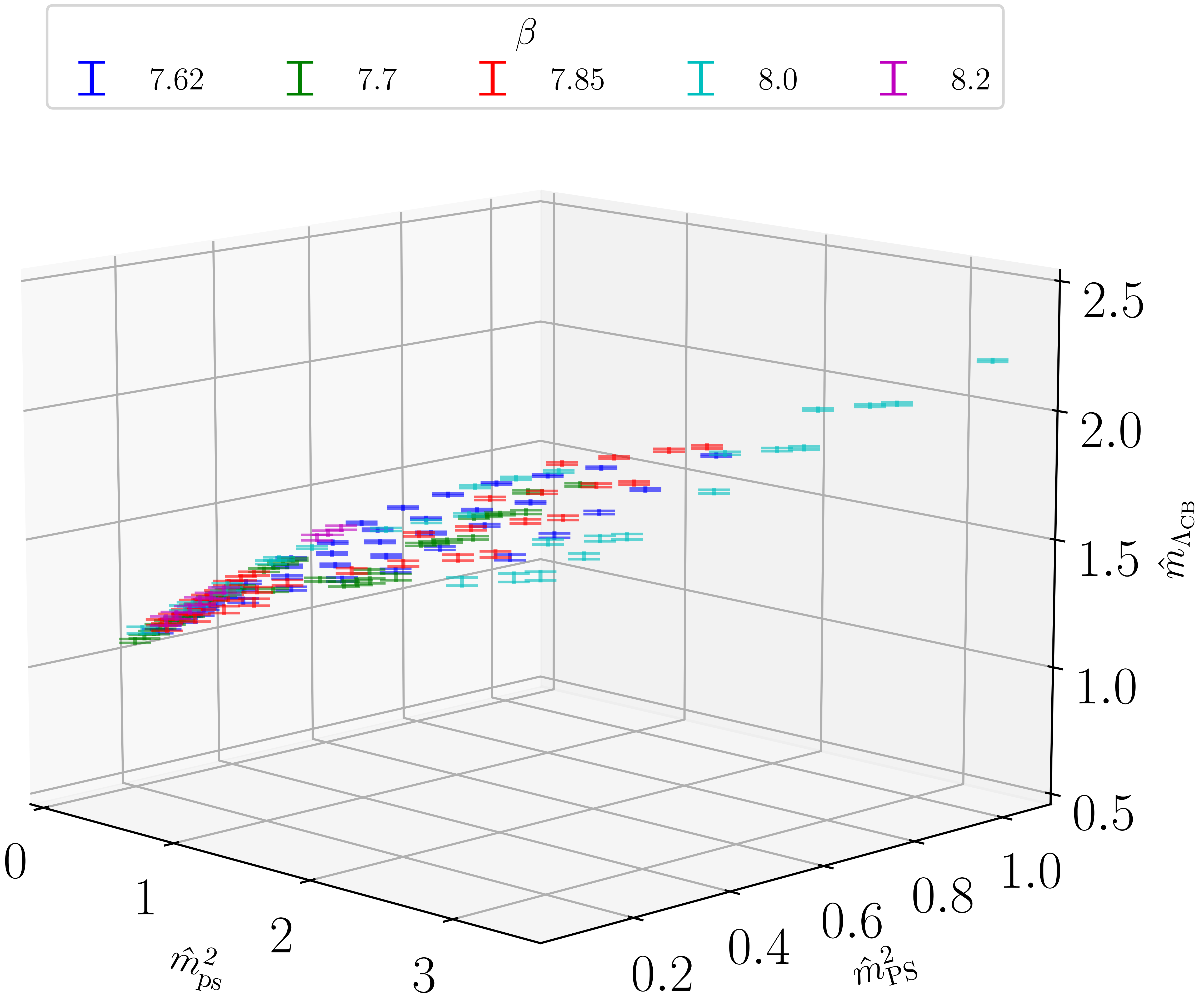


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = m^{\chi}_{CB} &+ F_2 \hat{m}^2_{\mathrm{PS}} + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a} \\
&+ F_3 \hat{m}^3_{\mathrm{PS}} + A_3 \hat{m}^3_{\mathrm{ps}} + L_{2F} \hat{a} \hat{m}^2_{\mathrm{PS}} + L_{2A} \hat{a} \hat{m}^2_{\mathrm{ps}} \\
&+ F_4 \hat{m}^4_{\mathrm{PS}} + A_4 \hat{m}^4_{\mathrm{ps}} + C_4 \hat{m}^2_{\mathrm{PS}} \hat{m}^2_{\mathrm{ps}}
\end{aligned}
$$



Results

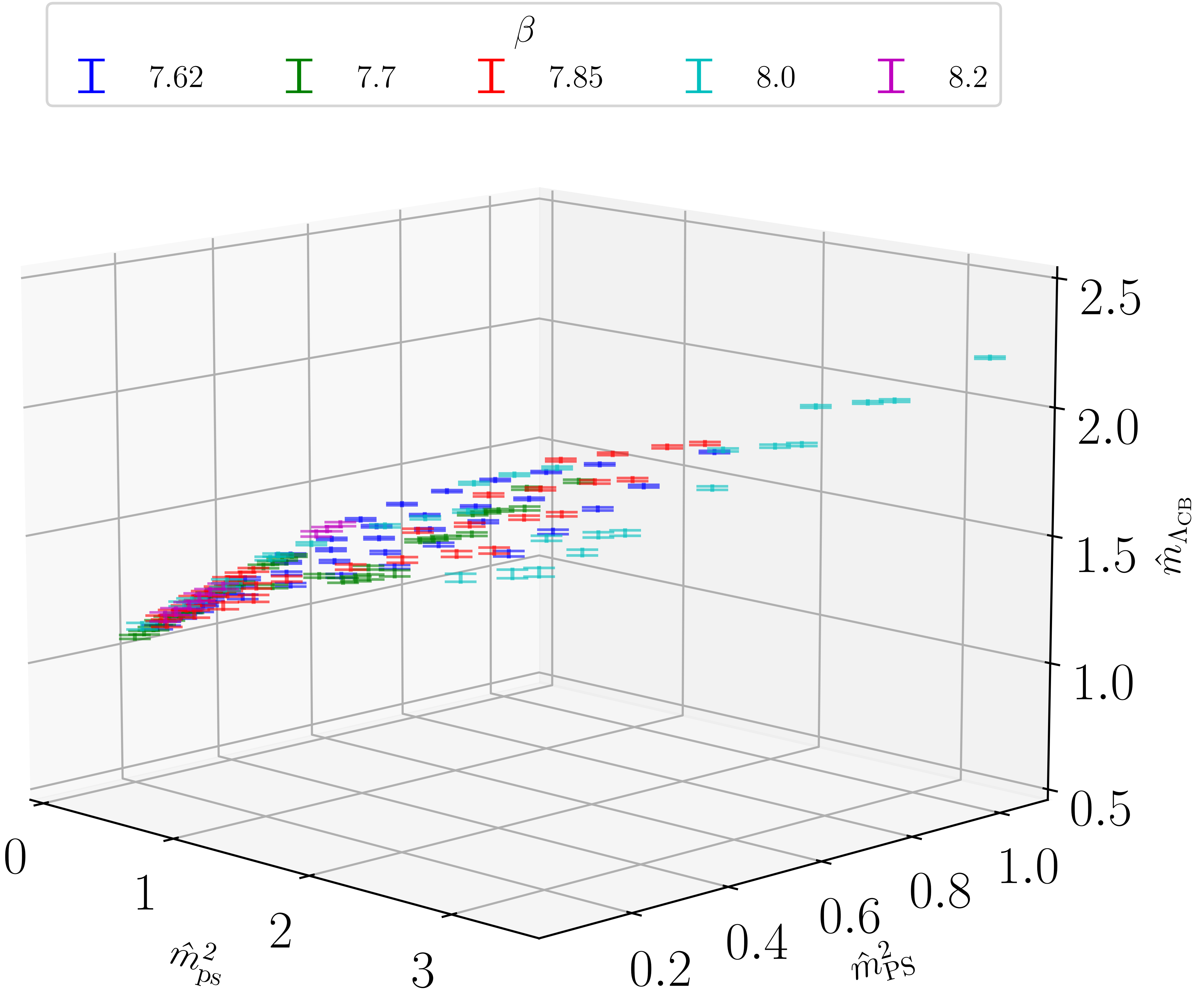
Fitting

- Apply tree-level baryon chiral perturbation theory

$$\begin{aligned} m_{\mathrm{CB}} = {} & m^{\chi}_{CB} + F_2 \hat{m}^2_{\mathrm{PS}} + A_2 {\hat{m}_{\mathrm{ps}}}^2 + L_1 \hat{a} \\ & + F_3 \hat{m}^3_{\mathrm{PS}} + A_3 \hat{m}^3_{\mathrm{ps}} + L_{2F} \hat{a} \hat{m}^2_{\mathrm{PS}} + L_{2A} \hat{a} \hat{m}^2_{\mathrm{ps}} \\ & + F_4 \hat{m}^4_{\mathrm{PS}} + A_4 \hat{m}^4_{\mathrm{ps}} + C_4 \hat{m}^2_{\mathrm{PS}} \hat{m}^2_{\mathrm{ps}} \end{aligned}$$

$\downarrow$

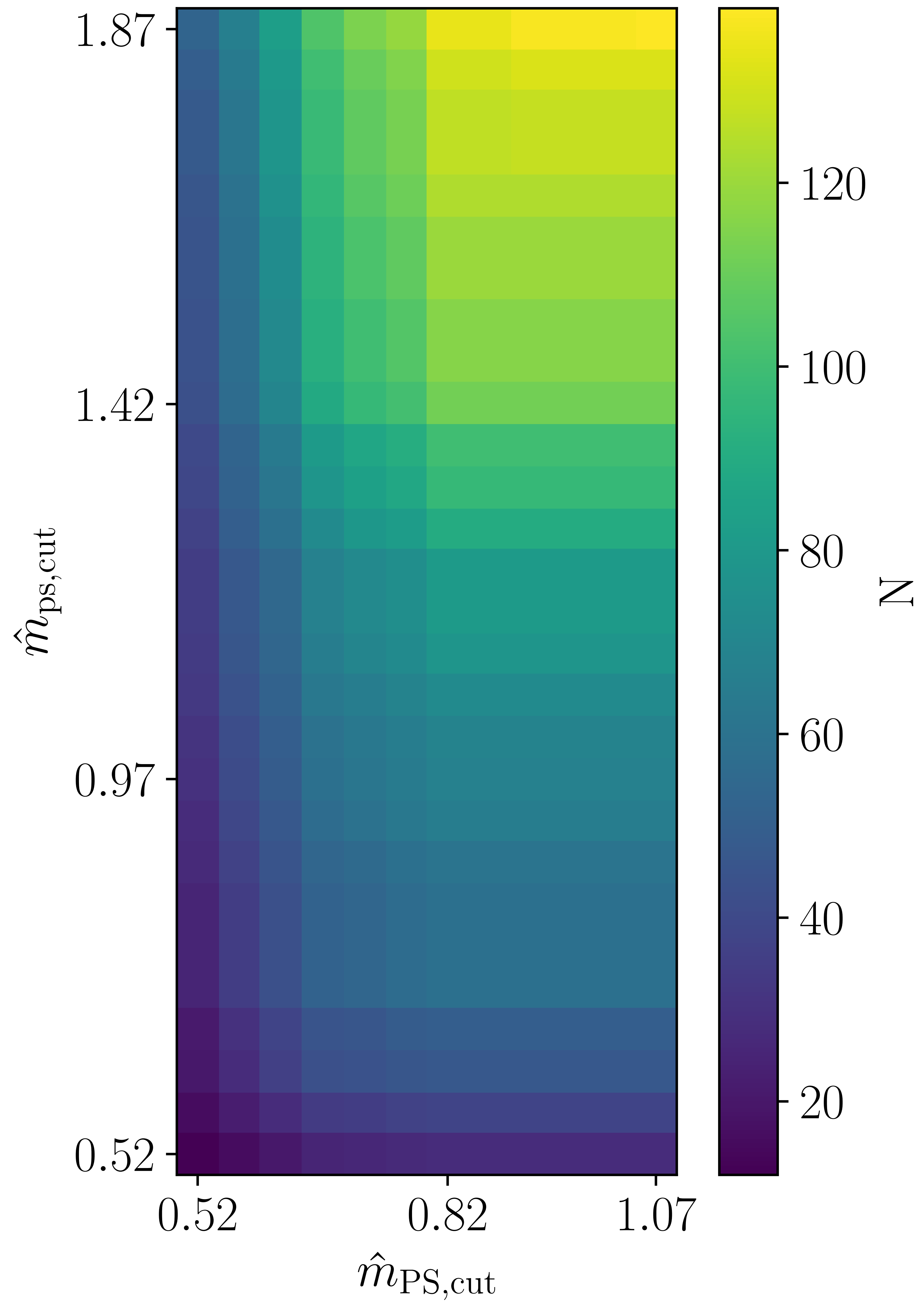
Returning large $\chi^2/N_{\mathrm{d.o.f.}}$



Results

Optimal search

- Try including different order of corrections
- Calculate AICs for each data set, and scan through all the possible cuts:
  - ➡ Fix the cut value for $\hat{m}_{\mathrm{PS}}$ and vary $\hat{m}_{\mathrm{ps}}$
  - ➡ Increase the fixed value of $\hat{m}_{\mathrm{PS}}$



Results

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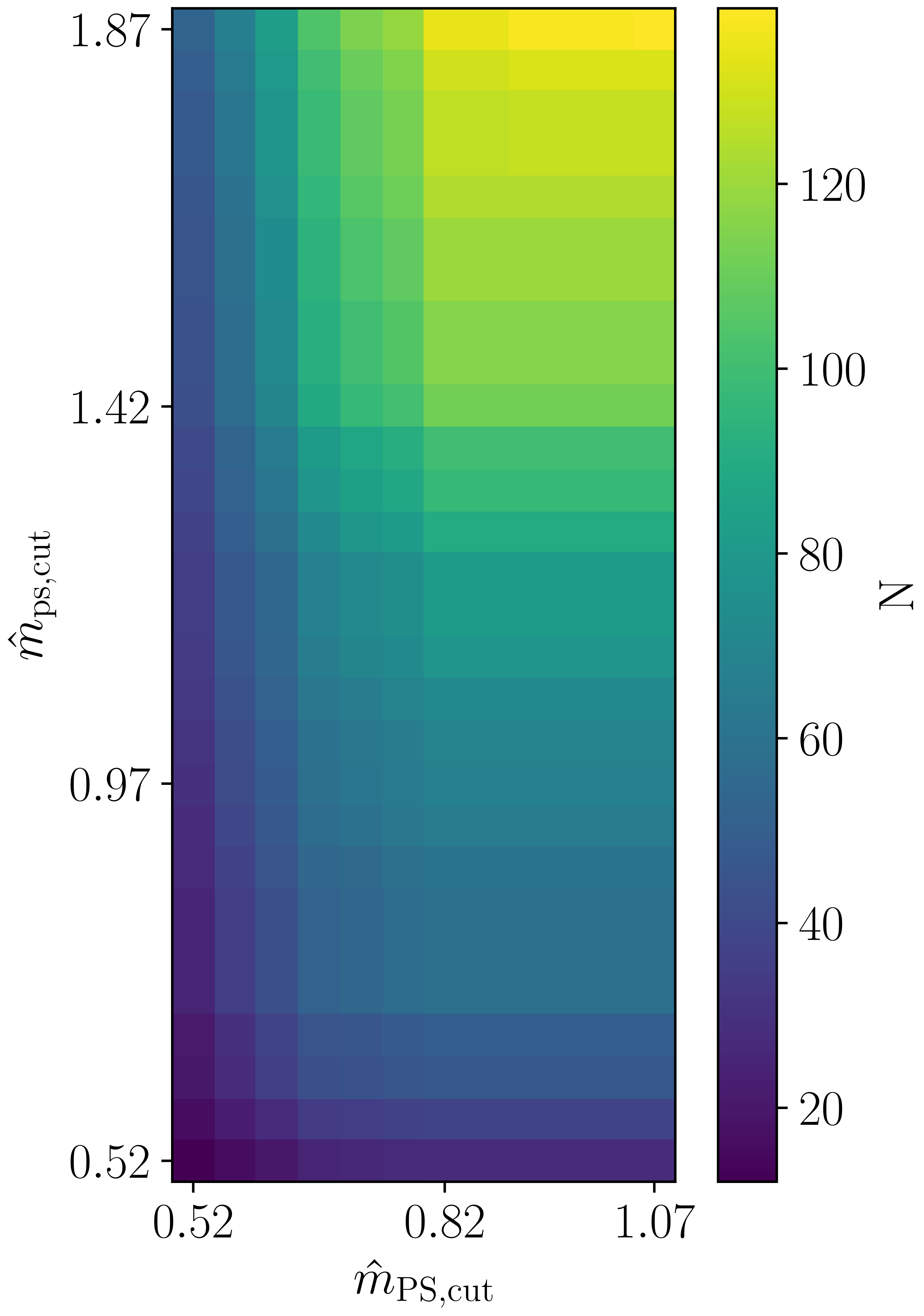
  - ➡ Fix the cut value for $\hat{m}_{\mathrm{PS}}$ and vary $\hat{m}_{\mathrm{ps}}$
  - ➡ Increase the fixed value of $\hat{m}_{\mathrm{PS}}$
- Goodness of a fit: Akaike information criterion (AIC)

$$\mathrm{AIC}(\mathrm{M}, N_{\mathrm{cut}}) \equiv \chi^2 + 2k + 2N_{\mathrm{cut}}$$

- Probability weight

$$W(\mathrm{M}, N_{\mathrm{cut}}) = \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{2}\mathrm{AIC}(\mathrm{M}, N_{\mathrm{cut}})\right]$$



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William I. Jay and Ethan T. Neil [2008.01069]

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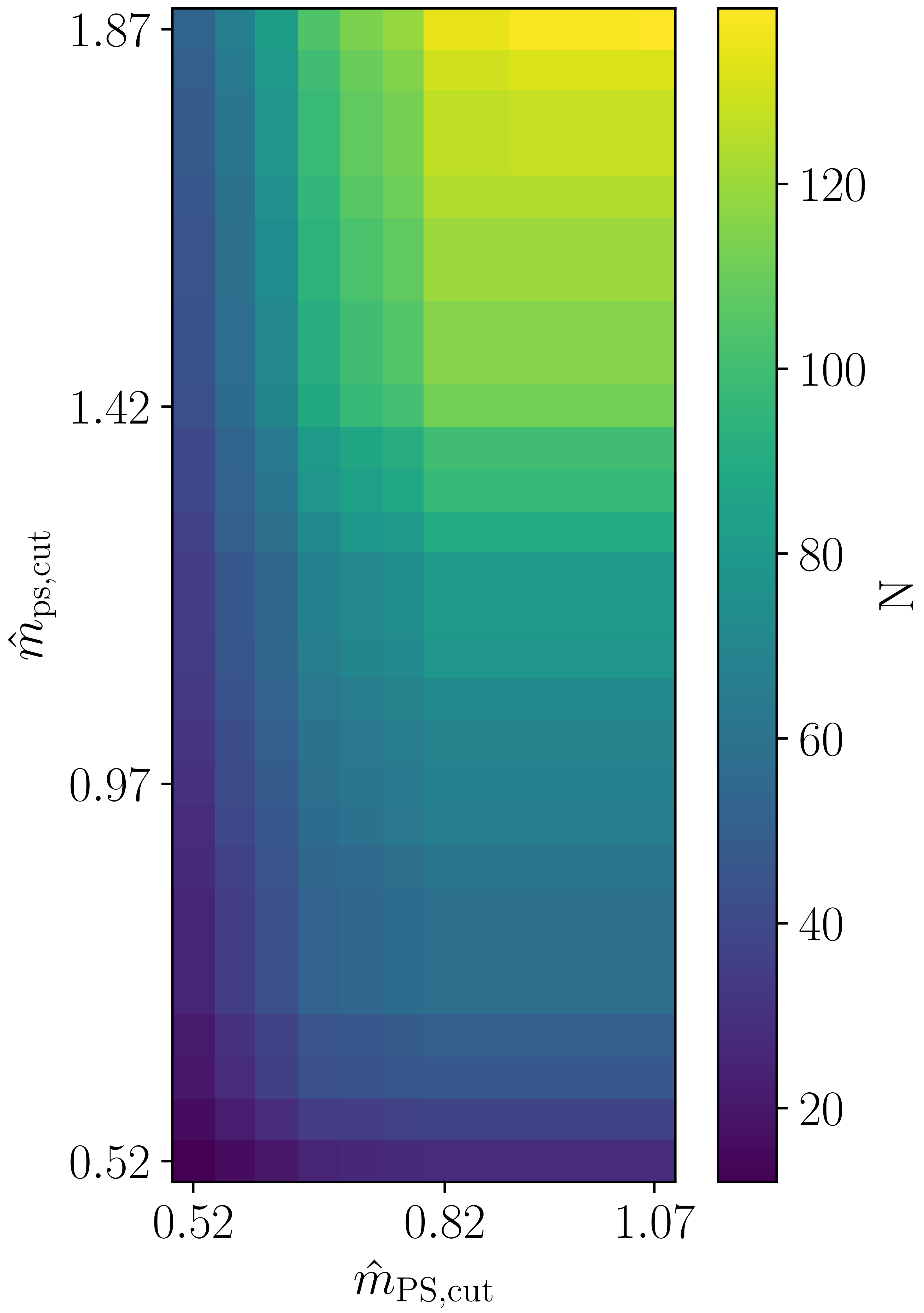
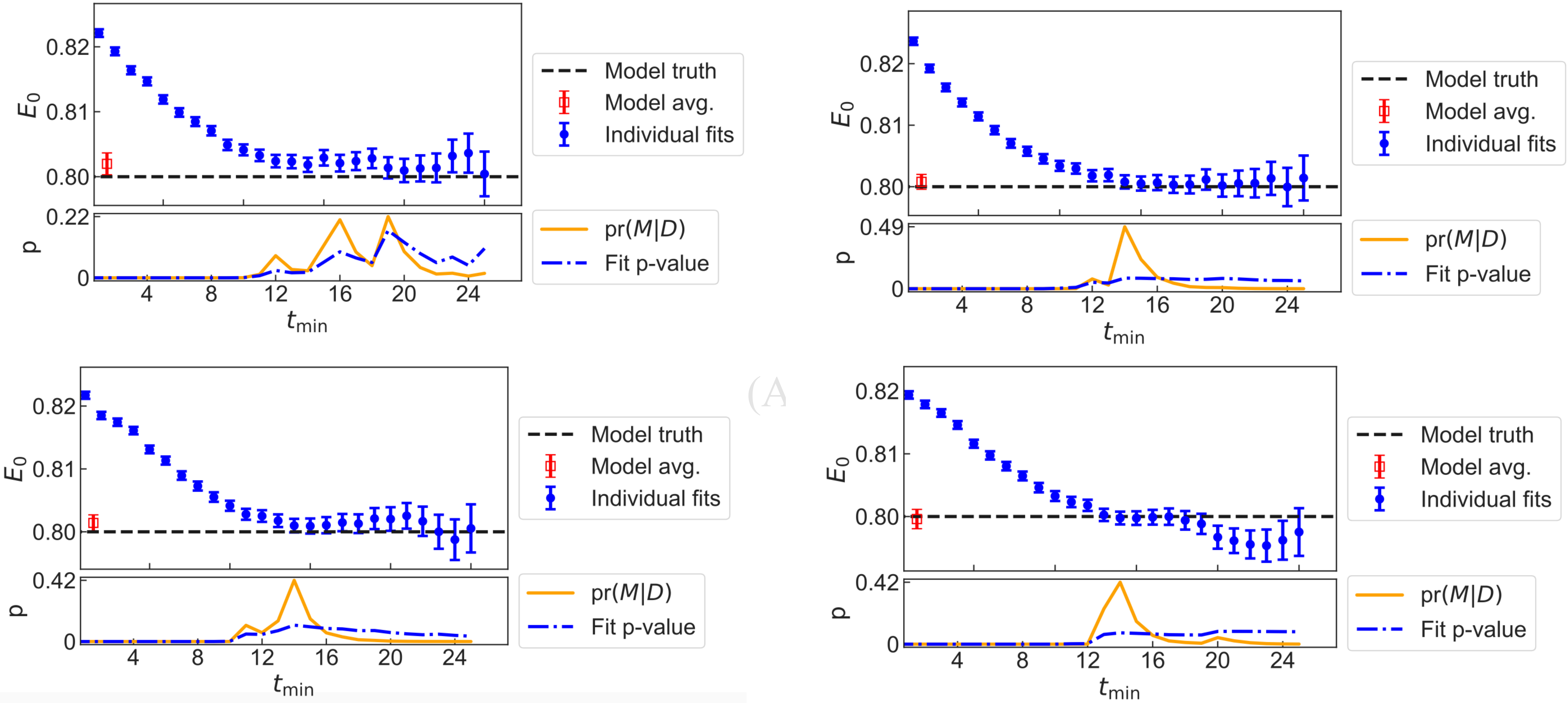


Results

William I. Jay and Ethan T. Neil [2008.01069]



Results

Optimal search

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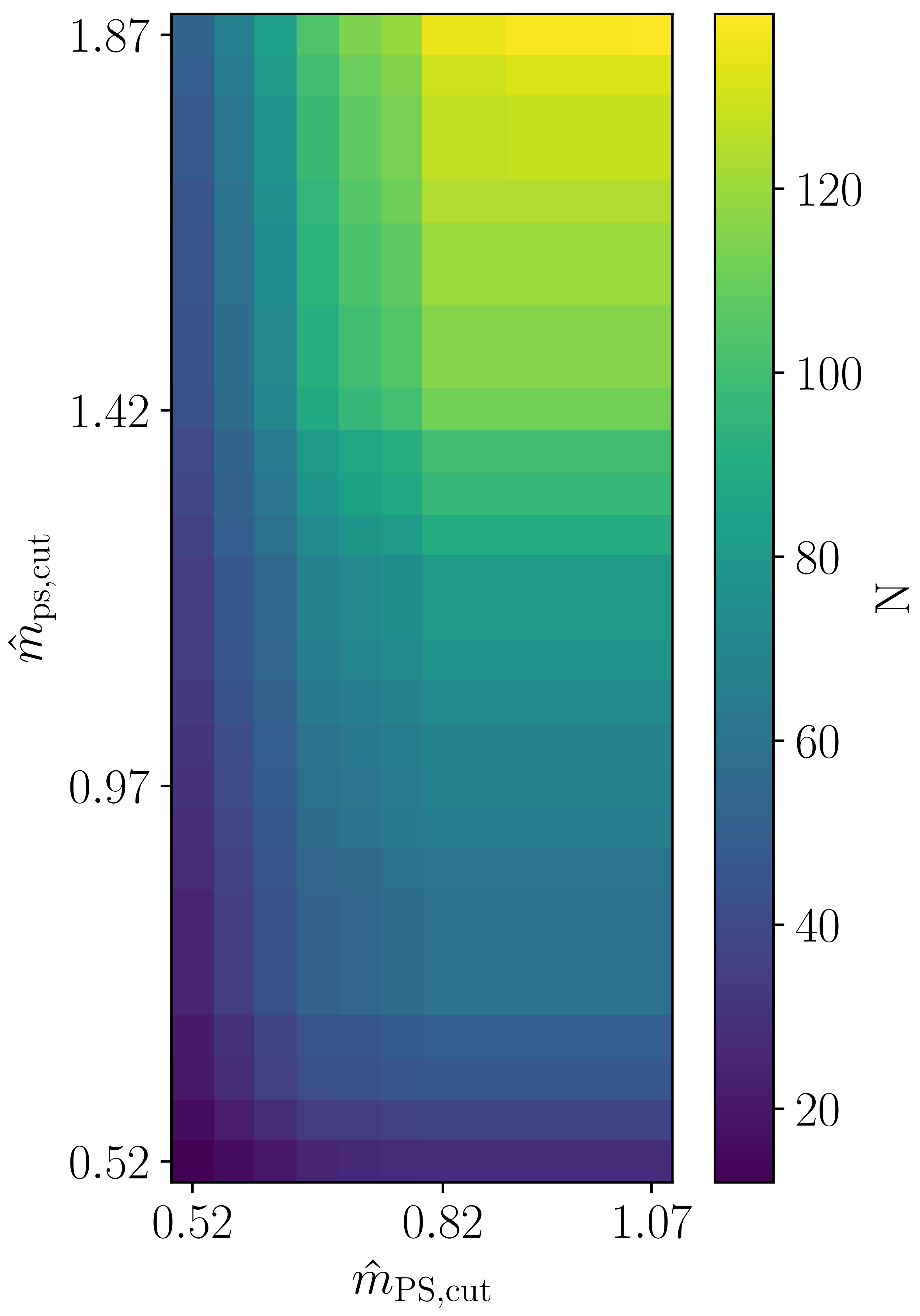


Results

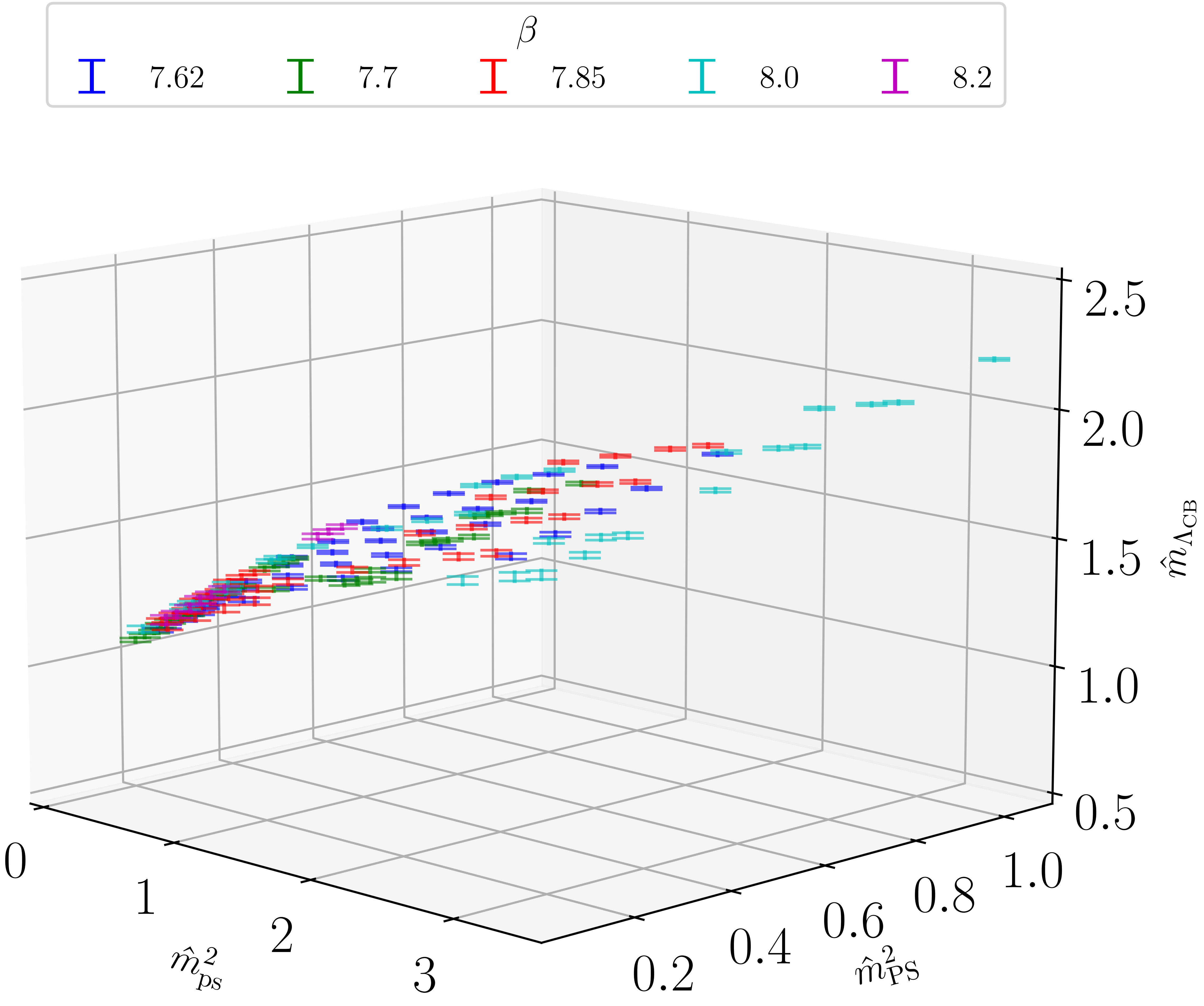
Fitting

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\rm CB} = m_{CB}^{\chi} &+ F_2\hat{m}_{\rm PS}^2 + A_2\hat{m}_{\rm ps}{}^2 + L_1\hat{a} \\
&+ F_3\hat{m}_{\rm PS}^3 + A_3\hat{m}_{\rm ps}^3 + L_{2F}\hat{a}\hat{m}_{\rm PS}^2 + L_{2A}\hat{a}\hat{m}_{\rm ps}^2 \\
&+ F_4\hat{m}_{\rm PS}^4 + A_4\hat{m}_{\rm ps}^4 + C_4\hat{m}_{\rm PS}^2\hat{m}_{\rm ps}^2
\end{aligned}
$$

↓

Still returning large $\chi^2/N_{\rm d.o.f.}$

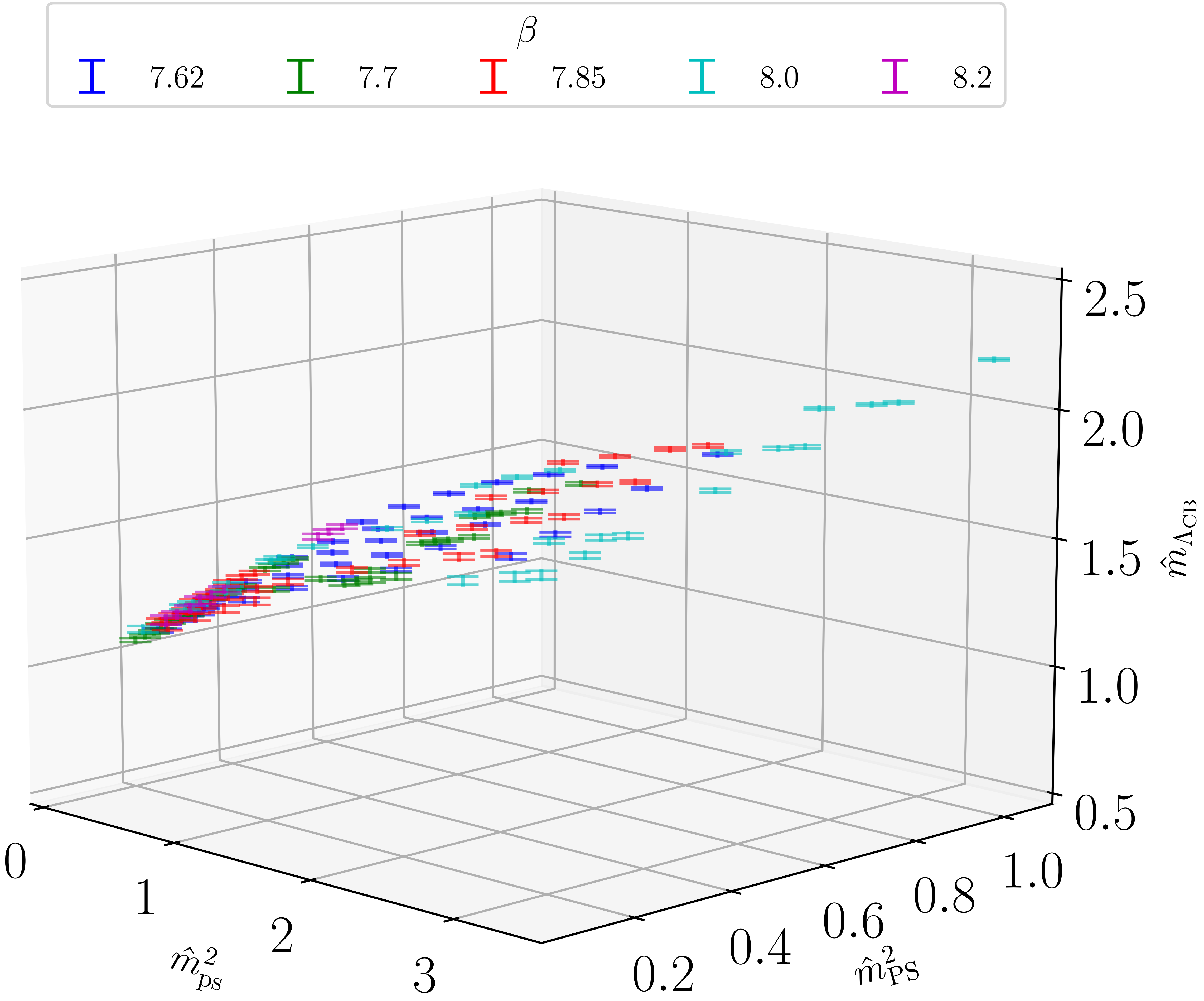


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$m_{\mathrm{CB}} = m^{\chi}_{CB} + F_2\hat{m}^2_{\mathrm{PS}} + A_2{\hat{m}_{\mathrm{ps}}}^2 + L_1\hat{a}$$
$$+ F_3\hat{m}^3_{\mathrm{PS}} + A_3\hat{m}^3_{\mathrm{ps}} + L_{2F}\hat{a}\hat{m}^2_{\mathrm{PS}} + L_{2A}\hat{a}\hat{m}^2_{\mathrm{ps}}$$
$$+ F_4\hat{m}^4_{\mathrm{PS}} + A_4\hat{m}^4_{\mathrm{ps}} + C_4\hat{m}^2_{\mathrm{PS}}\hat{m}^2_{\mathrm{ps}}$$

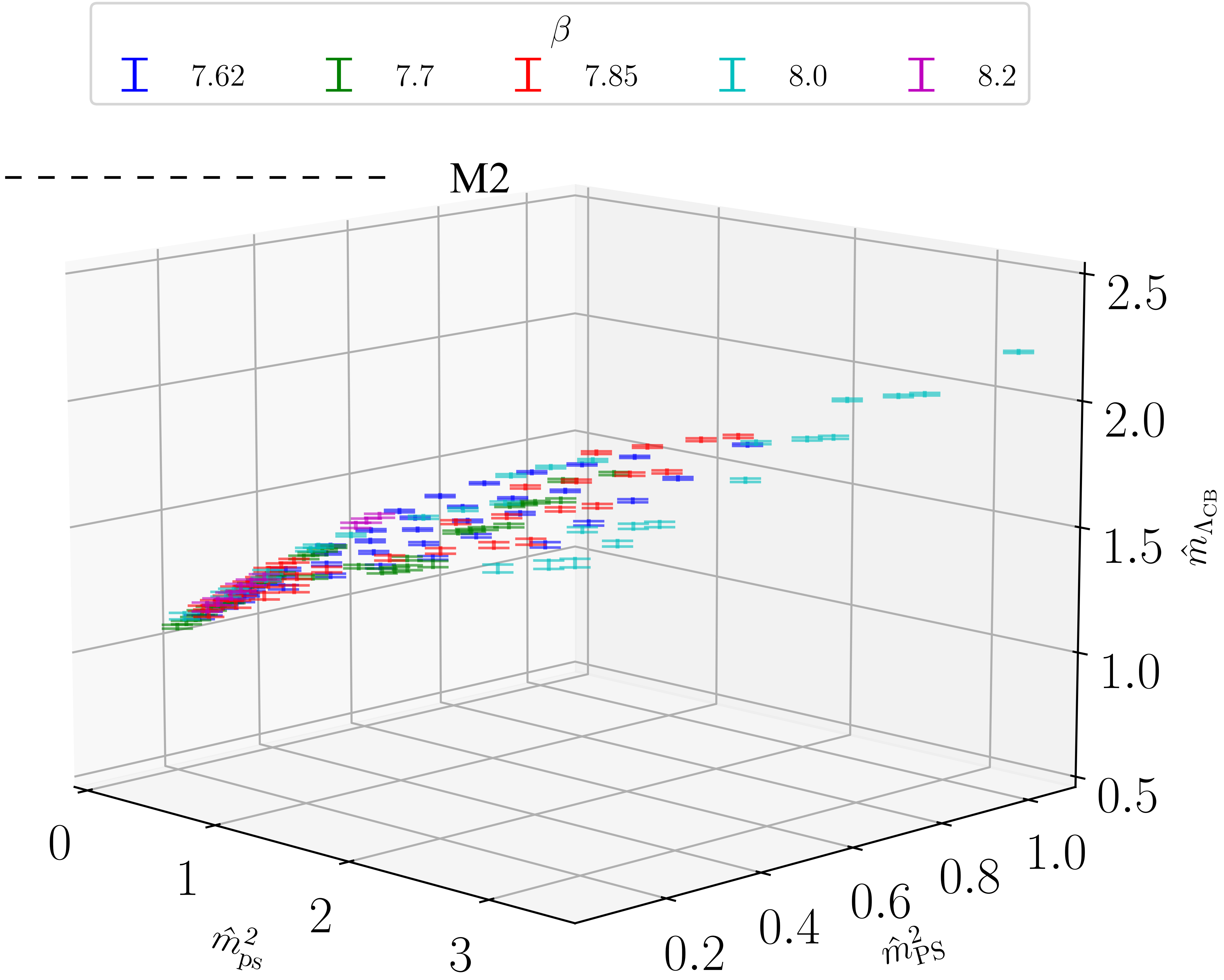


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$m_{\rm CB} = m^{\chi}_{CB} + F_2 \hat{m}^2_{\rm PS} + A_2 \hat{m}_{\rm ps}{}^2 + L_1 \hat{a} + F_3 \hat{m}^3_{\rm PS} + A_3 \hat{m}^3_{\rm ps} + L_{2F} \hat{a} \hat{m}^2_{\rm PS} + L_{2A} \hat{a} \hat{m}^2_{\rm ps} + F_4 \hat{m}^4_{\rm PS} + A_4 \hat{m}^4_{\rm ps} + C_4 \hat{m}^2_{\rm PS} \hat{m}^2_{\rm ps}$$

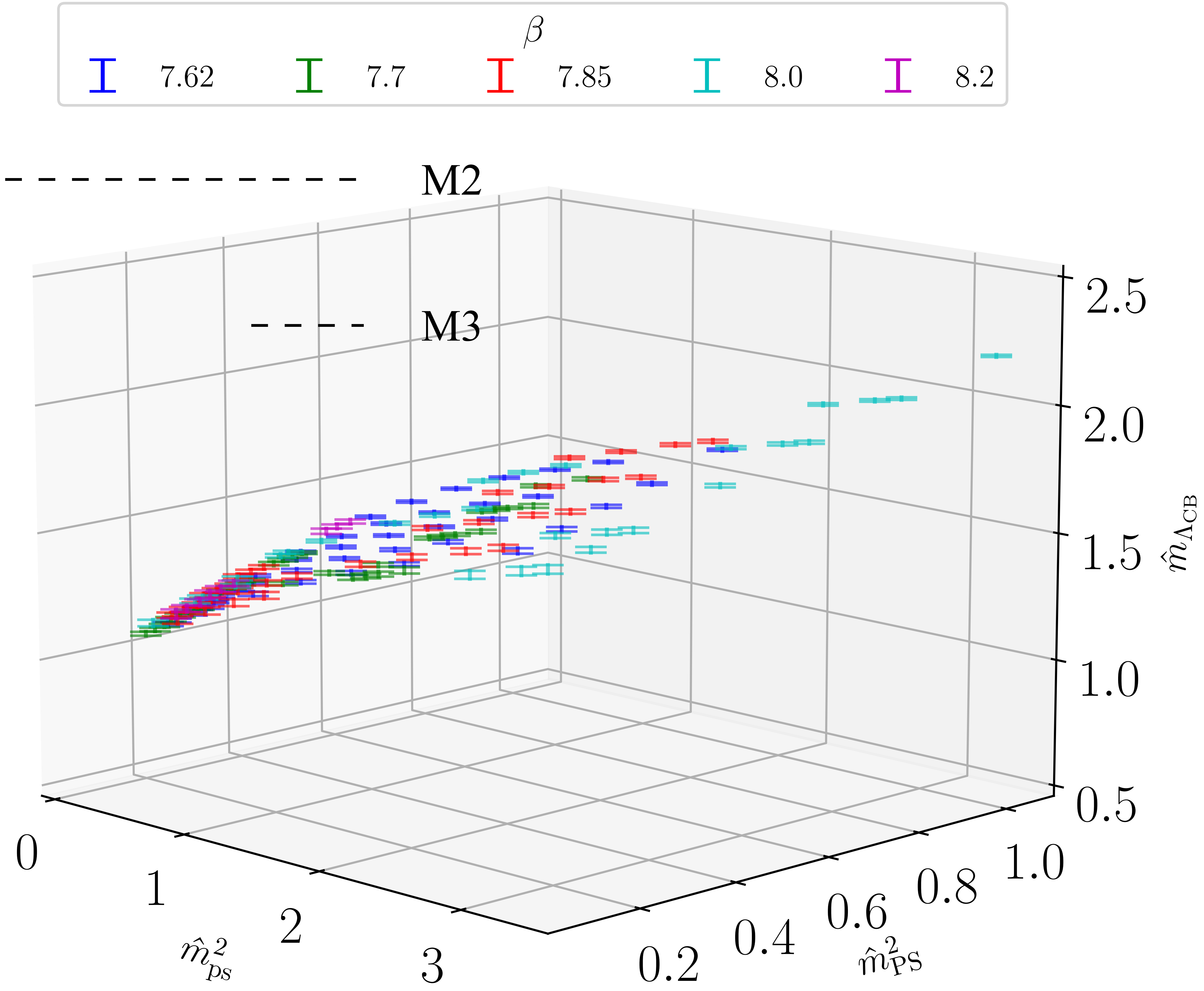


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = {} & m^{\chi}_{CB} + F_2\hat{m}^2_{\mathrm{PS}} + A_2\hat{m}_{\mathrm{ps}}{}^2 + L_1\hat{a} \\
& + F_3\hat{m}^3_{\mathrm{PS}} + A_3\hat{m}^3_{\mathrm{ps}} + L_{2F}\hat{a}\hat{m}^2_{\mathrm{PS}} + L_{2A}\hat{a}\hat{m}^2_{\mathrm{ps}} \\
& + F_4\hat{m}^4_{\mathrm{PS}} + A_4\hat{m}^4_{\mathrm{ps}} + C_4\hat{m}^2_{\mathrm{PS}}\hat{m}^2_{\mathrm{ps}}
\end{aligned}
$$



Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$m_{\mathrm{CB}} = m^{\chi}_{CB} + F_2 \hat{m}^2_{\mathrm{PS}} + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a}$$

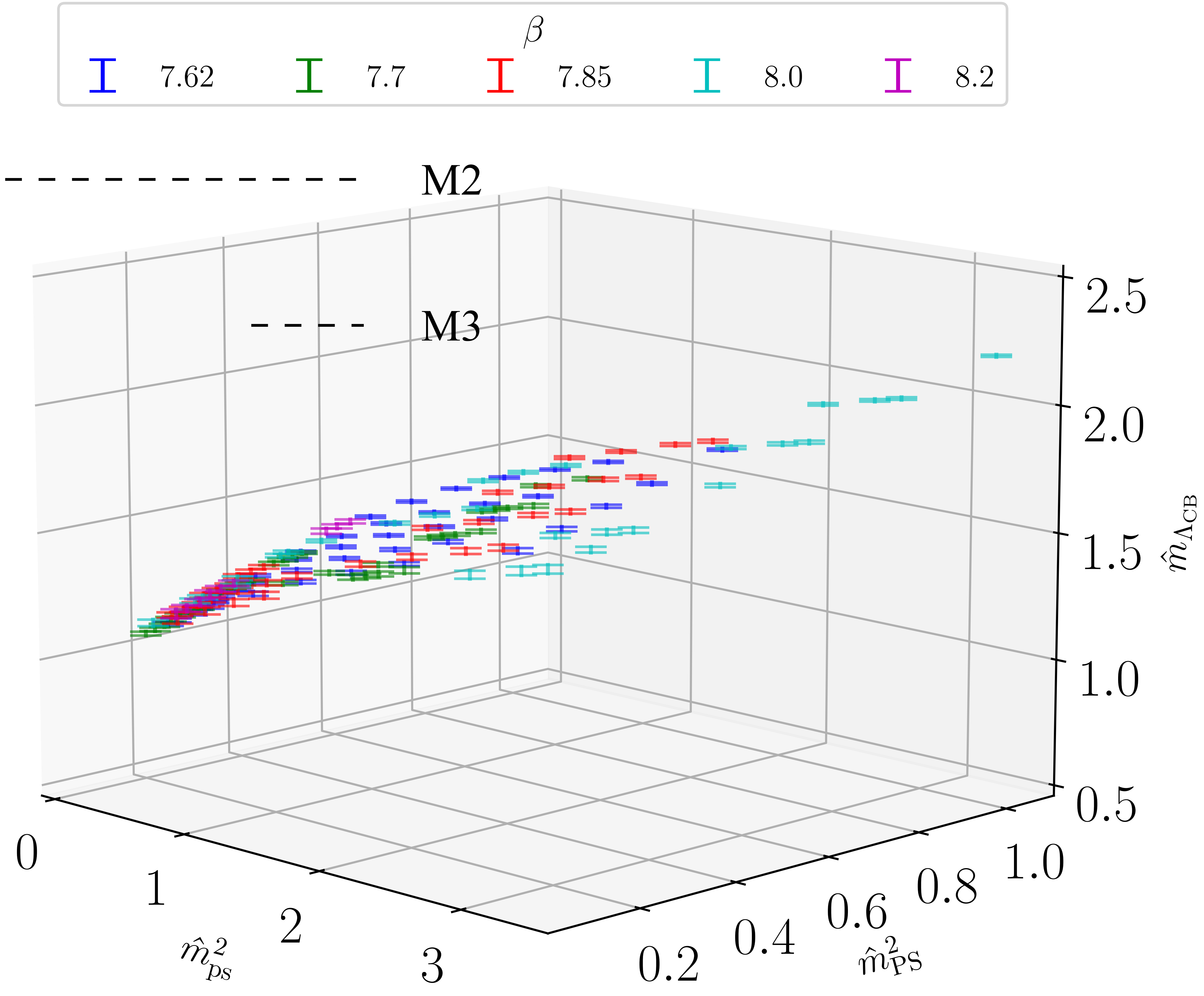
M2

$$+ F_3 \hat{m}^3_{\mathrm{PS}} + A_3 \hat{m}^3_{\mathrm{ps}} + L_{2F} \hat{a} \hat{m}^2_{\mathrm{PS}} + L_{2A} \hat{a} \hat{m}^2_{\mathrm{ps}}$$

M3

$$+ F_4 \hat{m}^4_{\mathrm{PS}} + A_4 \hat{m}^4_{\mathrm{ps}} + C_4 \hat{m}^2_{\mathrm{PS}} \hat{m}^2_{\mathrm{ps}}$$

MF4

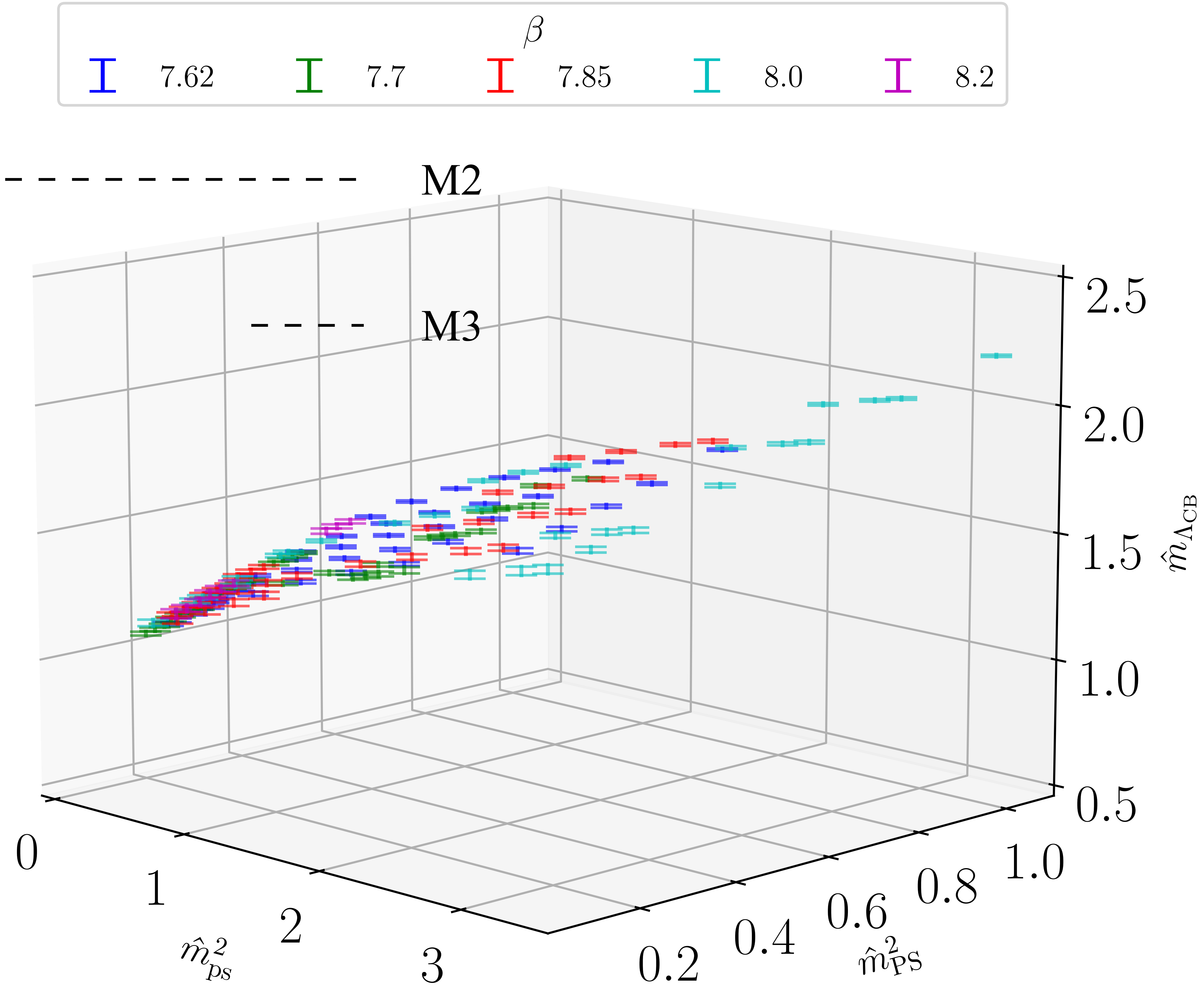


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = {} & m^{\chi}_{CB} + F_2\hat{m}^2_{\mathrm{PS}} + A_2\hat{m}_{\mathrm{ps}}{}^2 + L_1\hat{a} \qquad \text{- M2} \\
& + F_3\hat{m}^3_{\mathrm{PS}} + A_3\hat{m}^3_{\mathrm{ps}} + L_{2F}\hat{a}\hat{m}^2_{\mathrm{PS}} + L_{2A}\hat{a}\hat{m}^2_{\mathrm{ps}} \qquad \text{- - - - M3} \\
& \underbrace{+ F_4\hat{m}^4_{\mathrm{PS}}}_{\text{MF4}} \underbrace{+ A_4\hat{m}^4_{\mathrm{ps}}}_{\text{MA4}} + C_4\hat{m}^2_{\mathrm{PS}}\hat{m}^2_{\mathrm{ps}}
\end{aligned}
$$

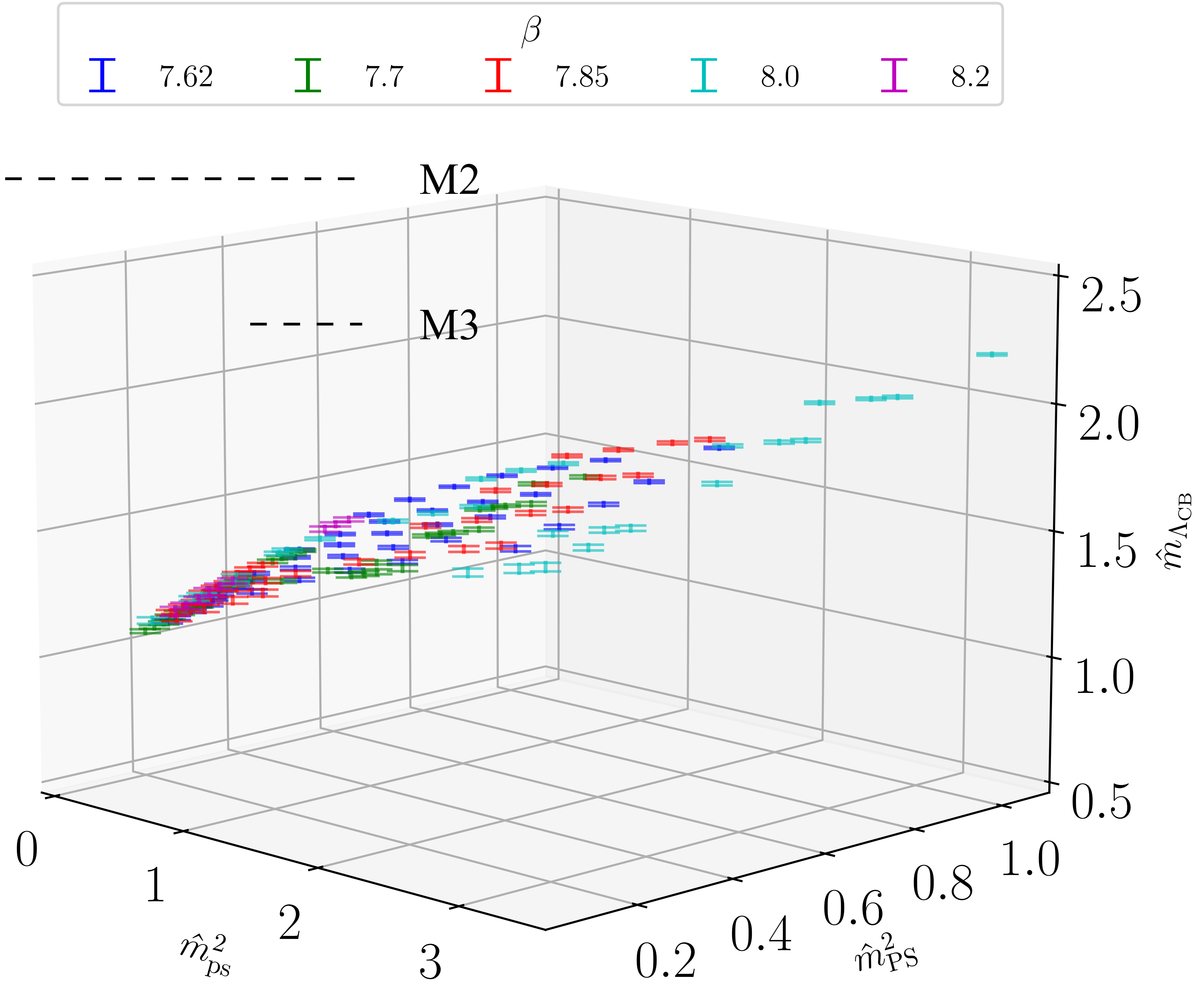


Results

Fitting

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = {} & m^{\chi}_{CB} + F_2 \hat{m}^2_{\mathrm{PS}} + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a} & \text{- M2} \\
& + F_3 \hat{m}^3_{\mathrm{PS}} + A_3 \hat{m}^3_{\mathrm{ps}} + L_{2F} \hat{a} \hat{m}^2_{\mathrm{PS}} + L_{2A} \hat{a} \hat{m}^2_{\mathrm{ps}} & \text{- - - M3} \\
& \underbrace{+ F_4 \hat{m}^4_{\mathrm{PS}}}_{\text{MF4}} \underbrace{+ A_4 \hat{m}^4_{\mathrm{ps}}}_{\text{MA4}} \underbrace{+ C_4 \hat{m}^2_{\mathrm{PS}} \hat{m}^2_{\mathrm{ps}}}_{\text{MC4}} &
\end{aligned}
$$



Results

Fittings of $\Lambda_{\rm CB}$

- Apply tree level baryon chiral perturbation theory

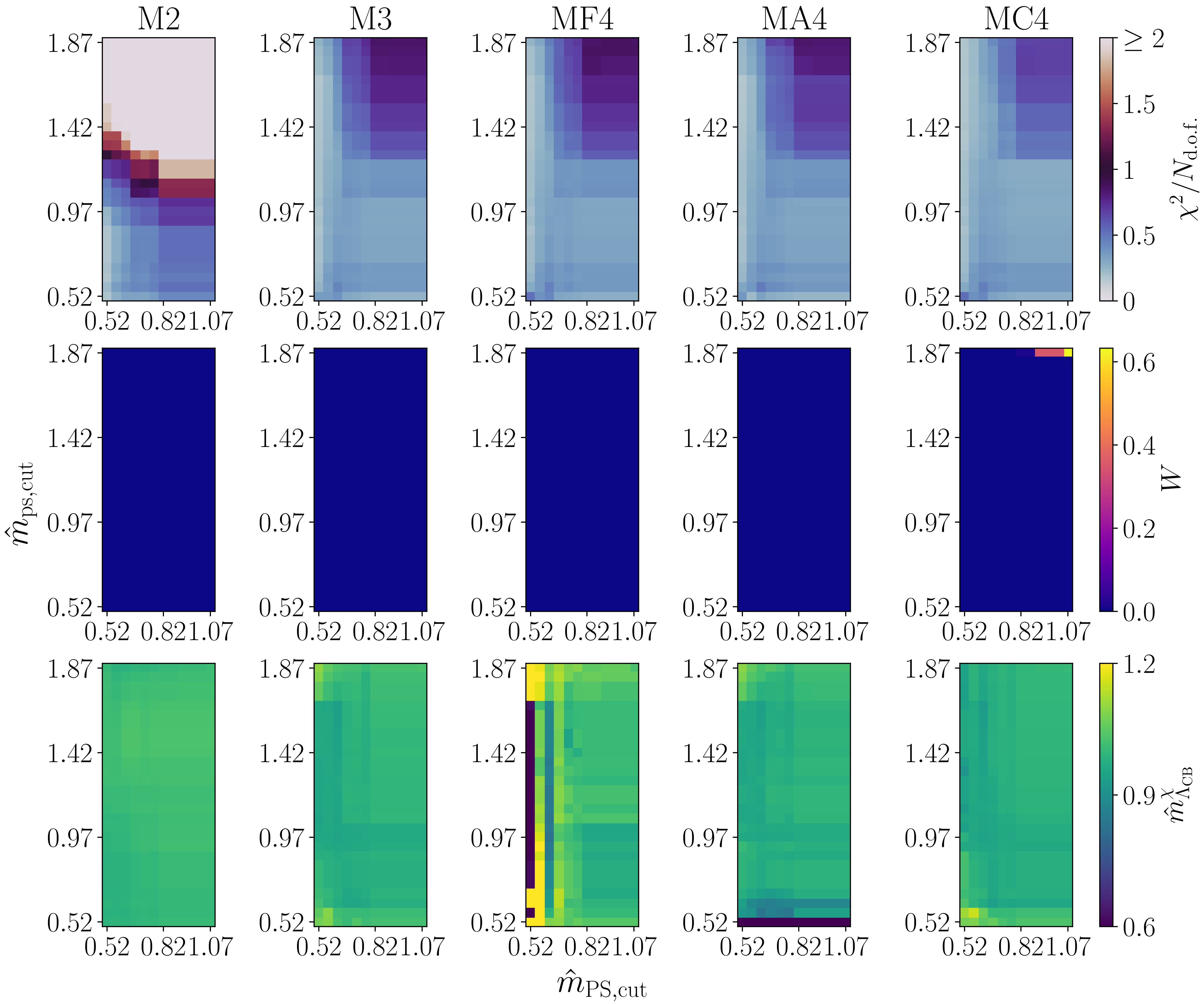
M2 $$m_{\rm CB} = m^{\chi}_{CB} + F_2 \hat{m}_{\rm PS}^2 + A_2 \hat{m}_{\rm ps}{}^2 + L_1 \hat{a}$$

M3 $$+ F_3 \hat{m}_{\rm PS}^3 + A_3 \hat{m}_{\rm ps}^3 + L_{2F} \hat{a} \hat{m}_{\rm PS}^2 + L_{2A} \hat{a} \hat{m}_{\rm ps}^2$$

$$\underset{\text{MF4}}{\underline{+ F_4 \hat{m}_{\rm PS}^4}} \underset{\text{MA4}}{\underline{+ A_4 \hat{m}_{\rm ps}^4}} \underset{\text{M2C}}{\underline{+ C_4 \hat{m}_{\rm PS}^2 \hat{m}_{\rm ps}^2}}$$

- probability weight

$$W(\mathrm{M}, N_{\rm cut}) = \frac{1}{\mathcal{N}} \exp\left[-\frac{1}{2} \mathrm{AIC}(\mathrm{M}, N_{\rm cut})\right]$$



Results

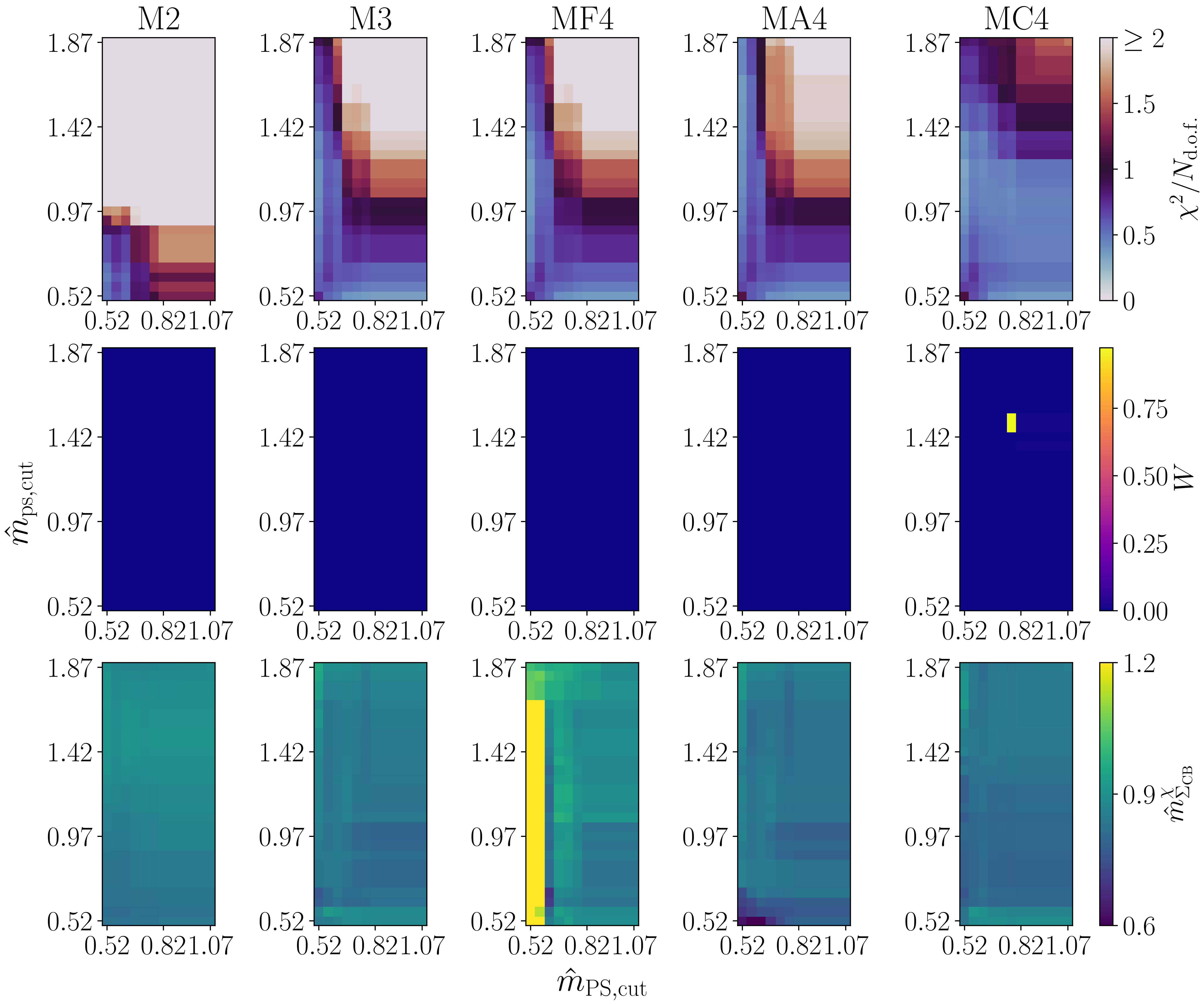
Fittings of Σ_{CB}

- Apply tree level baryon chiral perturbation theory

M2 $$m_{\mathrm{CB}} = m^{\chi}_{CB} + F_2\hat{m}^2_{\mathrm{PS}} + A_2\hat{m}_{\mathrm{ps}}{}^2 + L_1\hat{a}$$

M3 $$+F_3\hat{m}^3_{\mathrm{PS}} + A_3\hat{m}^3_{\mathrm{ps}} + L_{2F}\hat{a}\hat{m}^2_{\mathrm{PS}} + L_{2A}\hat{a}\hat{m}^2_{\mathrm{ps}}$$

$$\underbrace{+F_4\hat{m}^4_{\mathrm{PS}}}_{\text{MF4}} \underbrace{+A_4\hat{m}^4_{\mathrm{ps}}}_{\text{MA4}} \underbrace{+C_4\hat{m}^2_{\mathrm{PS}}\hat{m}^2_{\mathrm{ps}}}_{\text{M2C}}$$



Results

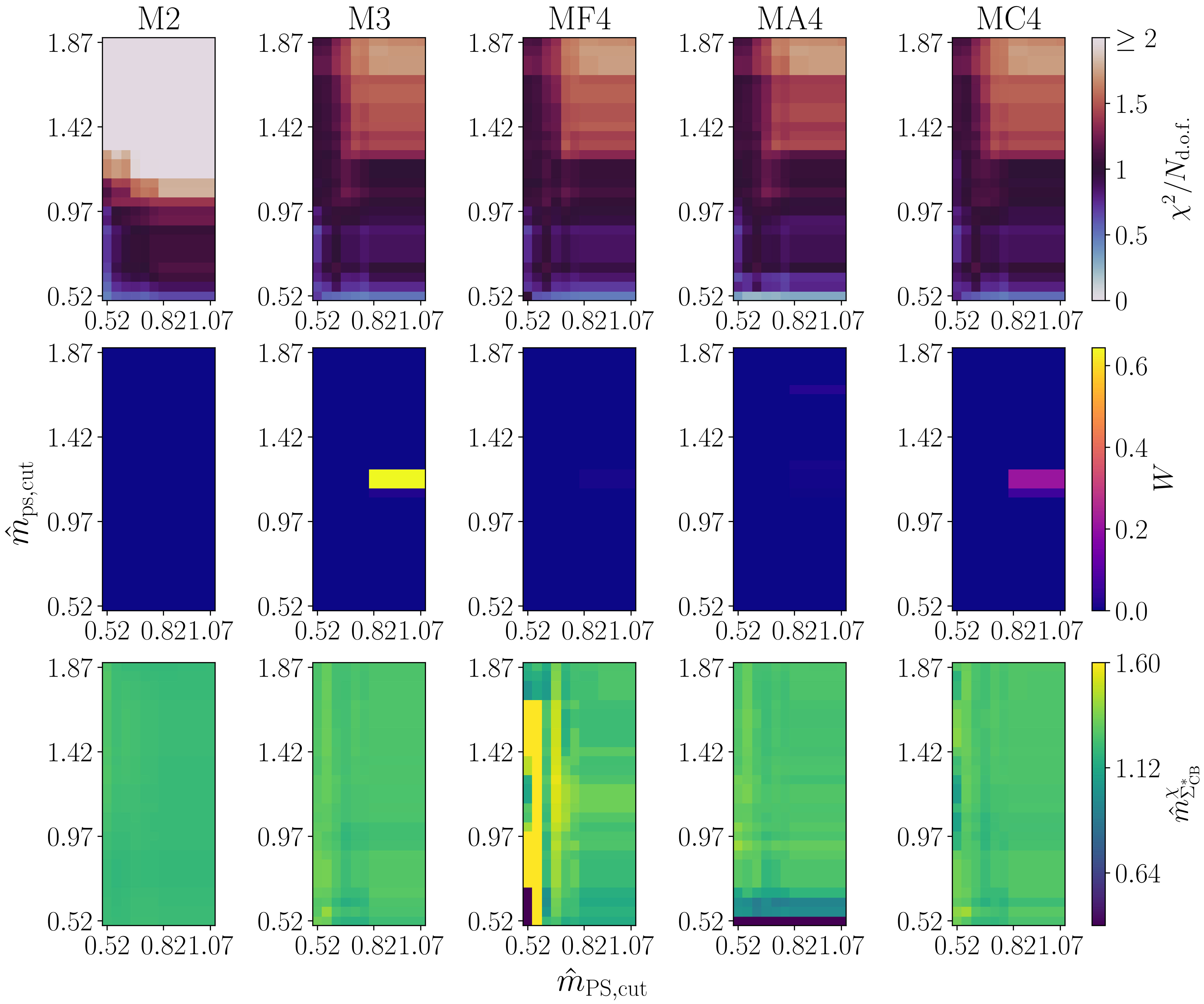
Fittings of $\Sigma^*_{\rm CB}$

- Apply tree level baryon chiral perturbation theory

M2 $$m_{\rm CB} = m^{\chi}_{CB} + F_2\hat{m}_{\rm PS}^2 + A_2\hat{m}_{\rm ps}{}^2 + L_1\hat{a}$$

M3 $$+F_3\hat{m}_{\rm PS}^3 + A_3\hat{m}_{\rm ps}^3 + L_{2F}\hat{a}\hat{m}_{\rm PS}^2 + L_{2A}\hat{a}\hat{m}_{\rm ps}^2$$

$$\underbrace{+F_4\hat{m}_{\rm PS}^4}_{\text{MF4}} \underbrace{+A_4\hat{m}_{\rm ps}^4}_{\text{MA4}} \underbrace{+C_4\hat{m}_{\rm PS}^2\hat{m}_{\rm ps}^2}_{\text{M2C}}$$



Results

Cross check

- Apply tree-level baryon chiral perturbation theory

$$
\begin{aligned}
m_{\mathrm{CB}} = m_{CB}^{\chi} &+ F_2 \hat{m}_{\mathrm{PS}}^2 + A_2 \hat{m}_{\mathrm{ps}}{}^2 + L_1 \hat{a} \\
&+ F_3 \hat{m}_{\mathrm{PS}}^3 + A_3 \hat{m}_{\mathrm{ps}}^3 + L_{2F} \hat{a} \hat{m}_{\mathrm{PS}}^2 + L_{2A} \hat{a} \hat{m}_{\mathrm{ps}}^2 \\
&+ F_4 \hat{m}_{\mathrm{PS}}^4 + A_4 \hat{m}_{\mathrm{ps}}^4 + C_4 \hat{m}_{\mathrm{PS}}^2 \hat{m}_{\mathrm{ps}}^2
\end{aligned}
$$

Results

Cross check

- At a fixed $\hat{m}_{\mathrm{PS}}^{as}$, the fitting function becomes

$$\begin{aligned} m_{\mathrm{CB}} = m_{CB}^{\chi} &+ F_2\hat{m}_{\mathrm{PS}}^2 + A_2\hat{m}_{\mathrm{ps}}^2 + L_1\hat{a} \\ &+ F_3\hat{m}_{\mathrm{PS}}^3 + A_3\hat{m}_{\mathrm{ps}}^3 + L_{2F}\hat{a}\hat{m}_{\mathrm{PS}}^2 + L_{2A}\hat{a}\hat{m}_{\mathrm{ps}}^2 \\ &+ F_4\hat{m}_{\mathrm{PS}}^4 + A_4\hat{m}_{\mathrm{ps}}^4 + C_4\hat{m}_{\mathrm{PS}}^2\hat{m}_{\mathrm{ps}}^2 \end{aligned}$$

Results

Cross check

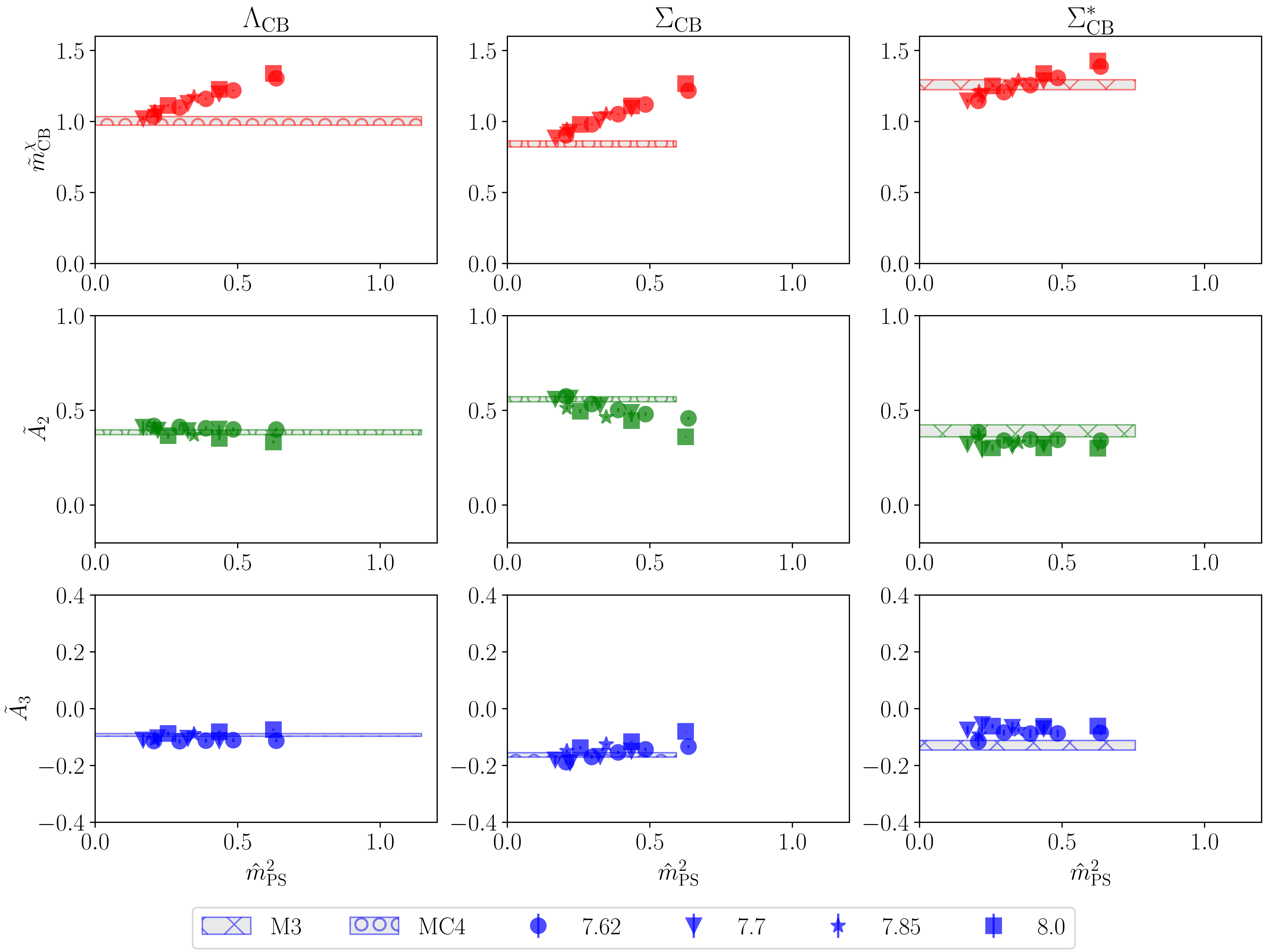
- At a fixed $\hat{m}^{as}_{\text{PS}}$, the fitting function becomes

$$\begin{aligned} m_{\text{CB}} = {} & m^{\chi}_{CB} + A_2 \hat{m}^{as\,2}_{\text{PS}} + L_1 \hat{a} + A_3 \hat{m}^{as\,3}_{\text{PS}} + L_{2A} \hat{a} \hat{m}^{as\,2}_{\text{PS}} + A_4 \hat{m}^{as\,4}_{\text{PS}} \\ & + F_2 \hat{m}^{f}_{\text{PS}}{}^{2} + C_4 \hat{m}^{f}_{\text{PS}}{}^{2} \hat{m}^{as\,2}_{\text{PS}} + L_{2F} \hat{a} \hat{m}^{f}_{\text{PS}}{}^{2} \\ & + F_3 \hat{m}^{f}_{\text{PS}}{}^{3} + F_4 \hat{m}^{f}_{\text{PS}}{}^{4} \end{aligned}$$

$$\Rightarrow \tilde{m}^{\chi}_{CB}(\hat{m}_{\text{ps}}, A, L, \hat{a}) + \tilde{F}_2(\hat{m}_{\text{ps}}, C, L, \hat{a}) \hat{m}^2_{\text{PS}} + \tilde{F}_3 \hat{m}^3_{\text{PS}} + F_4 \hat{m}^{f}_{\text{PS}}{}^{4}$$

Results

Cross check

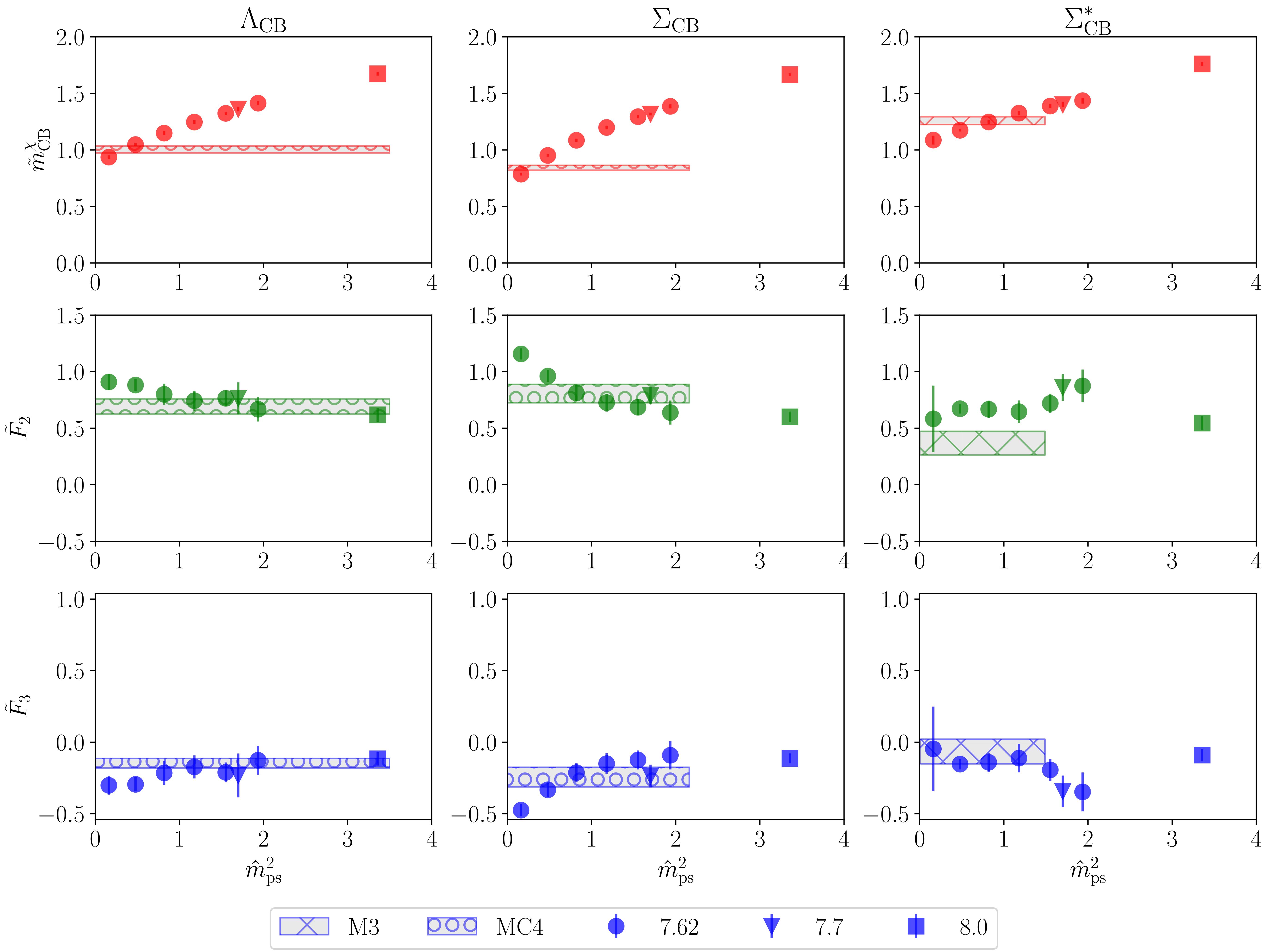


- At a fixed $\hat{m}_{\text{ps}}$

$$m_{\text{CB}} = \tilde{m}^\chi_{CB}(\hat{m}_{\text{ps}}, A, L, \hat{a})$$
$$+\tilde{F}_2(\hat{m}_{\text{ps}}, C, L, \hat{a})\hat{m}^2_{\text{PS}}+\tilde{F}_3\hat{m}^3_{\text{PS}}$$

Results

Cross check

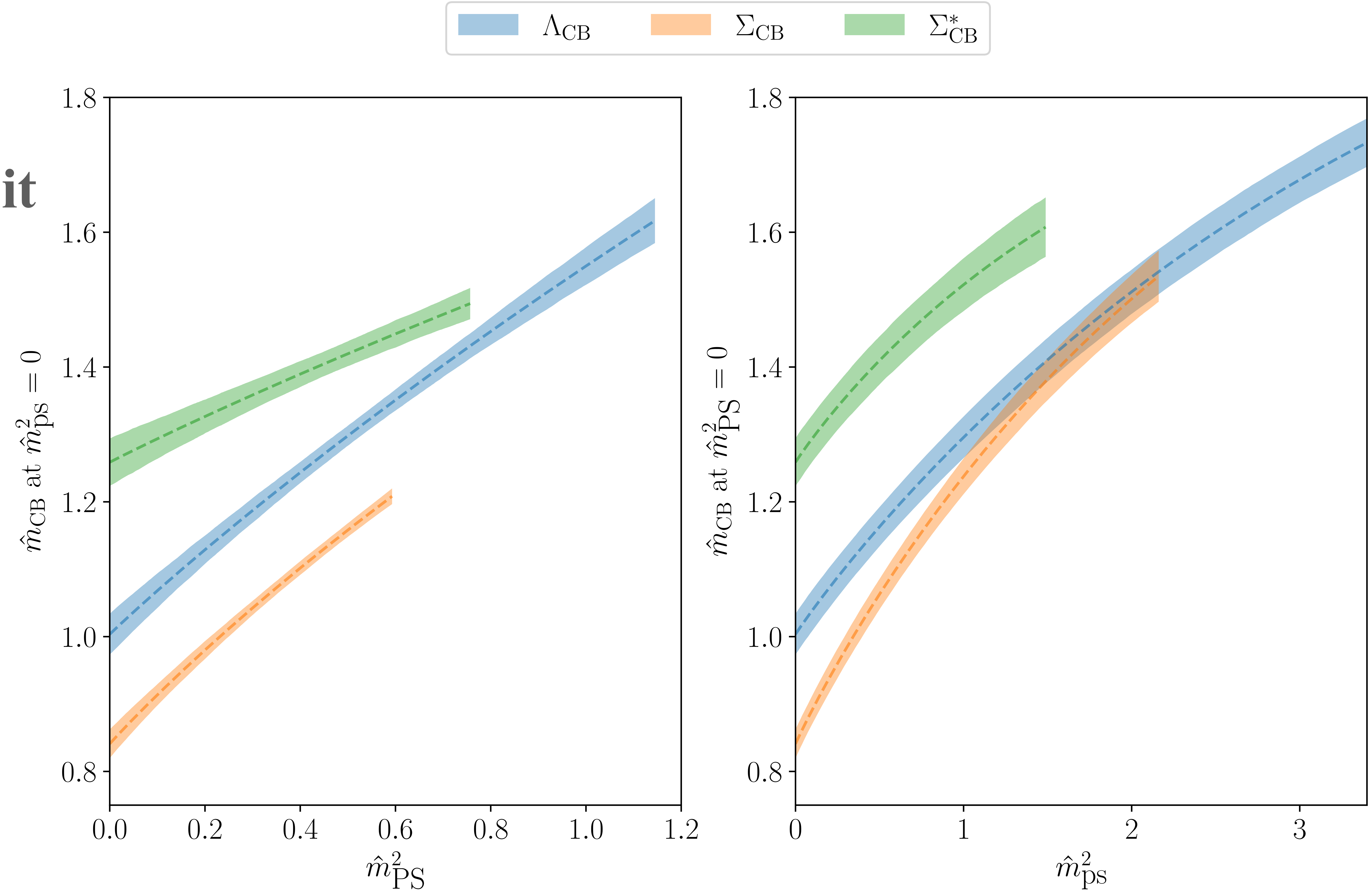


- At a fixed $\hat{m}_{\rm PS}$

$$m_{\rm CB} = \tilde{m}^{\chi}_{CB}(\hat{m}_{\rm PS}, F, L, \hat{a}) + \tilde{A}_2(\hat{m}_{\rm PS}, C, L, \hat{a})\hat{m}^2_{\rm ps} + \tilde{A}_3\hat{m}^3_{\rm ps}$$

Results

Massless-continuum limit

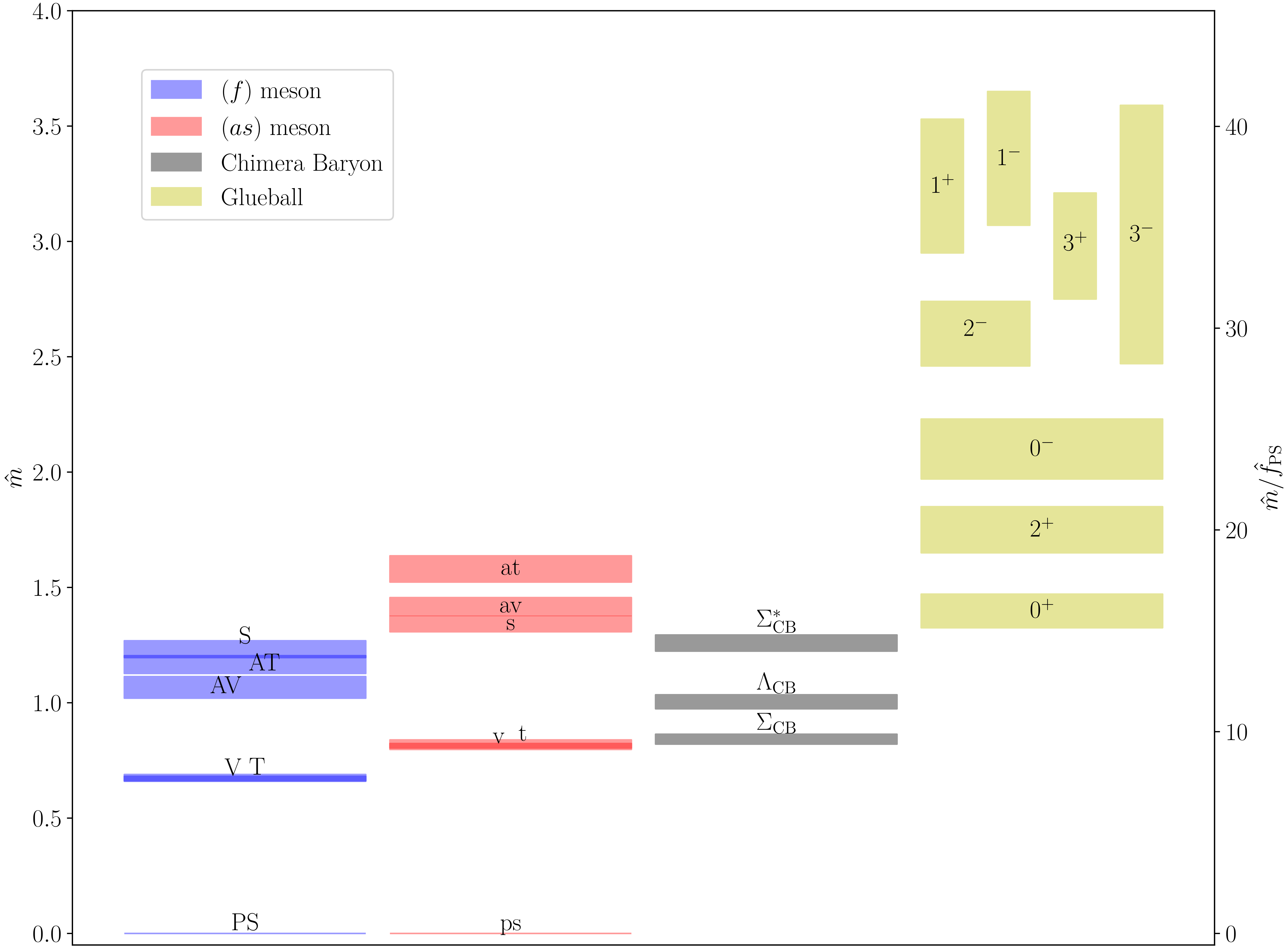


CB	Ansatz	$\hat{m}^{\chi}_{\rm CB}$	F_2	A_2	L_1	F_3	A_3	L_{2F}	L_{2A}	C_4
$\Lambda_{\rm CB}$	MC4	1.004(30)	0.692(67)	0.384(12)	-0.14(46)	-0.14(33)	-0.092(46)	0.091(76)	0.003(13)	-0.024(60)
$\Sigma_{\rm CB}$	MC4	0.842(21)	0.806(81)	0.558(13)	-0.14(33)	-0.24(68)	-0.162(77)	0.193(62)	-0.01(16)	-0.079(62)
$\Sigma^*_{\rm CB}$	M3	1.258(35)	0.36(10)	0.391(31)	-0.33(53)	-0.06(85)	-0.12(16)	0.335(86)	0.006(30)	-

Results

Massless-continuum limit

Comparison with masses of mesons in quenche approximation for fermions in the fundamenta (blue bands) and antisymmetric (red bands representation of $Sp(4)$, and glueballs (yellow at massless-continuum limit.



Summary and Outlook

☑ Composite Higgs model

☑ Chimera baryons

- Λ and Σ: Top partner candidates in our model
- Σ^* with spin-3/2

☑ Projection (Spin and Parity)

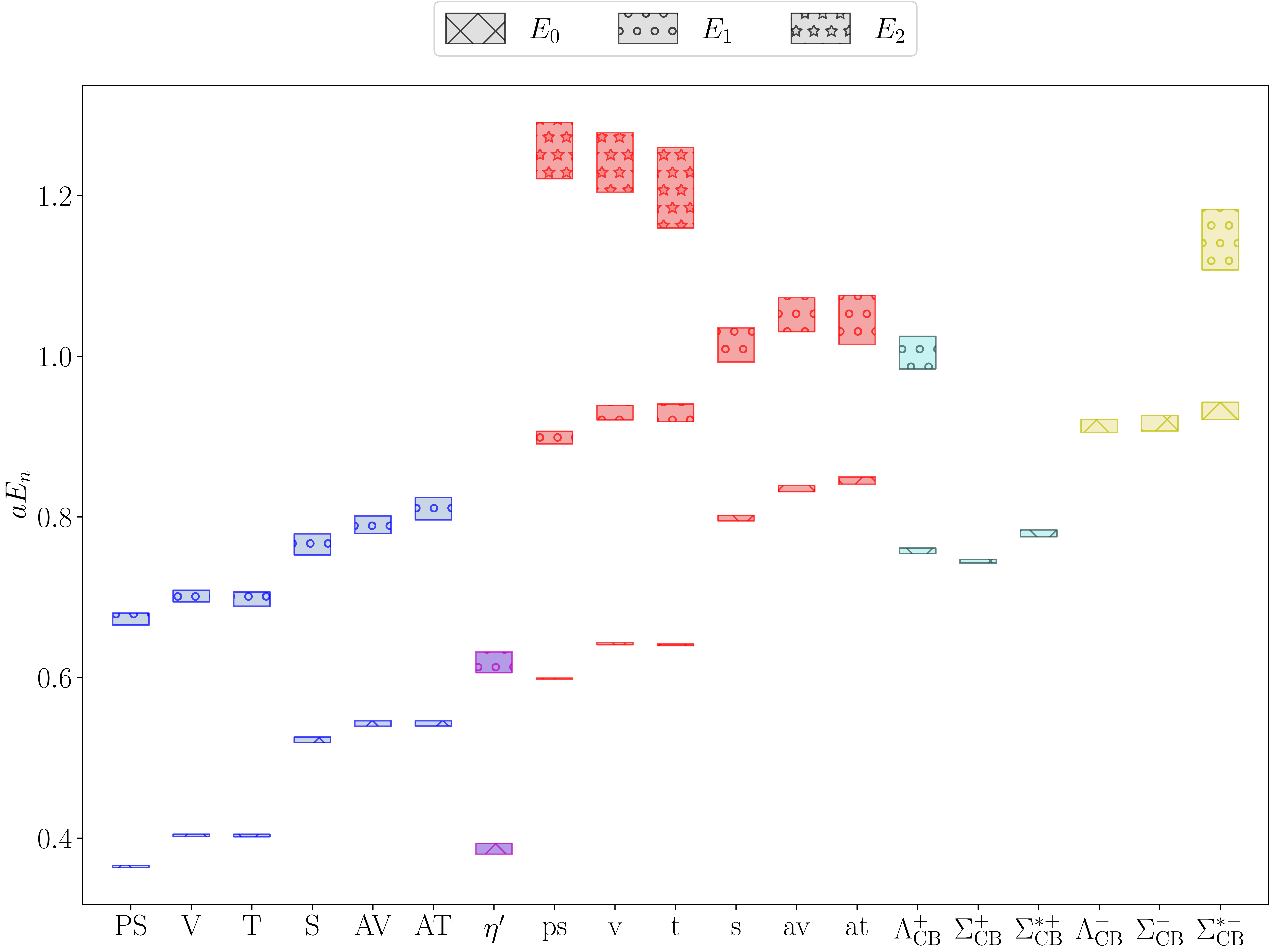
☑ The mass hierarchy of chimera baryons —— model building

☑ Chiral effective field theory

☐ Dynamical studies

END

Thank you


E_0
E_1
E_2
aE_n
1.2
1.0
0.8
0.6
0.4
PS
V
T
S
AV
AT
η'
ps
v
t
s
av
at
Λ^+_{CB}
Σ^+_{CB}
$\Sigma^{*+}_{\mathrm{CB}}$
Λ^-_{CB}
Σ^-_{CB}
$\Sigma^{*-}_{\mathrm{CB}}$

Lattice Method

- Generate the ensemble in quenched approximation with the standard Wilson action

$$S_g \equiv \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{2N} \mathrm{ReTr}\, \mathscr{P}_{\mu\nu} \right) \ , \ \text{with } \mathscr{P}_{\mu\nu}(x) \equiv U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x) \, .$$

•Consider the Wilson fermion for the spectroscopic measurements

$$D_m^R \psi_j^R(x) \equiv (4/a + m_0^R)\psi_j^R(x)$$
$$-\frac{1}{2a} \sum_\mu \left\{ (1-\gamma_\mu) U_\mu^R(x) \psi_j^R(x+\hat{\mu}) + (1+\gamma_\mu) U_\mu^{R,\dagger}(x-\hat{\mu}) \psi_j^R(x-\hat{\mu}) \right\} \ .$$

Lattice Method

Scale setting: gradient-flow

- Lüscher demonstrated that the action density can be related to the renormalised coupling with an extra dimension, *flow time t*:

[Martin Lüscher. 2009]

$$\frac{\mathrm{d}B_\mu(t,x)}{\mathrm{d}t} = D_\nu G_{\nu\mu}(t,x), \quad B_\mu(t,x)\,|_{t=0} = A_\mu(t,x)$$

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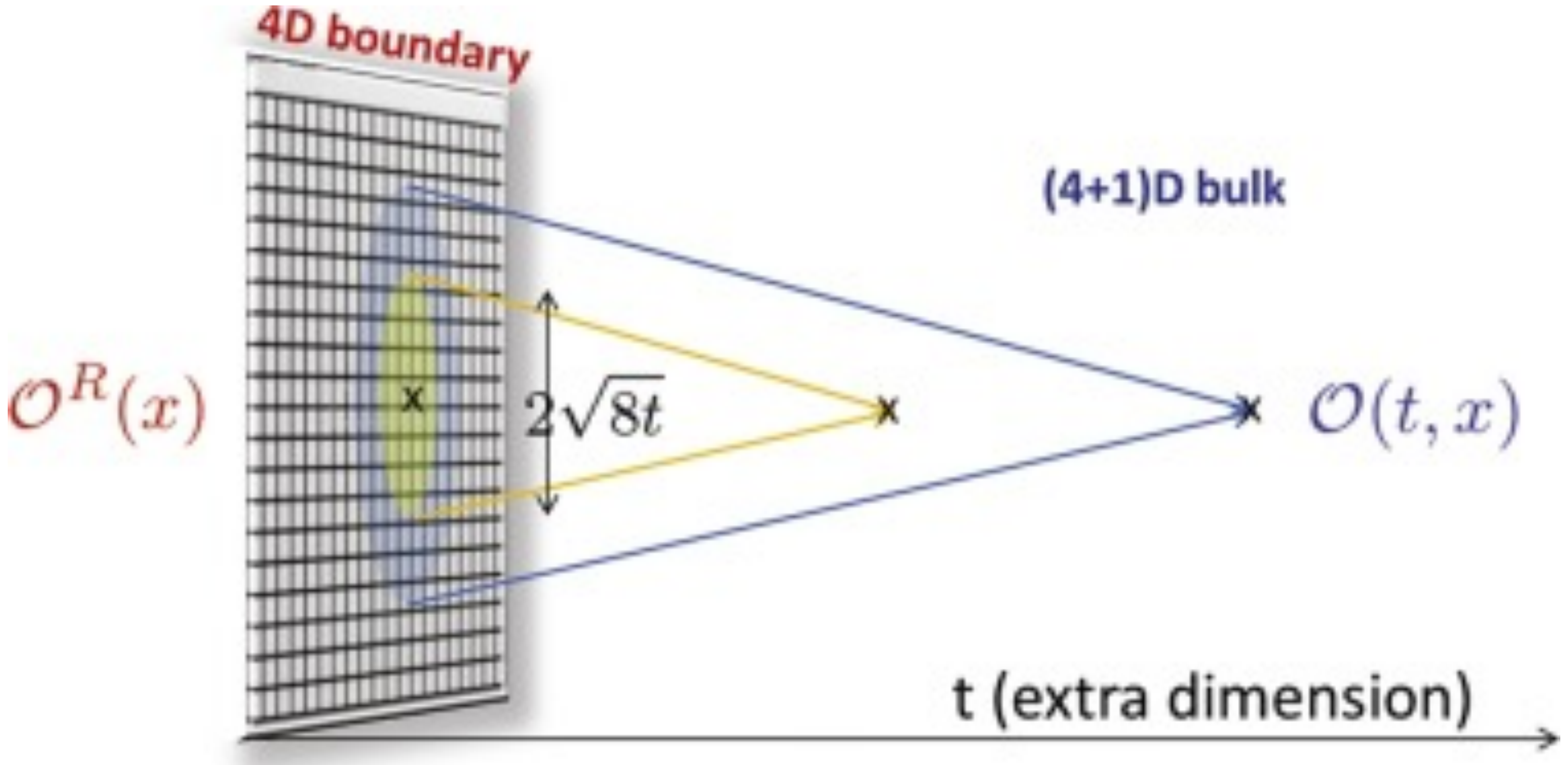
A diffusion process



[FlowQCD Collaboration]

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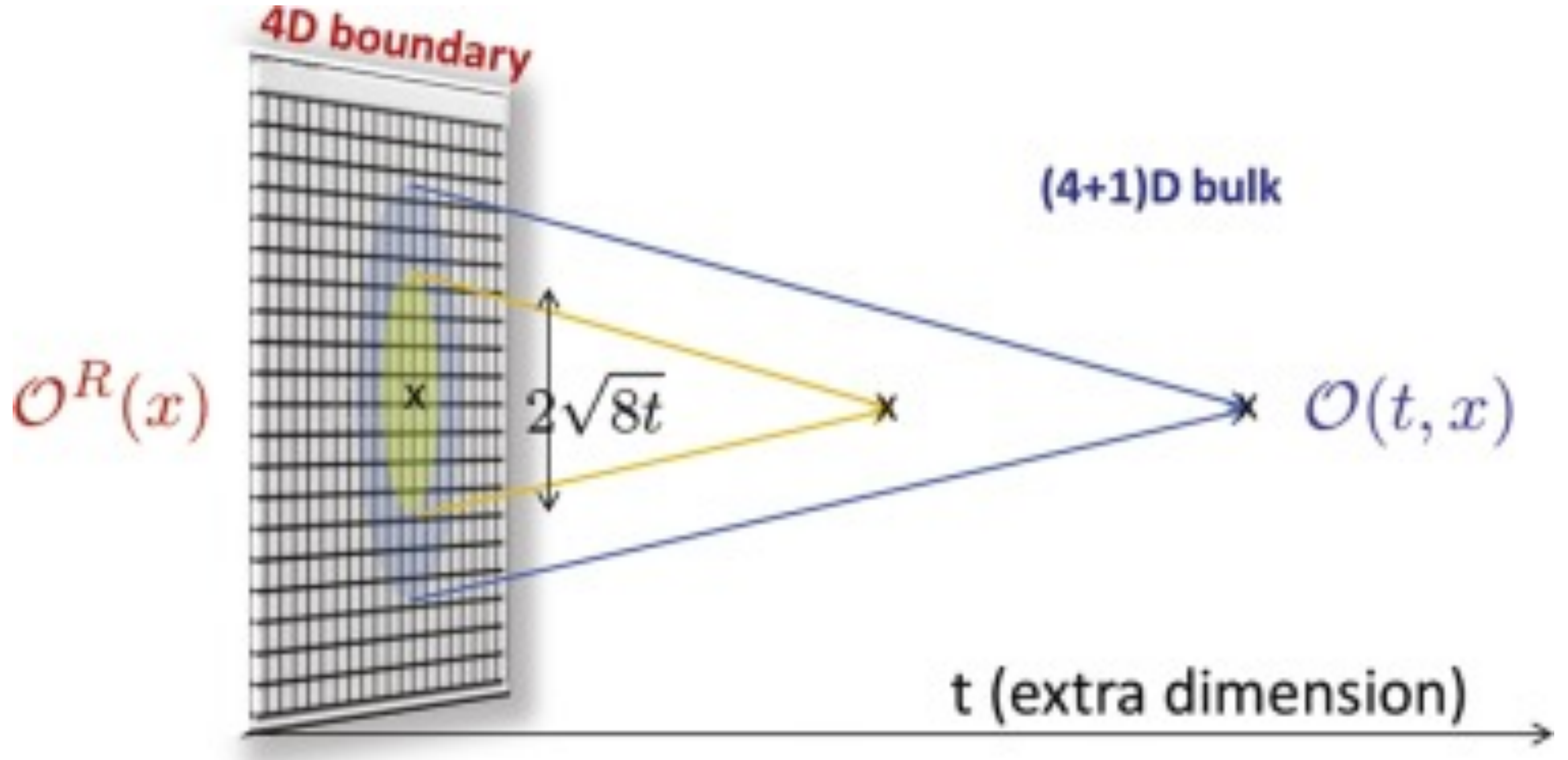
A diffusion process

- Setting the scale:

$$\alpha(\mu) = k_\alpha t^2 \langle E(t) \rangle \equiv k_\alpha \varepsilon(t)$$

with $\mu = 1/\sqrt{8t}$ and $E(t) = -\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})$.



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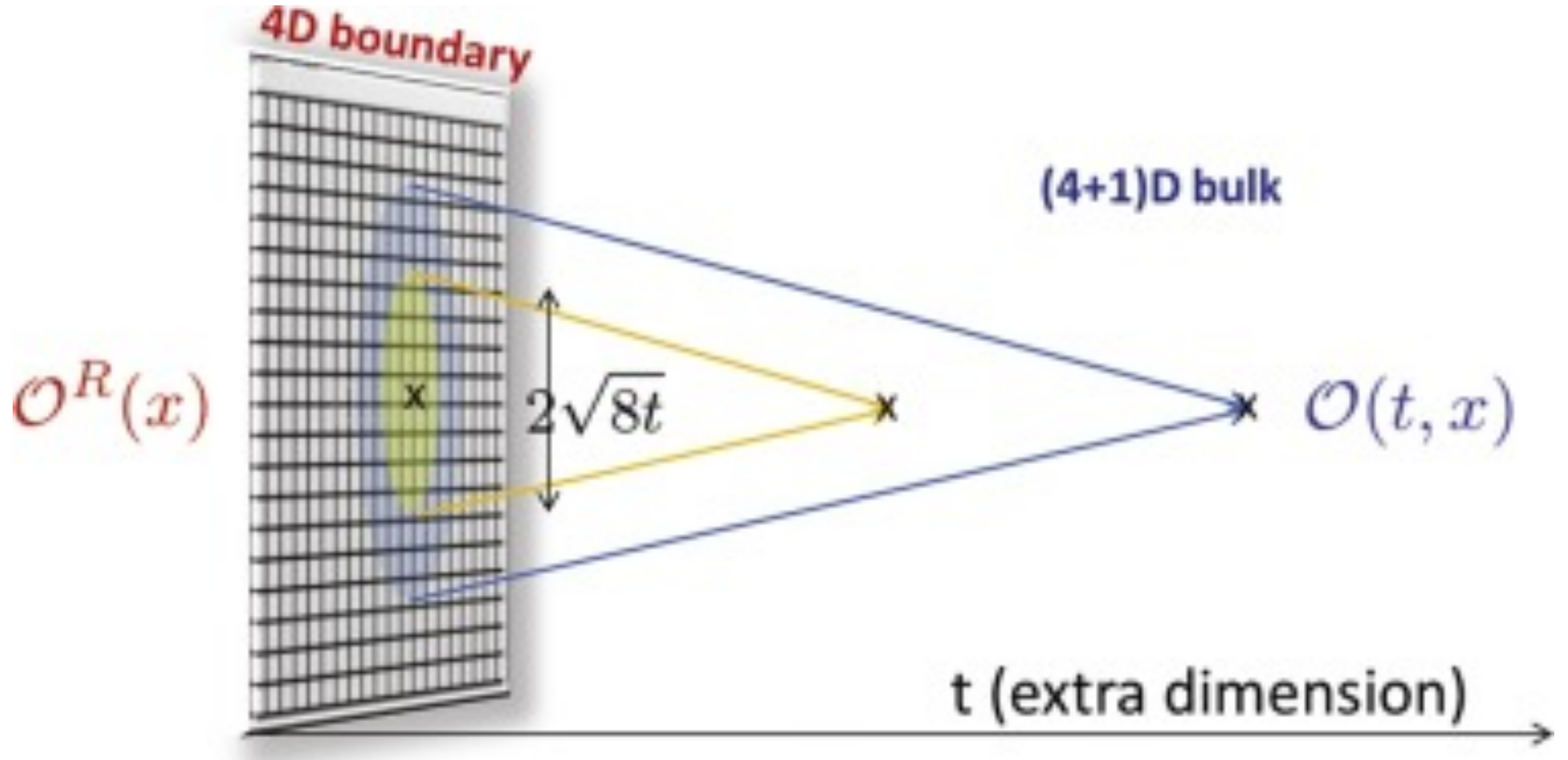

[FlowQCD Collaboration]

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renormalised coupling

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- Setting the scale:

$$\underbrace{\alpha(\mu)}_{\text{renormalised coupling}} = k_\alpha t^2 \langle E(t) \rangle \equiv \underbrace{k_\alpha}_{\text{analytically computable}} \varepsilon(t)$$

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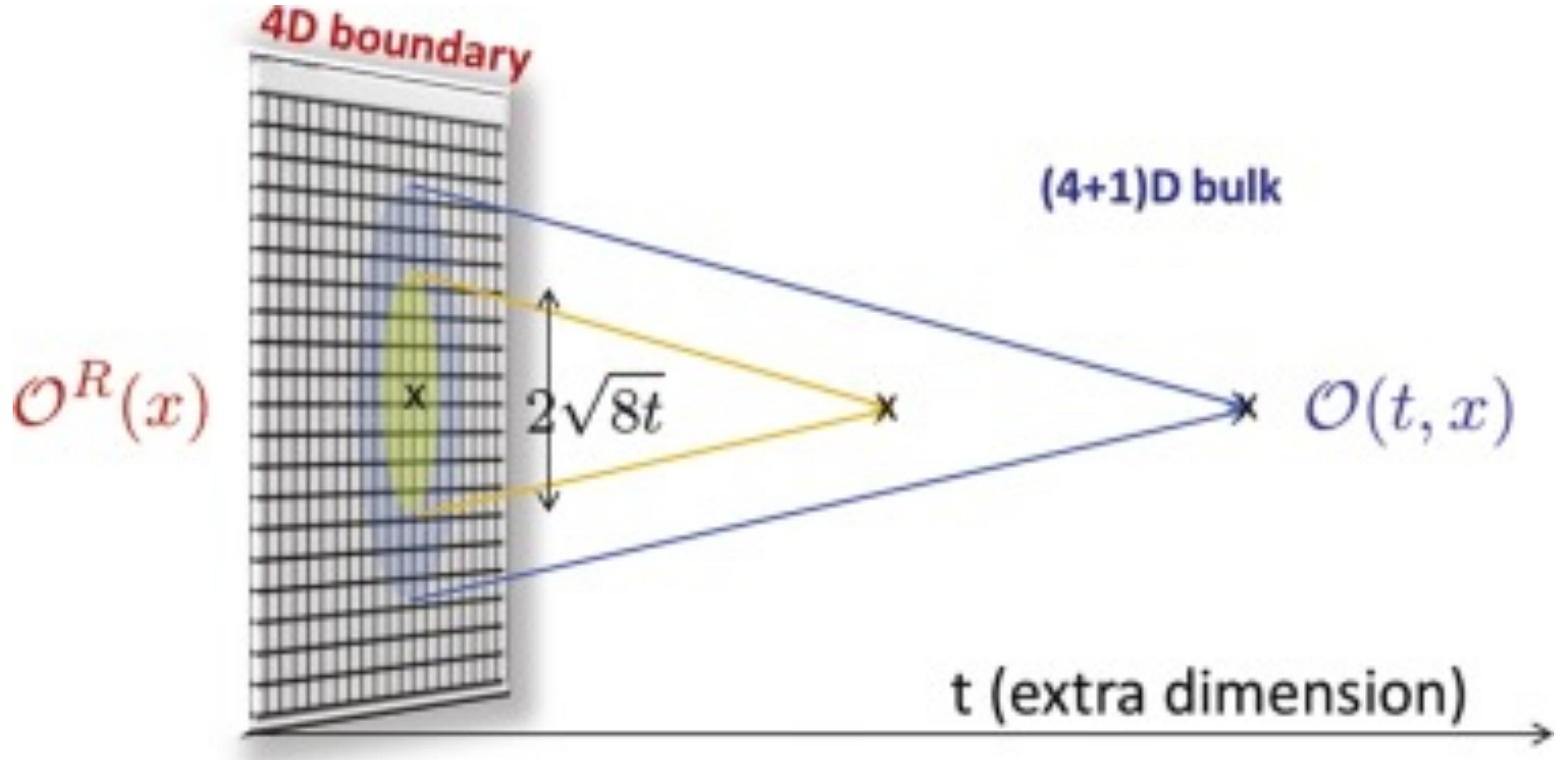

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renormalised coupling

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A constant one can choose

renormalised coupling

with $\mu = 1/\sqrt{8t}$ and $E(t) = -\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})$.

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renormalised coupling

$$\Rightarrow W(t) = t\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t}$$

with $\mu = 1/\sqrt{8t}$ and $E(t) = -\frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G_{\mu\nu})$.

$$\Rightarrow W(t)\,|_{t=\omega_0^2} = W_0$$

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Study Plan

- Meson spectrum with quenched fundamental and antisymmetric fermions
- Meson spectrum with $N_f = 2$ dynamical fundamental fermions
- Meson spectrum with $n_f = 3$ dynamical antisymmetric fermions
- Fully dynamical 2F + 3AS fermions
    - Chimera baryon (quenched studies)
    - 4-fermion operator matrix elements (relevant to generating Higgs mass)

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