

Phenomenology of compact objects from a first order phase transition in the dark sector

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[JTA, Po-Yan Tseng (曾柏彥), JHEP 08 (2023) 117
JTA, Danny Marfatia, Po-Yan Tseng, arxiv:24xx.xxxxx]

The Future is Flavourful

4th NCTS TG2.1 Future Workshop, NYCU Campus, Hsinchu, Taiwan ROC

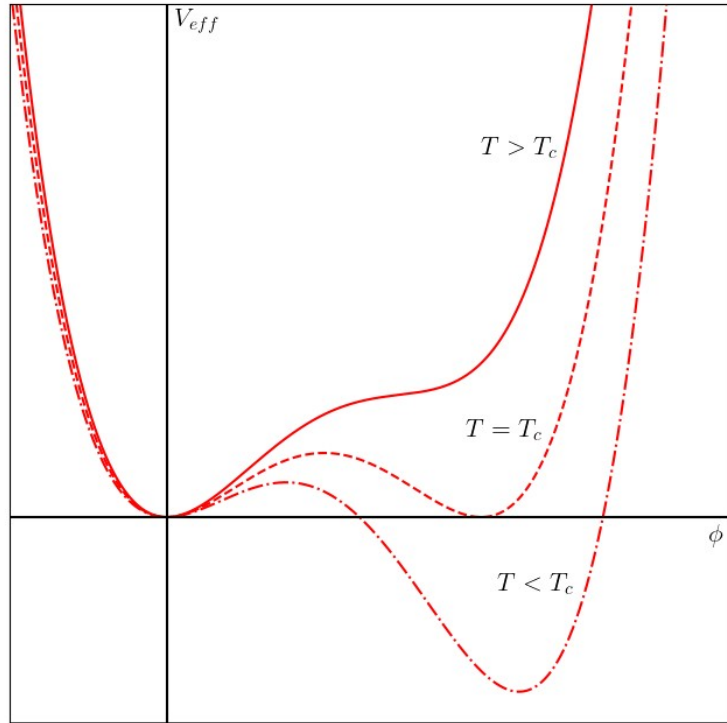
06 June 2024



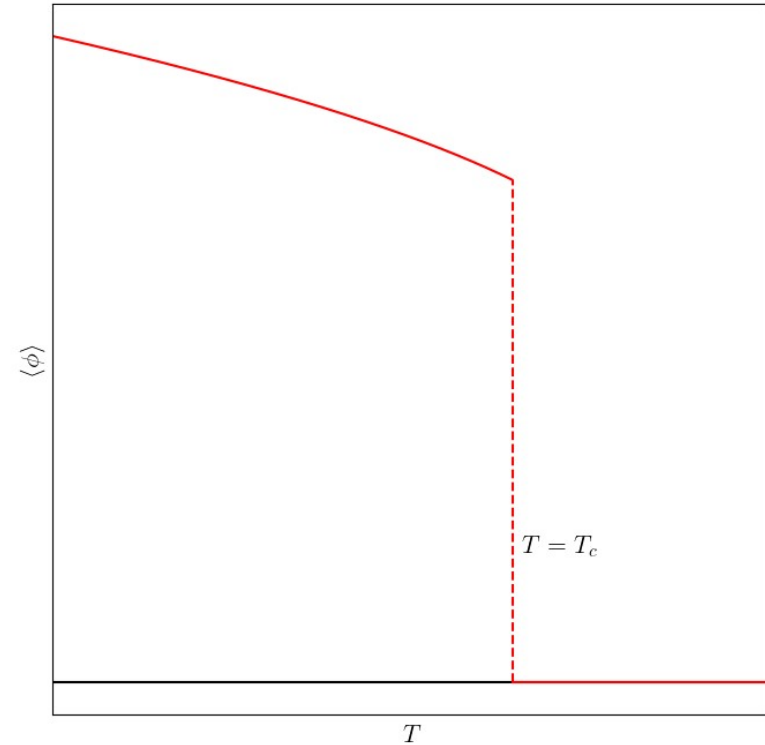
Outline

- Overview of phase transitions in cosmology
 - sGW production
 - Novel PBH formation mechanism: Fermi ball collapse from dark FOPT
- Pulsar timing
 - Doppler and Shapiro
 - Constraints: sGW and probe of compact objects
- Spin of PBH
- Conclusions

Phase transitions

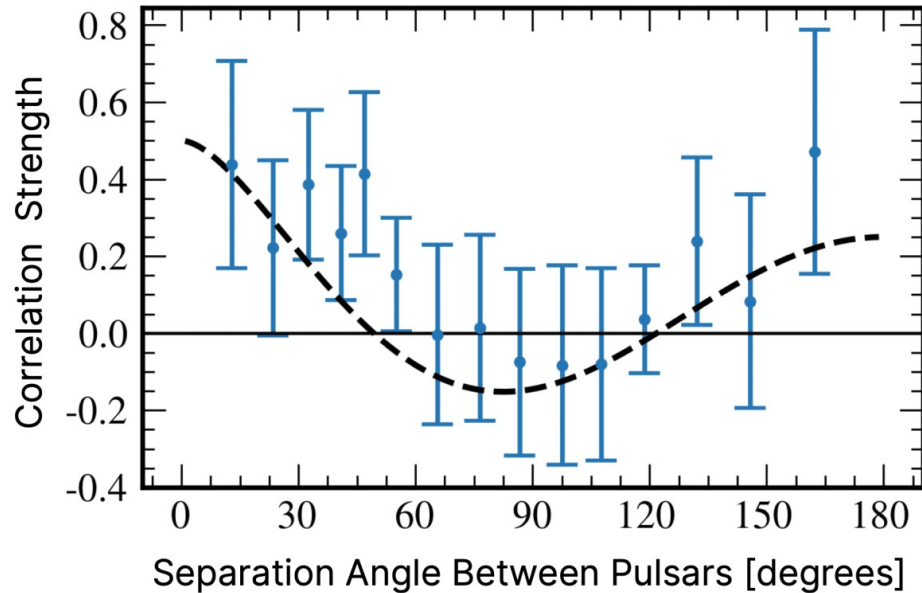


$$V_4(\phi, T) = \frac{\lambda}{4}\phi^4 - (AT + C)\phi^3 + D(T^2 - T_0^2)\phi^2.$$



$$\phi_+(T) = \frac{3(AT + C) + \sqrt{9(AT + C)^2 - 8\lambda D(T^2 - T_0^2)}}{2\lambda}.$$

Phase transitions in cosmology

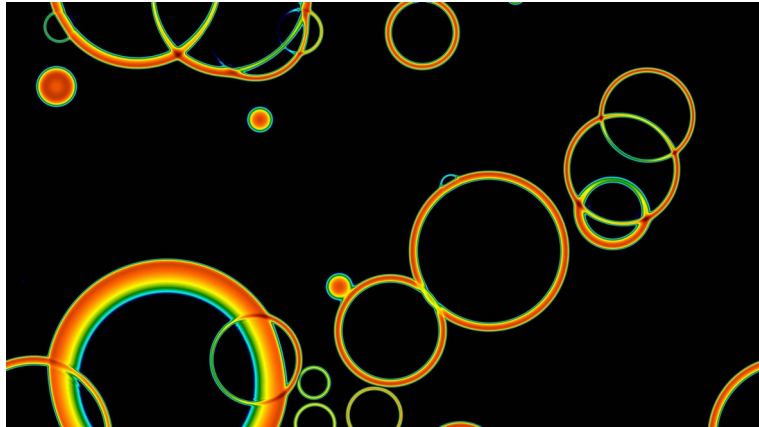


Retrieved from:
<https://nanograv.org/15yr/Summary/Background>

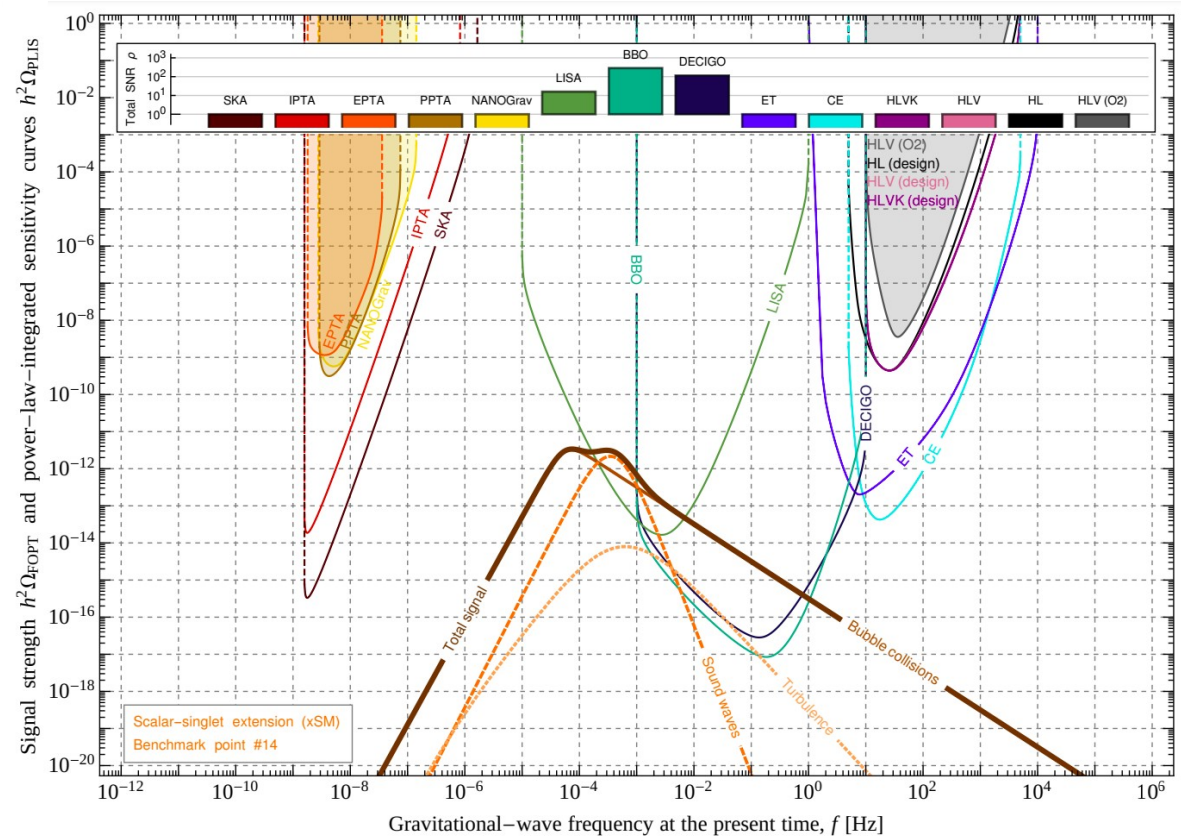


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<https://nanograv.org/news/nanograv-finds-possible-first-hints-low-frequency-gravitational-wave-background-0>

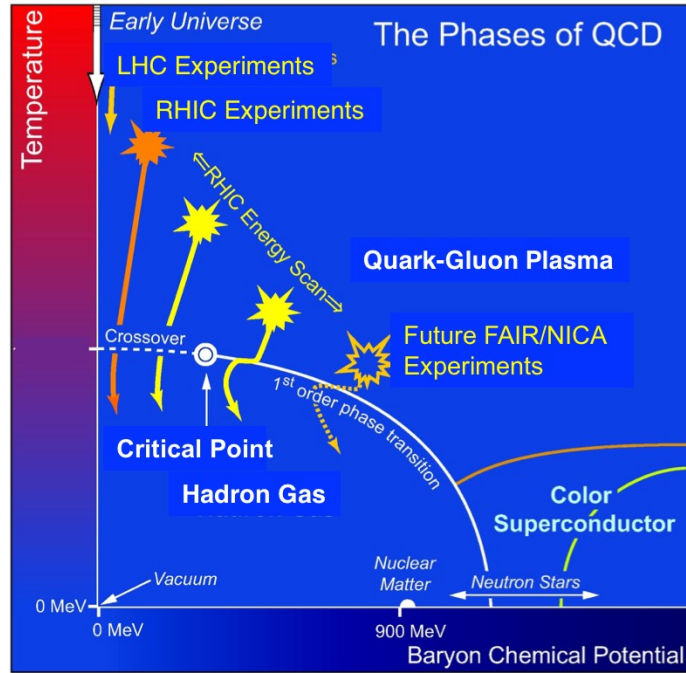
Phase transitions in cosmology



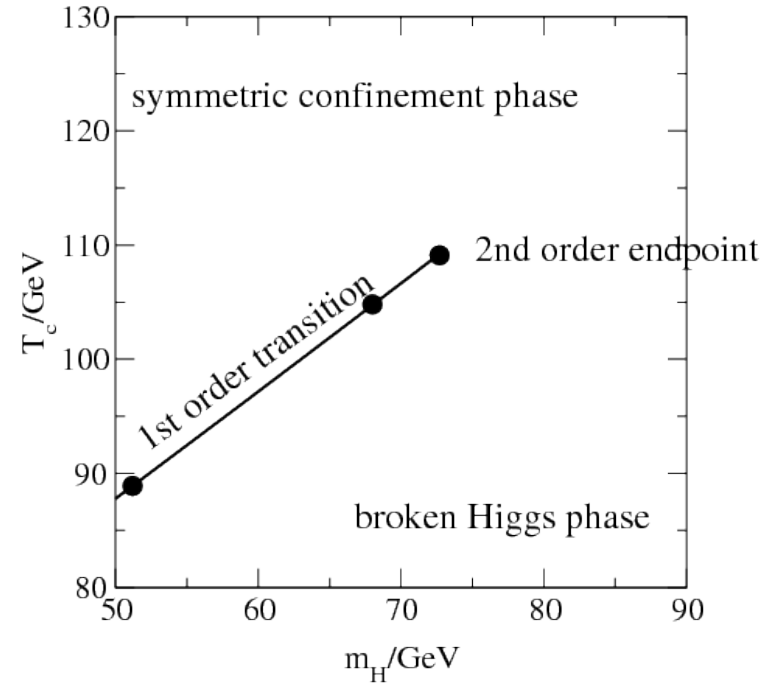
Retrieved from: arXiv:1705.01783



Phase transitions in cosmology



Retrieved from:
<https://cds.cern.ch/record/2729160/files/phases.png>



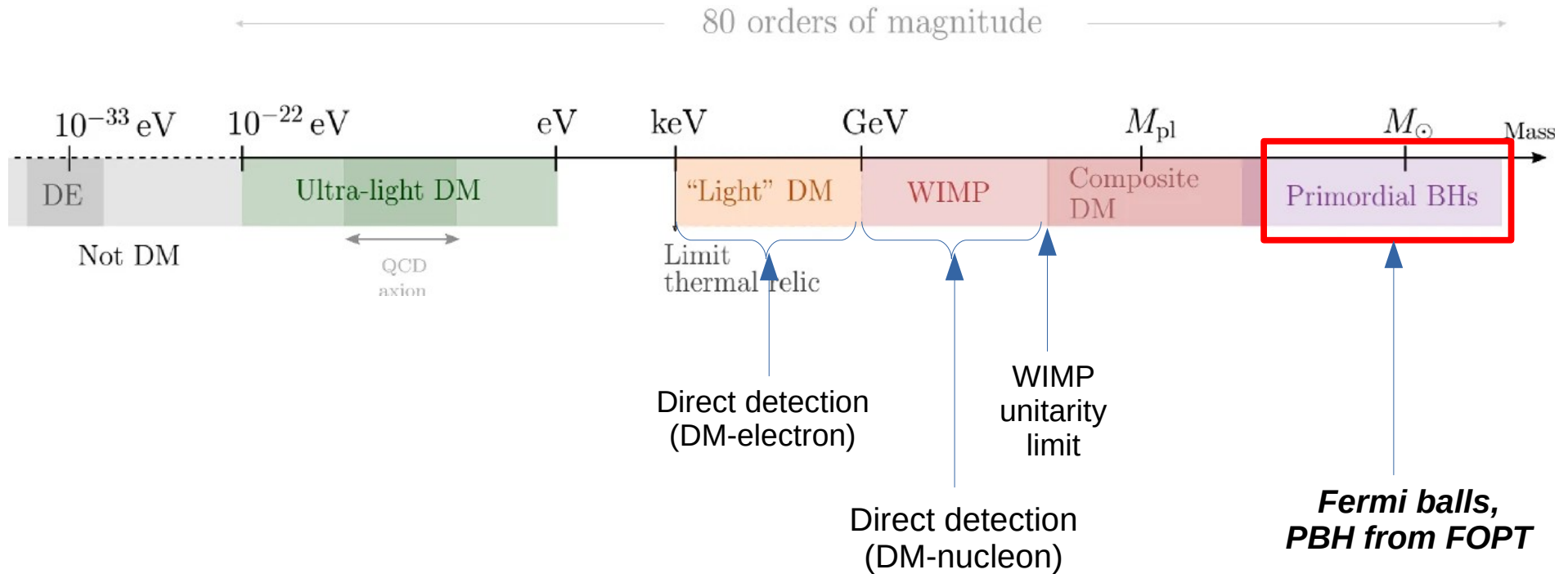
Retrieved from:
https://cds.cern.ch/record/471776/files/sm_simple.png

Take-away: all PT in SM are crossovers, strong FOPT may generate huge signal to explain NANOGrav, need physics beyond SM (BSM)

Outline

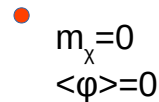
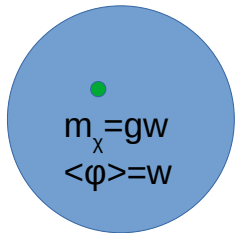
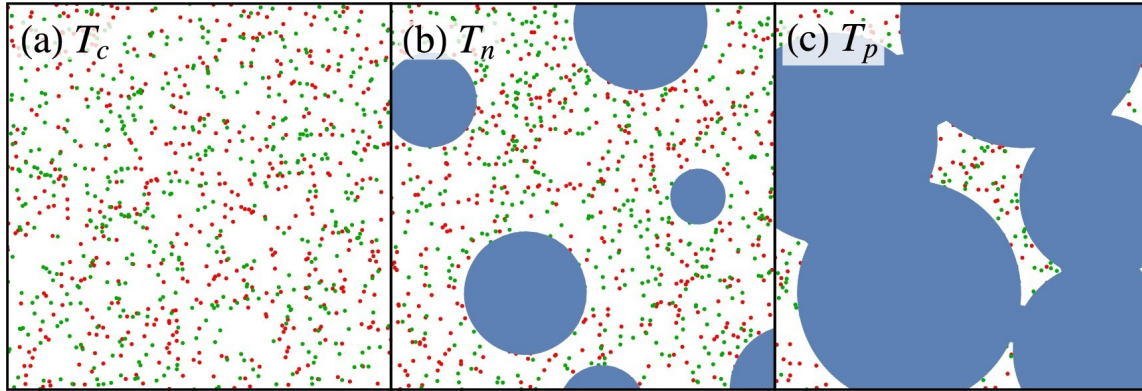
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Dark matter



Retrieved from:
<https://link.springer.com/article/10.1007/s00159-021-00135-6/figures/1>

Fermi ball and PBH DM^[2]



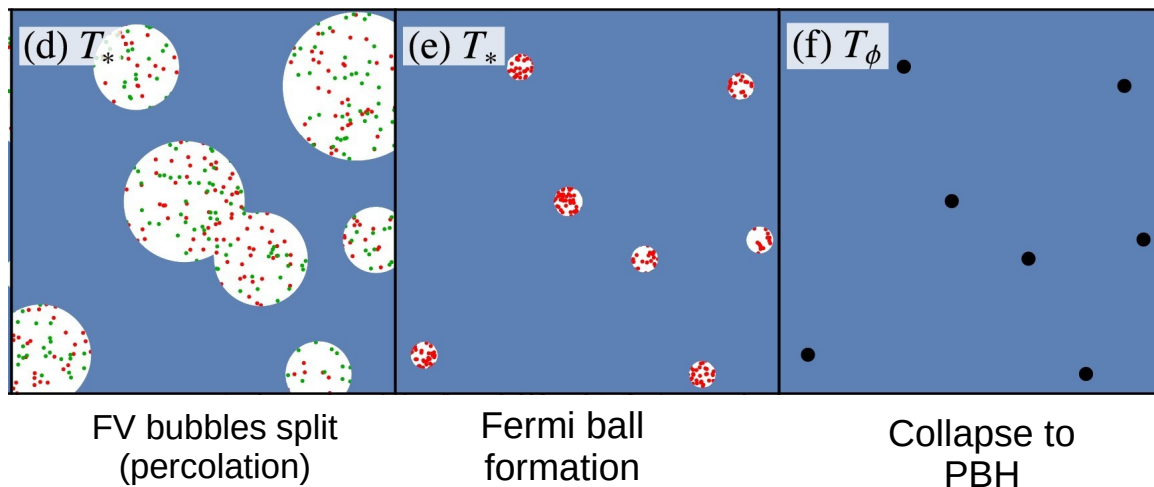
$gw \gg T_*$

$$L = \frac{1}{2} (\partial \phi)^2 - V_{eff}(\phi, T) + \bar{\chi} (i \gamma^\mu \partial_\mu - m_\chi) \chi - g_\chi \phi \bar{\chi} \chi$$

Dark sector
[temp. = $T(t)$]

Visible sector
[temp. = $T_{SM}(t)$]

Fermi ball and PBH DM^[2]



$$U_{tot} = \frac{3\pi}{4} \left(\frac{3}{2\pi}\right)^{2/3} \frac{Q_{FB}^{4/3}}{R} + \frac{4\pi}{3} \Delta V R^3$$

$$Q_{FB} = \frac{\eta_\chi s_v(t_*)}{f_{FV}(t_*)} A \frac{4\pi R_*^3}{3} \quad \begin{array}{l} U(1) \\ \text{charge in} \\ \text{FV bubble} \end{array}$$

$$M_{FB} = Q_{FB} (12\pi^2 \Delta V_{eff}(T_*))^{1/4}$$

$$V_{Yukawa} = \frac{-g_\chi^2}{4\pi r} \exp(-M_\phi r)$$

$$M_\phi^2 = \frac{d^2 V_{eff}(0, T)}{dT^2} = 2D(T^2 - T_0^2)$$

$$M_\phi^{-1} > \frac{R_{FB}}{Q_{FB}^{1/3}}$$

Criterion
for FB
collapse to
PBH

Panels (d)-(f) taken from:
[2]Kawana, Kiyoharu, and Ke-Pan
Xie. (2022)

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Fermi ball and PBH DM^[2]

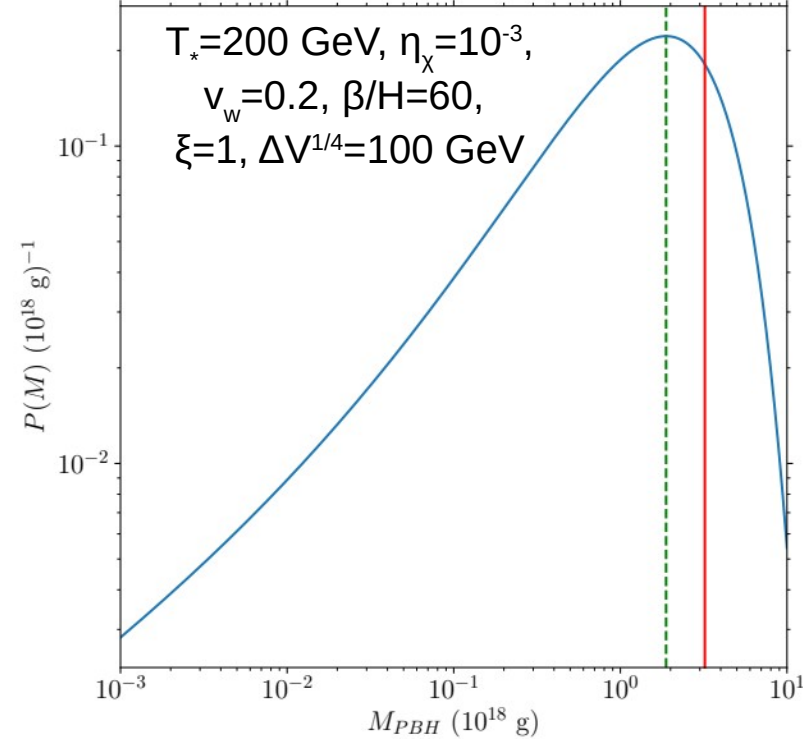
$$\langle M \rangle \simeq (4.07 \times 10^{-8} M_{\odot}) \left(\frac{10.63}{g_*} \right)^{1/4} \left(\frac{0.1 \text{ MeV}}{T_*} \right)^2 \left(\frac{\xi}{0.1} \right)^2$$

$$\times \left(\frac{\eta_{\chi}}{10^{-7}} \right) \left(\frac{v_w}{1} \right)^3 \left(\frac{2.5 \times 10^2}{\beta/H_*} \right)^3 \left(\frac{\alpha_{tr}}{0.1} \right)^{1/4} \left[\frac{\mathcal{F}(T_*/T_c)}{0.308} \right]^{1/4}$$

$$\omega_{PBH,*} \simeq 0.434 \left(\frac{\alpha_{tr}}{0.1} \right)^{1/4} \left(\frac{g_*}{10.63} \right)^{1/4} \left(\frac{T_*}{0.1 \text{ MeV}} \right)$$

$$\left(\frac{0.1}{\xi} \right) \left(\frac{\eta_{\chi}}{10^{-7}} \right) \left[\frac{\mathcal{F}(T_*/T_c)}{0.308} \right]^{1/4}$$

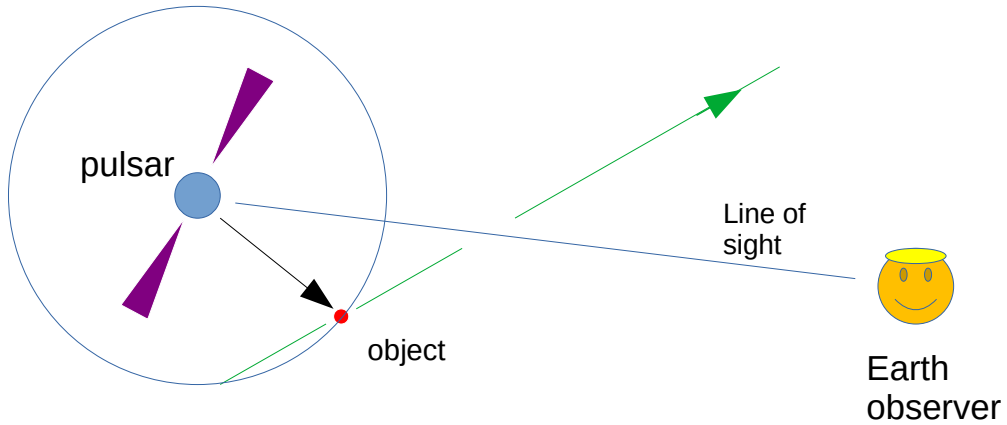
$$F(x) \equiv \frac{1-x}{1-3x/4}$$



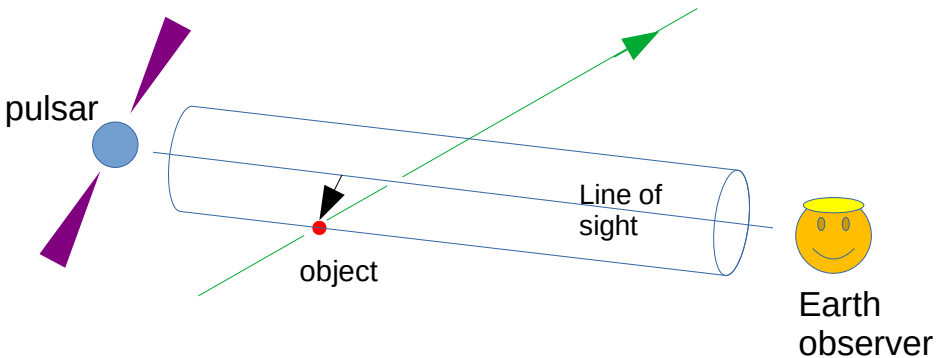
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Pulsar timing: Doppler & Shapiro



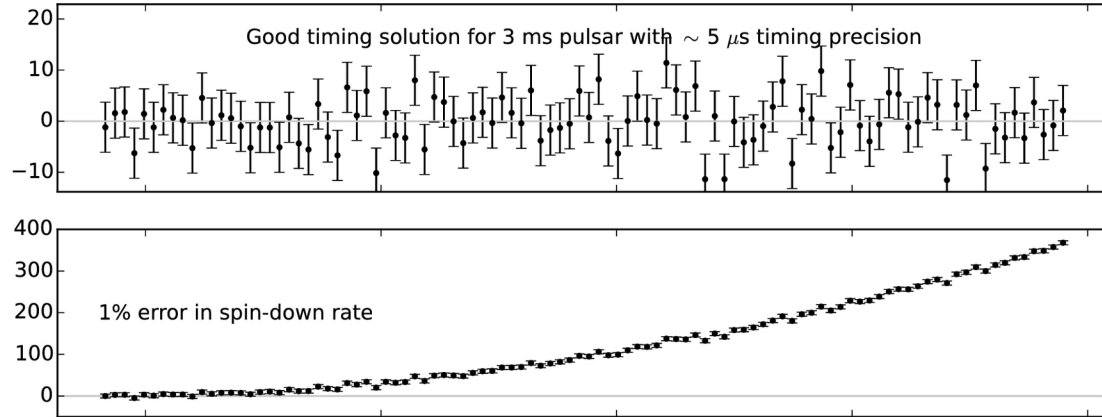
$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{1}{c} \hat{d} \cdot \int \vec{\nabla} \Phi dt$$



$$\left(\frac{\delta\nu}{\nu}\right)_S = -\frac{2}{c^3} \vec{v} \cdot \int \vec{\nabla} \Phi dz$$

$$\delta\phi = \int^t dt' \delta\nu(t')$$

Pulsar timing: Detector properties



Retrieved from: <https://www.cv.nrao.edu/~sransom/web/Ch6.html>

$$\phi(t) = \phi_0 + \nu t + \frac{\dot{\nu}}{2} t^2 + \frac{\ddot{\nu}}{6} t^3 + \dots$$

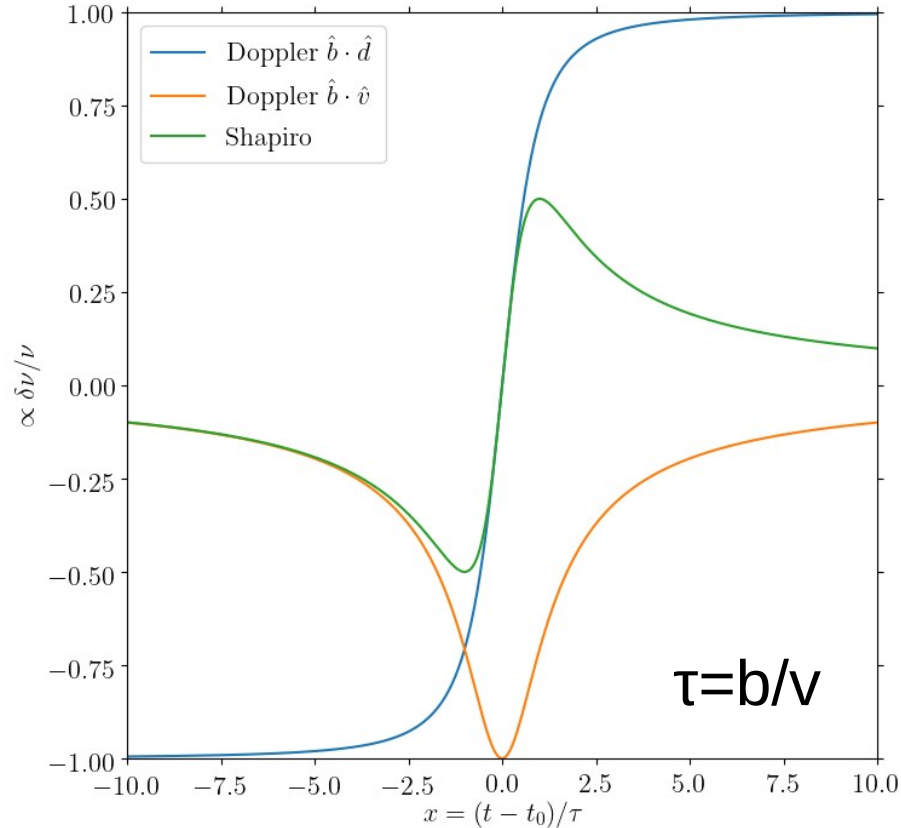
$$SNR = \frac{|\ddot{\nu} / \nu|}{\sigma_{\ddot{\nu} / \nu}} \quad (?)$$

$$\sigma_{\ddot{\nu} / \nu} = 6 \sqrt{\frac{2800 \Delta t}{T} \frac{t_{rms}}{T^3}} \quad (\sim 2.8 \times 10^{-33} \text{ Hz}^2)$$

^[3]Uncertainty in $\ddot{\nu} / \nu$

PTA property	Value
No. of pulsars	200
RMS timing residual	50 ns
Cadence	2 weeks
Total observation time	20 years

Pulsar timing: monochromatic, pointlike PBHs



$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{G_N M_{PBH}}{v^2 c \tau_D} \frac{1}{\sqrt{1+x_D^2}} (x_D \hat{b} - \hat{v}) \cdot \hat{d}$$

$$\left(\frac{\delta\nu}{\nu}\right)_S = \frac{4G_N M_{PBH}}{c^3 \tau_S} \frac{x_S}{1+x_S^2}$$

$$SNR = \frac{|\ddot{v}/v|}{\sigma_{\ddot{v}/v}} \quad (\sim 10^{-34} \text{ Hz}^2)$$

$$\sigma_{\ddot{v}/v} = 6 \sqrt{\frac{2800 \Delta t}{T} \frac{t_{rms}}{T^3}} \quad (\sim 2.8 \times 10^{-33} \text{ Hz}^2;$$

SNR scales with N_p
for Doppler)

Dynamical ($\tau \ll T$) vs Static ($\tau \gg T$)^[4]

Pulsar timing: sensitivity limits^[5,6]

$$\delta\phi = \int^t dt' \delta\nu(t')$$

$$\text{SNR}_I^2 = \frac{1}{\nu_I^2 t_{rms}^2 \Delta t} \int_0^{T_{obs}} dt h_I^2(t).$$

$$h_I(t) = \sum_{i=1}^N \delta\phi_{I,i}(t) - \delta\phi_{0,I}(t)$$

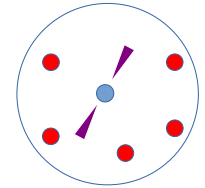
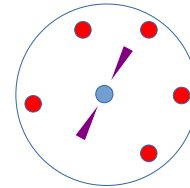
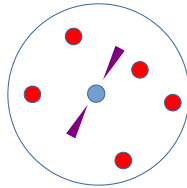
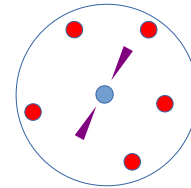
Sensitivity criterion: 90% of mock universes have max SNR > 4

(f, M) -> size of simulation volume

$f\rho_{DM}/M$ -> # density

Maxwell-Boltzmann -> velocity assignment

NB: We developed a parallelizable FORTRAN code to perform the simulation on a 72-core cluster

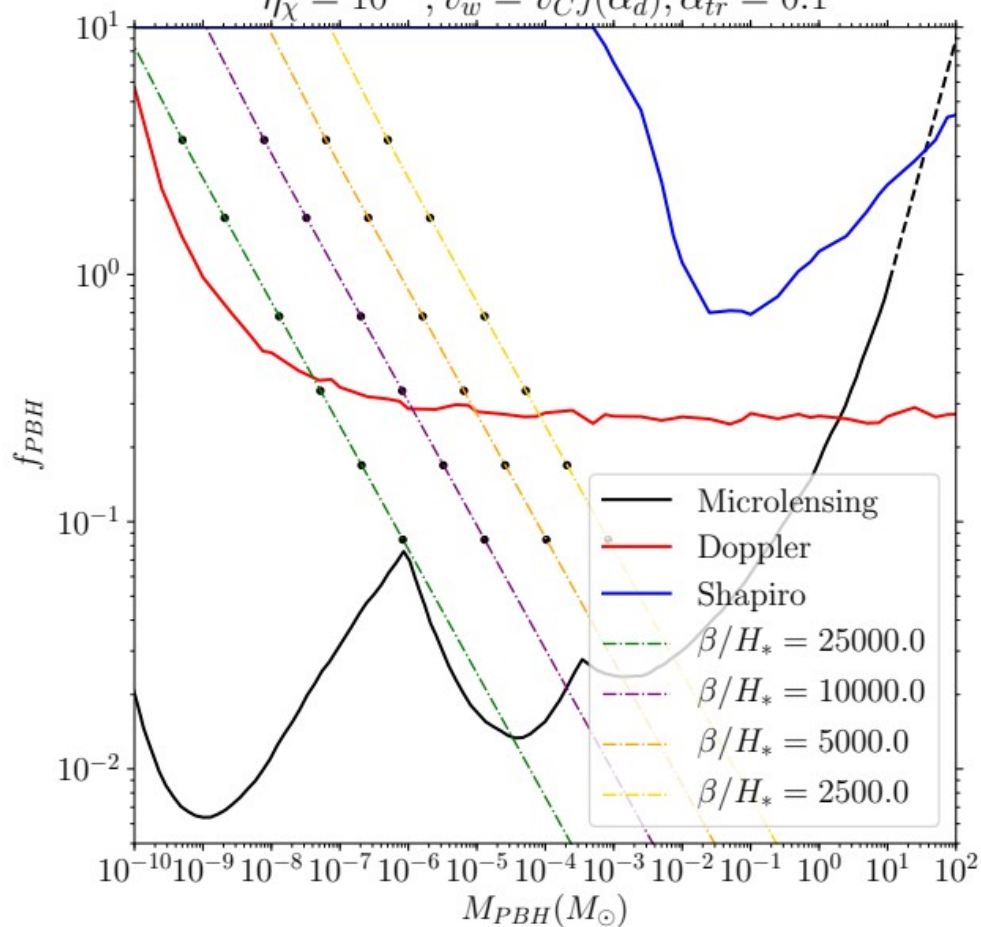


^[5]Lee, Vincent SH, et al. (2021)

^[6]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. (2020)

Monochromatic PBH mass (benchmark)

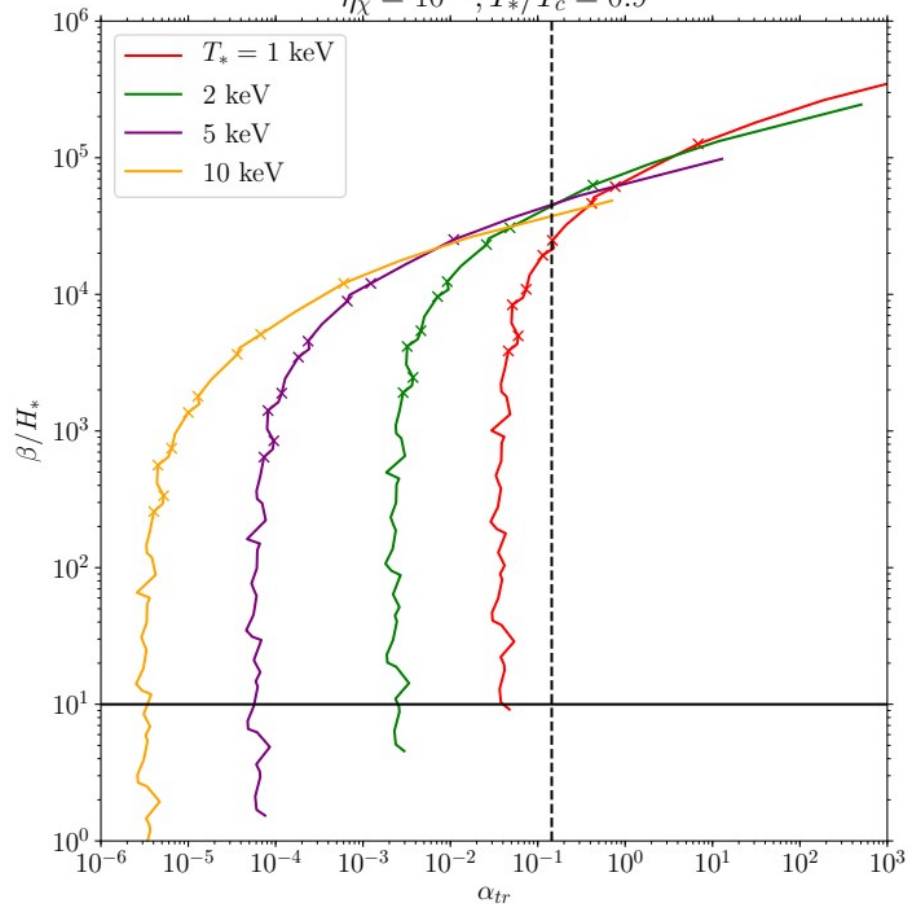
$$\eta_\chi = 10^{-5}, v_w = v_{CJ}(\alpha_d), \alpha_{tr} = 0.1$$



Novel PBH formation scenario

w/ P(M)

$$\eta_\chi = 10^{-5}, T_*/T_c = 0.9$$



^[4]Dror, Jeff A., et al. (2019)

^[7]D. Croon, D. McKeen, N. Raj and Z. Wang, (2020)

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Complementary signal: Stochastic GWs

- GW through sound waves, nonrunaway regime
- Assess sensitivity reach using some SNR
- Peak-integrated sensitivity curves (PISC)^[1] as a means to calculate SNR

$$\Omega_s(f)h^2 = \Omega_s^{peak} h^2 \mathcal{S}_s(f, f_s)$$

$$\mathcal{S}_s(f, f_s) = \left(\frac{f}{f_s}\right)^3 \left[\frac{7}{4 + 3(f/f_s)^2}\right]^{7/2}$$

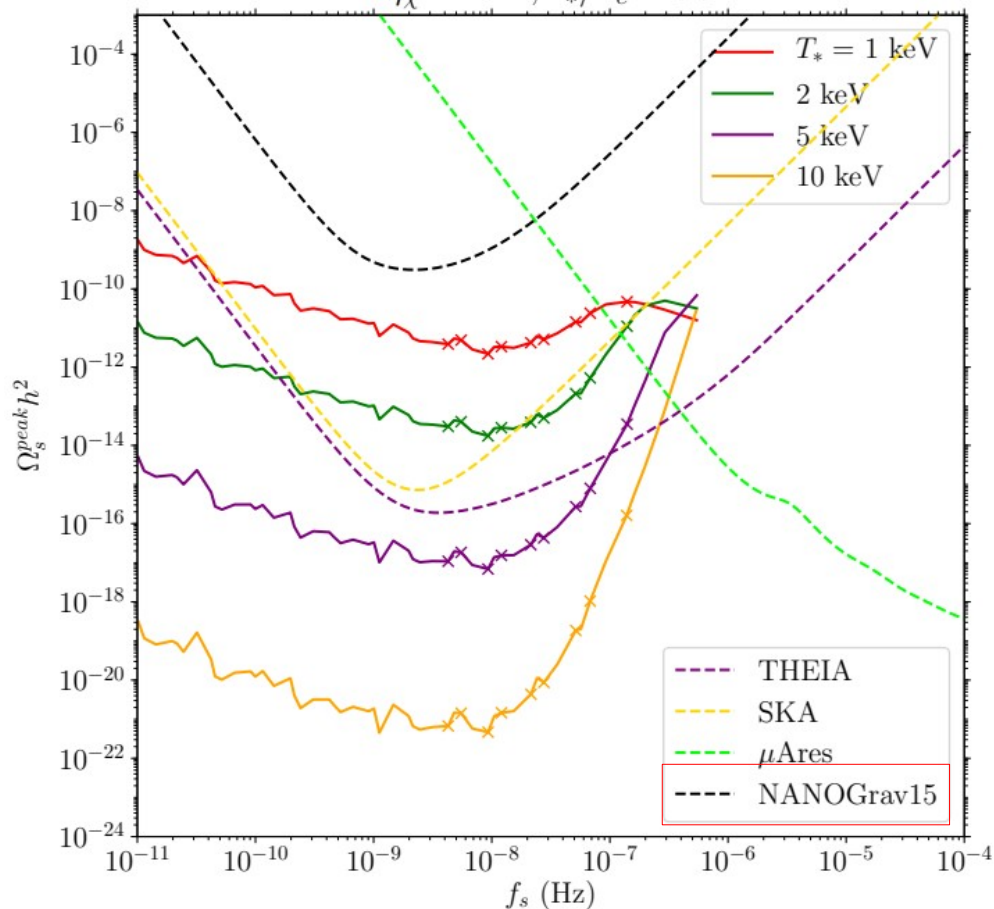
$$\rho^2 \equiv n_{det} \tau_{obs, GW} \int_{f_{min}}^{f_{max}} df \left[\frac{\Omega_{sig}(f)h^2}{\Omega_{noise}(f)h^2} \right]^2$$

$$\rho(f_s) = \frac{\Omega_{peak} h^2}{\Omega_{PIS}(f_s) h^2}$$

$$(\Omega_{PIS} h^2)^{-2} (f_s) \equiv n_{det} \tau_{obs, GW} \int_{f_{min}}^{f_{max}} df \left[\frac{\mathcal{S}(f, f_s)}{\Omega_{noise}(f) h^2} \right]^2$$

^[1]Schmitz, Kai. (2021)

$$\eta_\chi = 10^{-5}, T_*/T_c = 0.9$$



Sensitivity criterion: sGW SNR > 1

$$\Omega_s^{peak} h^2 \simeq 2.65 \times 10^{-6} \left(\frac{v_w}{\beta/H_*} \right) \left[\frac{100}{g_{*\rho,v}(T_*)} \right]^{1/3} \left(\frac{\kappa_s \alpha_{tr}}{1 + \alpha_{tr}} \right)^2 \left(1 + \frac{g_{*\rho,d}}{g_{*\rho,v}} \xi^4 \right)$$

$$f_s \simeq 1.9 \times 10^{-2} \text{ mHz} \left[\frac{g_{*v}(T_*/\xi)}{100} \right]^{1/6} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{\beta/H_*}{v_w} \right) \left(1 + \frac{g_{*\rho,d}}{g_{*\rho,v}} \xi^4 \right)^{1/2} \frac{1}{\xi}$$

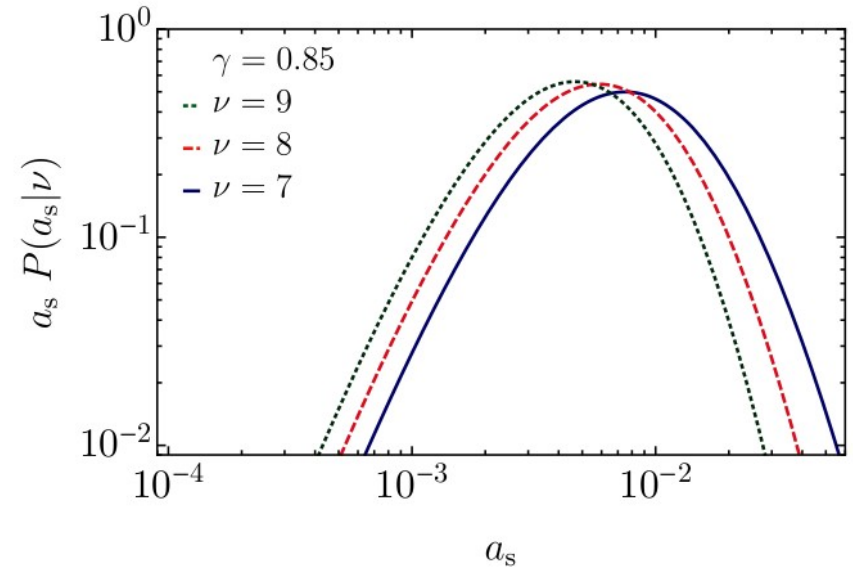
SKA is sensitive to ~ 1 keV
(~ 0.1 keV) for $\eta_\chi = 10^{-5}$ (10^{-4})

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Previous work on PBH spin

- Peebles 1969: galaxy rotation in MD
- Choptuik criticality (Choptuik '93; Baumgarte & Gundlach '16; Chiba & Yokoyama '17)
- 1st vs 2nd order approach (Gruzinov, Mirbabayi, Noreña '19; de Luca et al '19; Harada et al '21)
- *Assume that PBHs are formed from the enhancement of primordial power spectrum*



Taken from: De Luca, Valerio, et al. (2019)

Our approach

- Spin is induced by cosmological perturbations
- Gaussian scale invariant
- Assume spherical proto-object
- Cosmological perturbations could be modified by FOPT^[9]

$$\langle \mathcal{R}_k \mathcal{R}_{k'} \rangle = (2\pi)^3 P_{\mathcal{R}}(k) \delta^{(3)}(\vec{k} - \vec{k}'),$$

$$P_{\mathcal{R}}(\eta = 0, k) = \frac{2\pi^2}{k^3} A_s \left(\frac{k}{k_s} \right)^{n_s - 1}.$$

$$A_s = (2.196 \pm 0.060) \times 10^{-9},$$

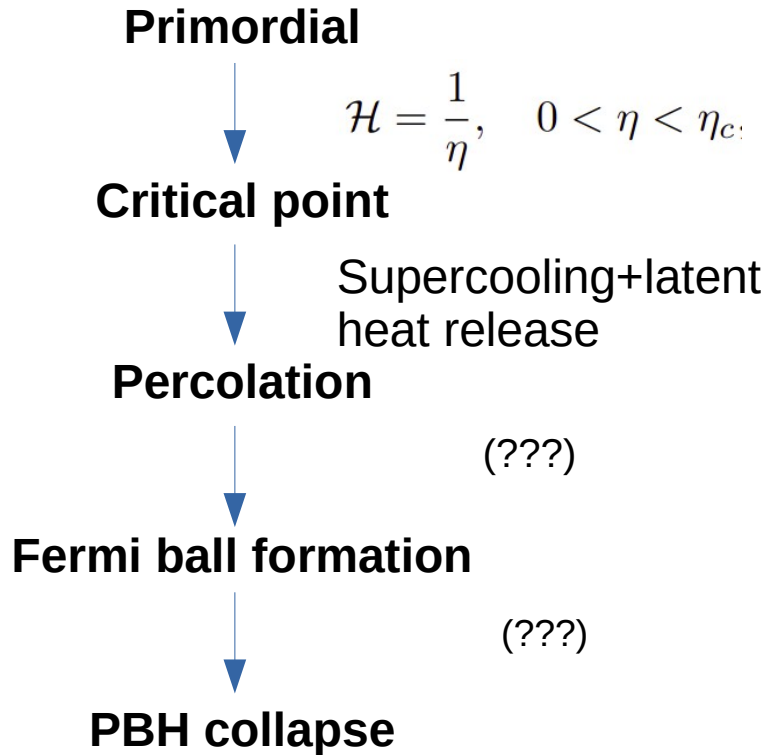
$$\vec{J}_{F,CM} = \int \rho_F a \vec{x} \times \vec{v} \sqrt{g_3} d^3 \vec{x} - \vec{R}_{CM} \times \vec{P}_{CM}$$

$$\begin{aligned} \vec{J}_{F,CM}(\eta) &= 4\pi \bar{\rho}_F R^5 \int d\Pi_k d\Pi_{k'} (\vec{k} \times \vec{k}') \delta_{eff}(\vec{k}, \eta) \\ &\times \left[\mathcal{F}(|\vec{k} + \vec{k}'| x_0) - 3\mathcal{F}(k x_0) \mathcal{G}(k' x_0) \right] \hat{\psi}_F(\vec{k}', \eta) / k'. \end{aligned}$$

$$\vec{s} \equiv \frac{\vec{J}}{G_N M^2}.$$

^[9]Schmid, Christoph, Dominik J. Schwarz, and Peter Widerin. (1999)

Cosmological evolution



$$\frac{da}{dx} = ah$$

$$\frac{dy}{dx} = -hy \left\{ 1 - \frac{3FTB'(T)}{4\rho_{\chi+TV}} + \frac{1}{h} \frac{dF}{dx} \left[\frac{B(T) - TB'(T)}{4\rho_{\chi+TV}} \right] \right\} \left[1 - \frac{FT^2B''(T)}{4\rho_{\chi+TV}} \right]^{-1}$$

$$\frac{dg_1}{dx} = -8\pi a^3 \frac{\Gamma(T_c y)}{H_c^4} v_w^3$$

$$\frac{dg_{i+1}}{dx} = \frac{g_i}{a}, \quad 1 \leq i \leq 2$$

$$\frac{dF}{dx} = g^3 \frac{F}{a}$$

$$\frac{d\tilde{\eta}}{dx} = \frac{1}{a},$$

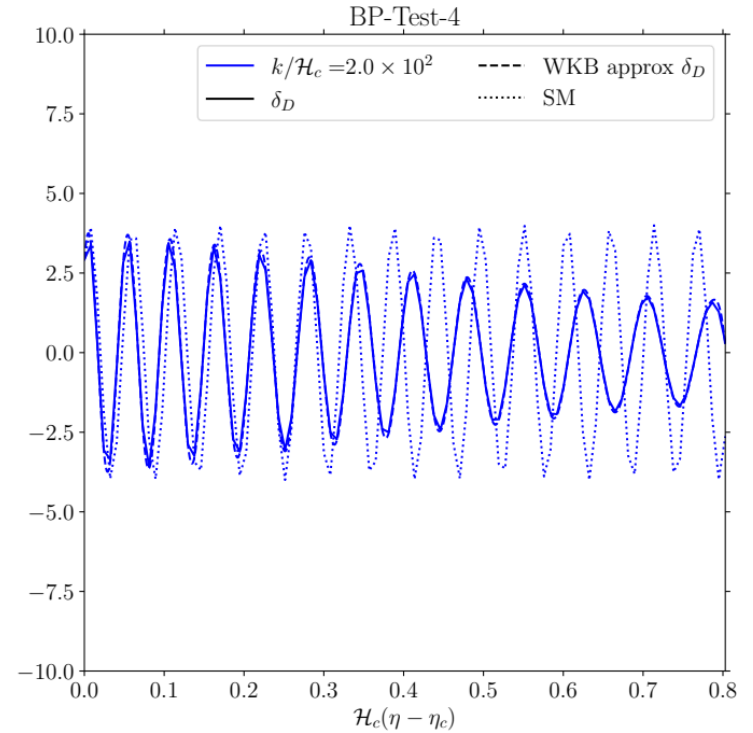
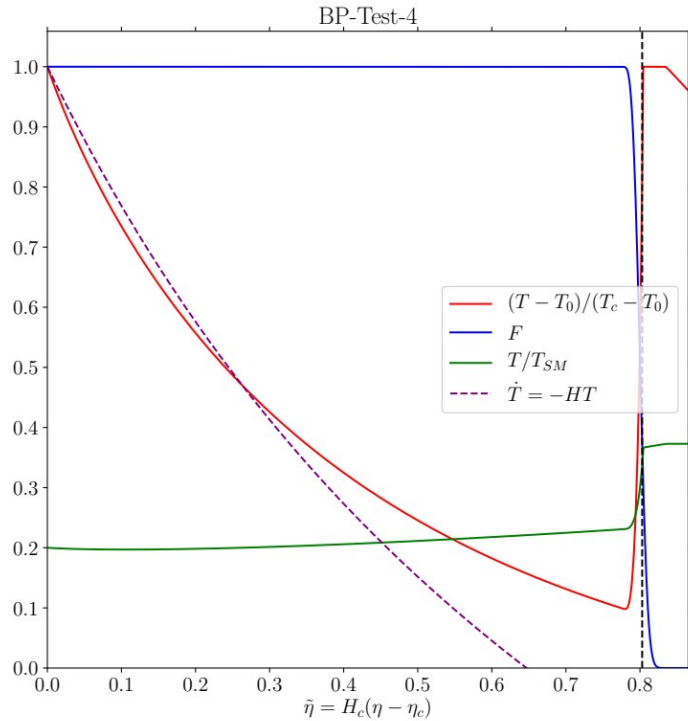
Background evolution

$$\frac{\delta'_F}{\mathcal{H}} - \frac{k}{\mathcal{H}} \hat{\psi}_F + 3(1 + w_F) A_k + 3(c_{s,F}^2 - w_F) \delta_F = 0,$$

$$\frac{\hat{\psi}'_F}{\mathcal{H}} + (1 - 3w_F) \hat{\psi}_F + c_{s,F}^2 \frac{k}{\mathcal{H}} \delta_F + (1 + w_F) \frac{k}{\mathcal{H}} A_k = 0.$$

Perturbation evolution

Cosmological evolution

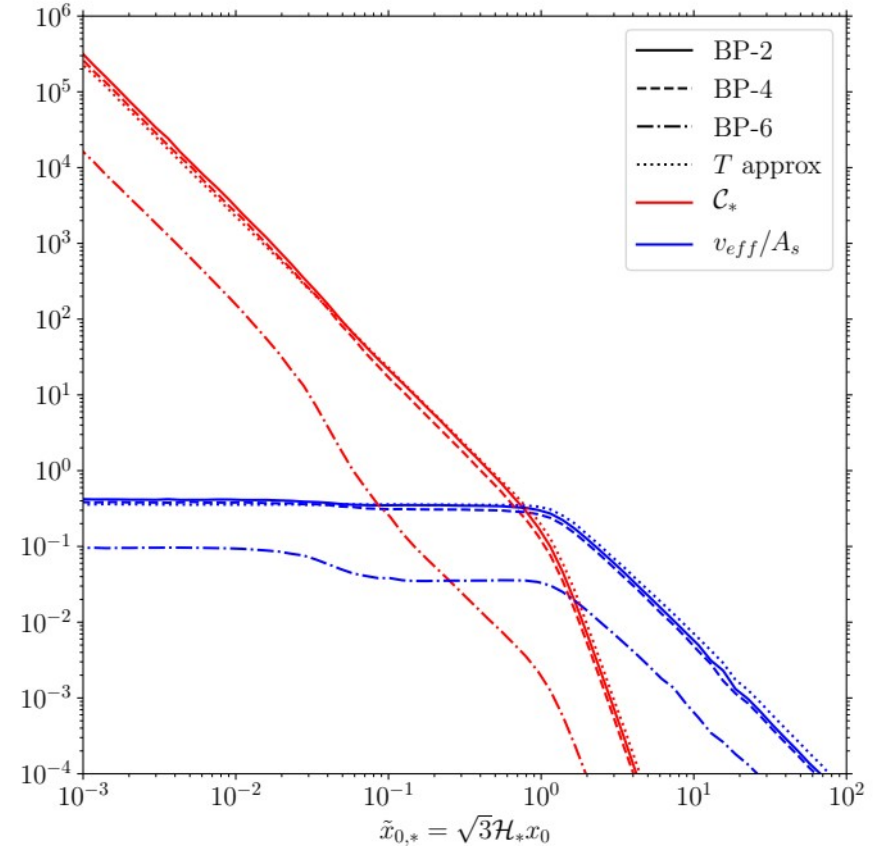


Preliminary results

$$s_{rms,*} = 6 \left(1 + \frac{\rho_{SM,*}}{\rho_{D,*}} \right) \frac{v_{eff,*}}{\tilde{x}_{0,*}^2},$$

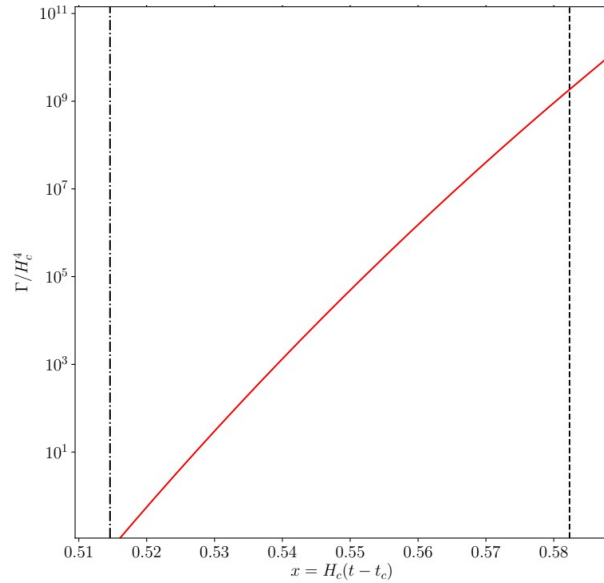
$$v_{eff} = \frac{3}{4} A_s \tilde{x}_0(\eta) \mathcal{C}^{1/2}(\tilde{x}_0, x_0),$$

$$\mathcal{C}(\tilde{x}_0, x_0) \equiv A_s^{-2} \int_0^\infty dz z \int_0^\infty dz' z' \\ \times \int_{-1}^1 dx (1-x^2) W^2(z, z'; \tilde{x}_0) \Delta_{\mathcal{R}}^2(k) \Delta_{\mathcal{R}}^2(k'),$$



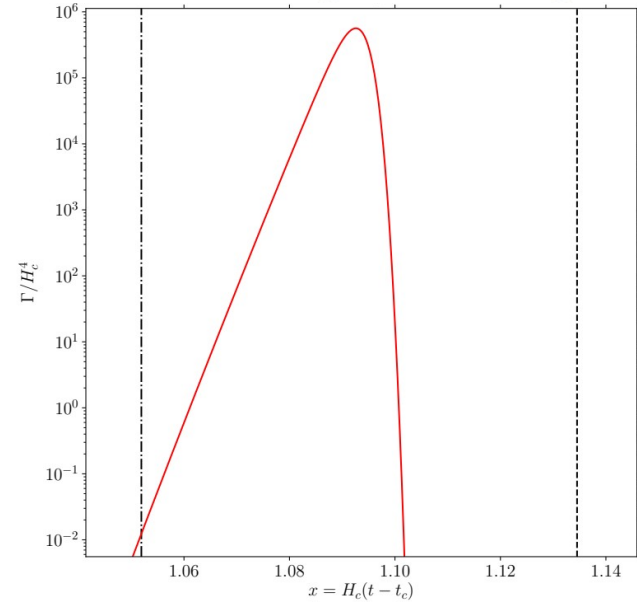
Preliminary results

$$\Gamma(t) \approx \Gamma_0 \exp[-\beta_0(t - t_0)].$$



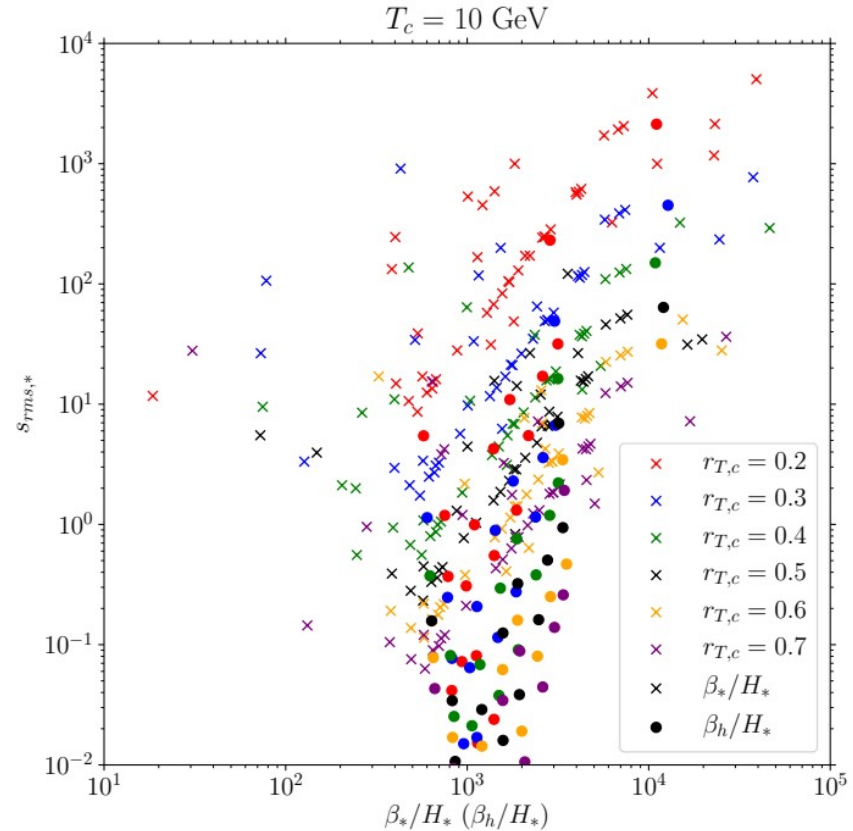
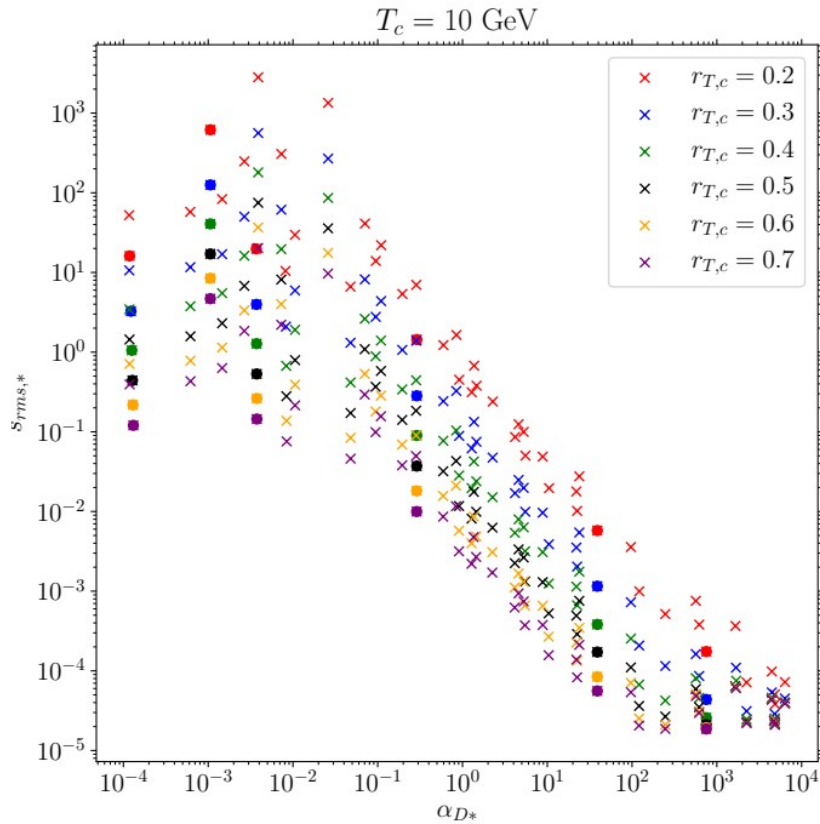
$$\frac{s_{rms,*}}{r_{T,c}^4} \approx \left(\frac{4}{3F_*}\right)^{2/3} v_{eff,*} \left(\frac{\beta_0/H_c}{v_w}\right)^2 \frac{\pi^2}{30} \frac{g_{*SM}}{\frac{3\pi^2}{20} + F_* \frac{L_c}{T_c^4}},$$

$$\Gamma(t) \approx \Gamma_h \exp\left[-\frac{\beta_h^2}{2}(t - t_h)^2\right].$$



$$\frac{s_{rms,*}}{r_{T,c}^4} \approx \frac{2\pi^3}{15} v_{eff,*} \left(\frac{2}{3F_*}\right)^{2/3} \left(\frac{\Gamma_h/H_c^4}{\beta_h/H_c}\right)^{2/3} \frac{g_{*SM}}{\frac{3\pi^2}{20} + F_* \frac{L_c}{T_c^4}},$$

Preliminary results



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Conclusions

- FOPT in early Universe is a subject rich in phenomenology
- Interesting playground to probe fundamental physics and inspiration for exotic physics
- In focus: novel PBH formation mechanism
- Pulsar timing can probe this novel PBH formation scenario
- Sensitivity range: PBH mass of $10^{-8}\sim 10^{-4}$, GW frequency of nHz \sim μ Hz
- Provided a first step in calculating spin of PBH from this novel formation mechanism

Acknowledgments

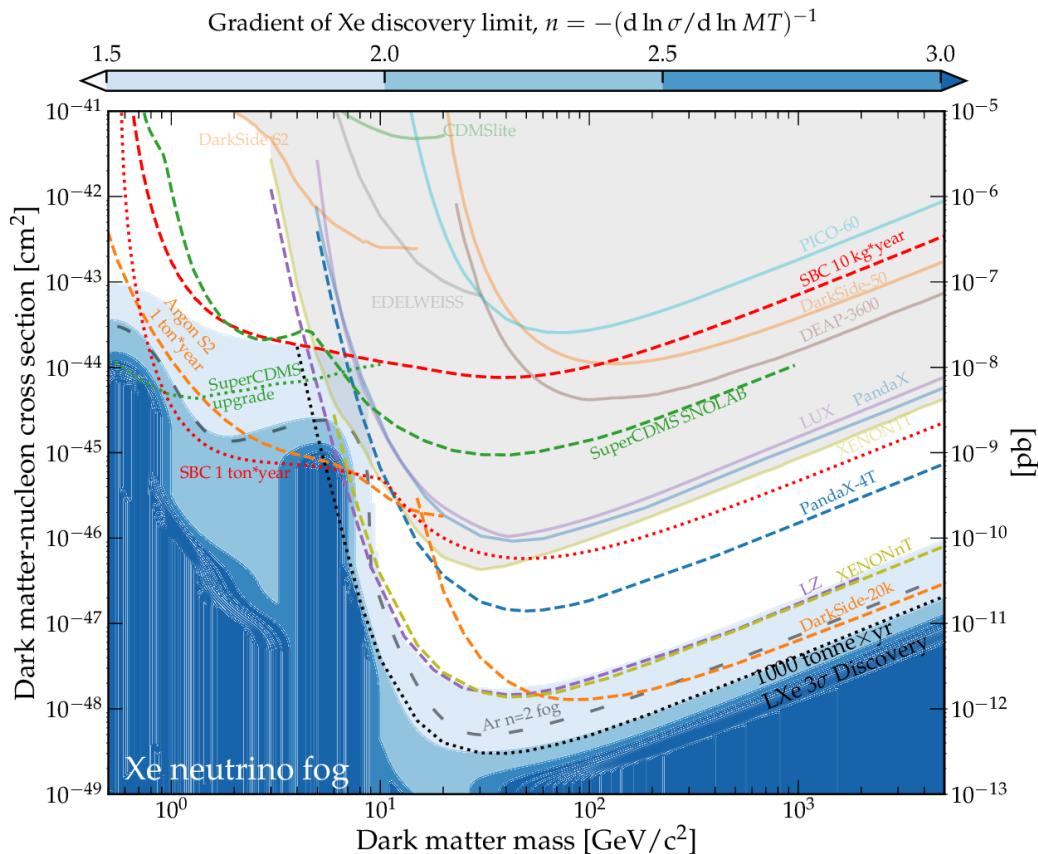
- NSTC grant # NSTC 111-2811-M-007-018-MY2
- NTHU IoA CICA cluster

Thank you for your attention!

感謝各位的聆聽！

Extra slides

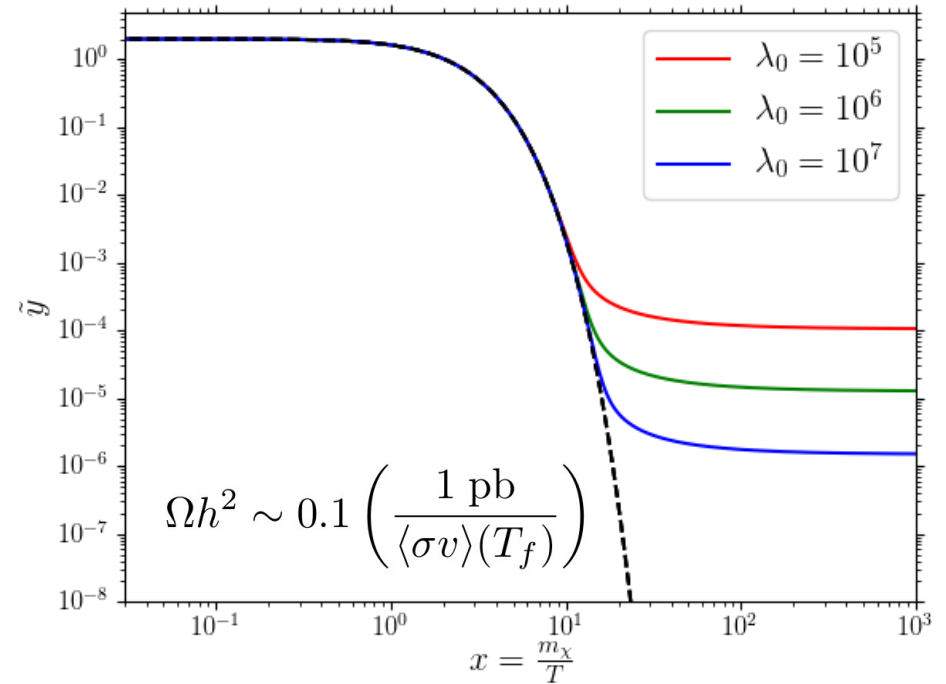
(WIMP) dark matter



Retrieved from: <https://lss.fnal.gov/archive/2022/conf/fermilab-conf-22-180-v.pdf>

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WIMP thermal production



$$\langle \sigma v \rangle \approx (2.5 \times 10^{-9} \text{ GeV}^{-2}) \left(\frac{\alpha_X}{0.5 \times 10^{-2}} \right)^2 \left(\frac{100 \text{ GeV}}{m} \right)^2$$

WIMP unitarity limit: $m < 20 \text{ TeV}$

Primordial black holes

- cf. BH from stellar collapse ($\sim 1-10 M_{\text{sol}}$)
- Can be formed at any mass^[1]
- Potential DM candidate
- Proposed formation mechanisms
 - Collapse of overdense regions from primordial fluctuations
 - Critical collapse and Choptuik scaling
 - PBH from FOPT
 - Softening of fluid EoS (QCD PT)
 - Bubble wall collisions during FOPT^[2]
 - ***Collapse of Fermi balls from filtered out DM during dark FOPT***^[3]

$$\rho_{\text{BH}} = \frac{3}{8\pi G_N} \frac{1}{R_s^2} \qquad \bar{\rho} = \frac{3}{8\pi G_N} \frac{1}{(1/H)^2}$$

$$M_H = \frac{1}{2G_N H} = \frac{t}{G_N} \approx 10^{-18} M_{\text{sol}} \left(\frac{t}{10^{-23} \text{s}} \right)$$

$$T_{\text{Hawking}} = 8.62 \times 10^{-12} \text{eV} \left(\frac{M_{\text{sol}}}{M} \right)$$

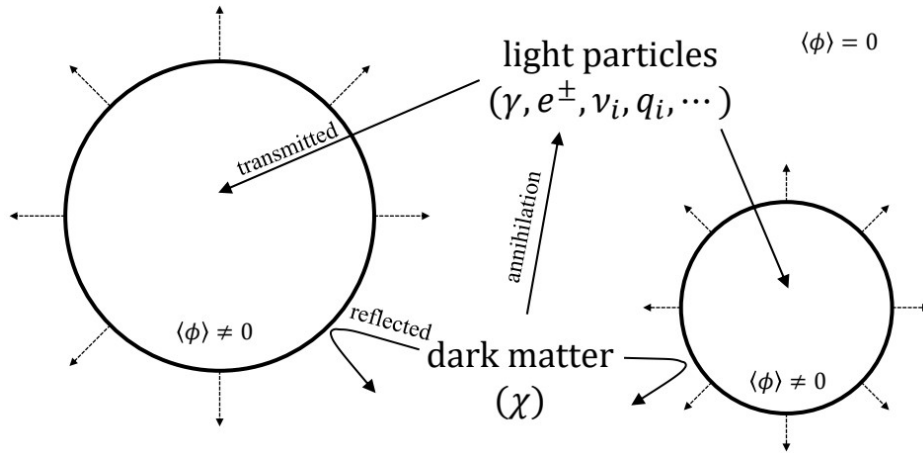
$$\tau = 10^{64} \text{y} \left(M / M_{\text{sol}} \right)^3$$

^[1]Zeldovich Ya., and Novikov I. D. (1974)

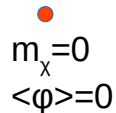
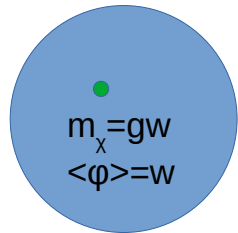
^[2]Kodama, Hideo, Misao Sasaki, and Katsuhiko Sato. (1982)

^[3]Kawana, Kiyoharu, and Ke-Pan Xie. (2022)

Filtered out dark matter



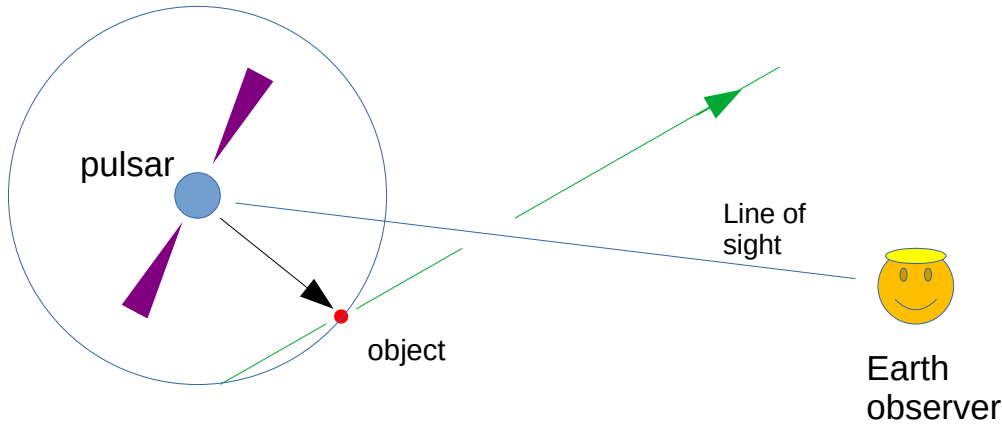
Taken from: Dongjin Chway, Tae Hyun Jung, Chang Sub Shin, Phys. Rev. D 101, 095019 (2020)



$$gw \gg T_*$$

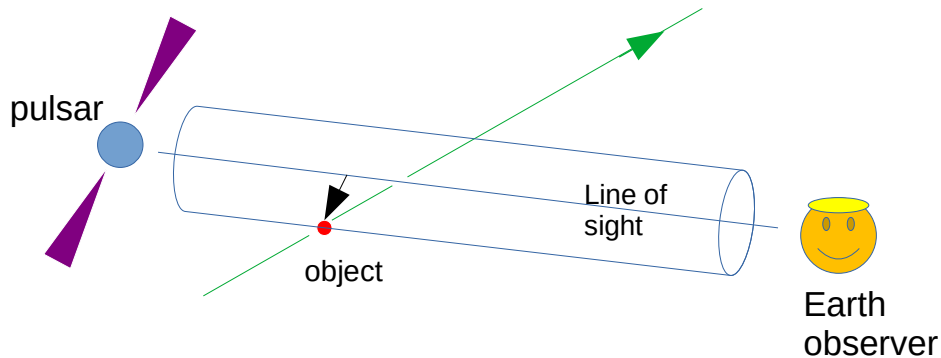
- DM mass is dynamically generated from FOPT
- Key ingredient: mass difference $\gg T$
- DM number density is Boltzmann suppressed like in WIMP freeze out

Pulsar timing: Doppler & Shapiro



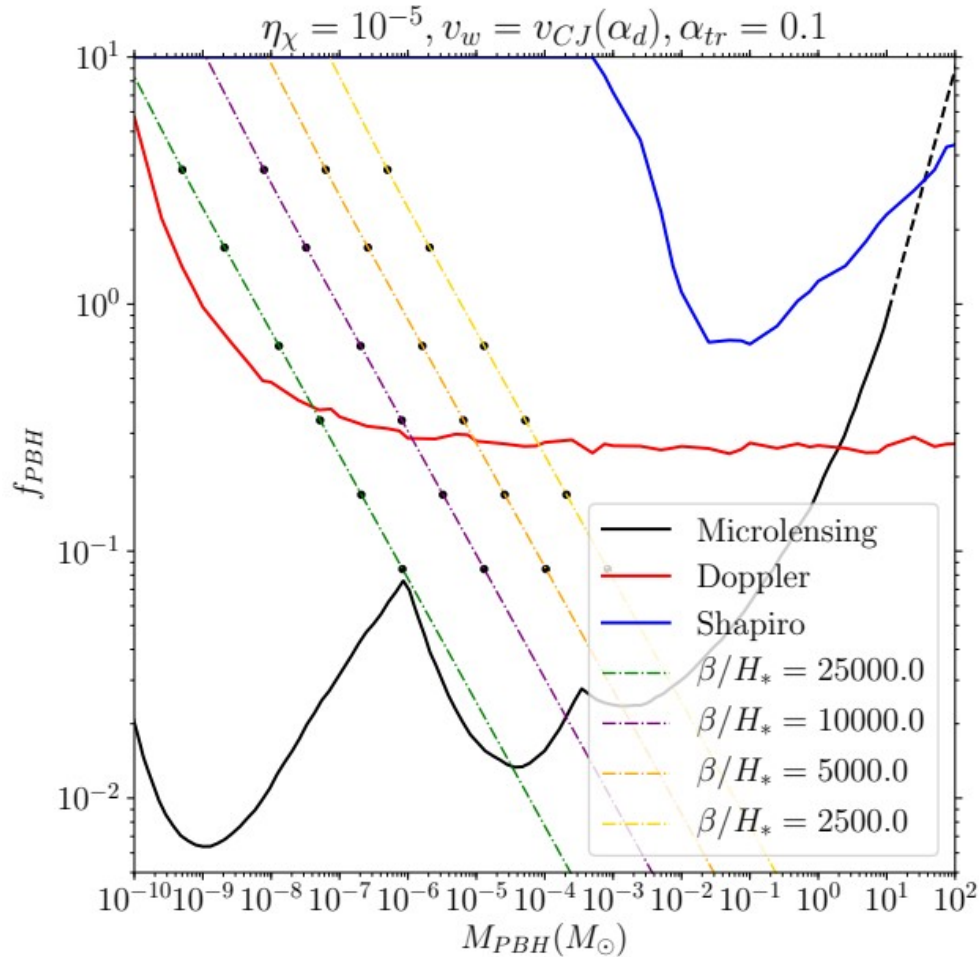
$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{1}{c} \hat{d} \cdot \int \vec{\nabla} \Phi dt$$

$$\Gamma_D \sim \pi \nu n^{1/3} \sim 0.1 f^{1/3} \left(\frac{M}{10^{-8} M_{sun}}\right)^{-1/3} yr^{-1}$$



$$\left(\frac{\delta\nu}{\nu}\right)_S = -\frac{2}{c^3} \vec{v} \cdot \int \vec{\nabla} \Phi dz$$

$$\Gamma_S \sim \pi^{1/2} \nu (nL)^{1/2} \sim 0.17 f^{1/2} \left(\frac{M}{10^{-4} M_{sun}}\right)^{-1/2} \left(\frac{L}{10 \text{ kpc}}\right)^{1/2} yr^{-1}$$

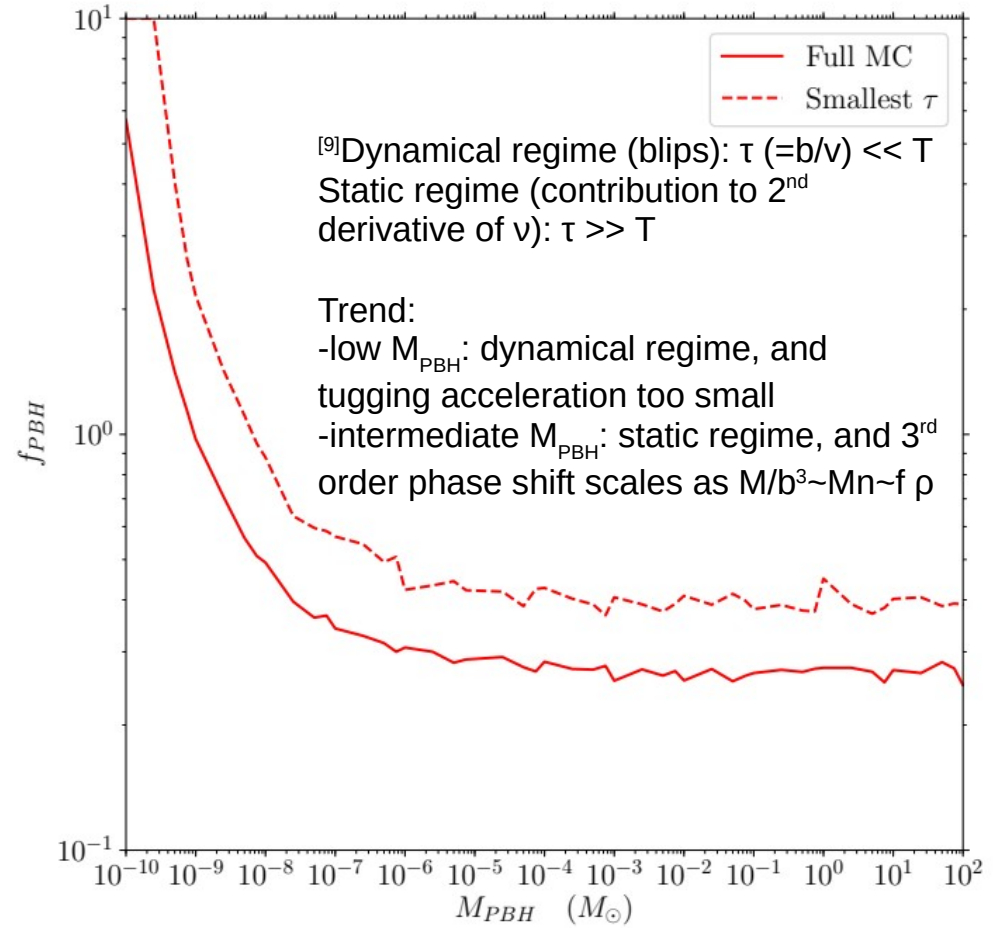


Microlensing

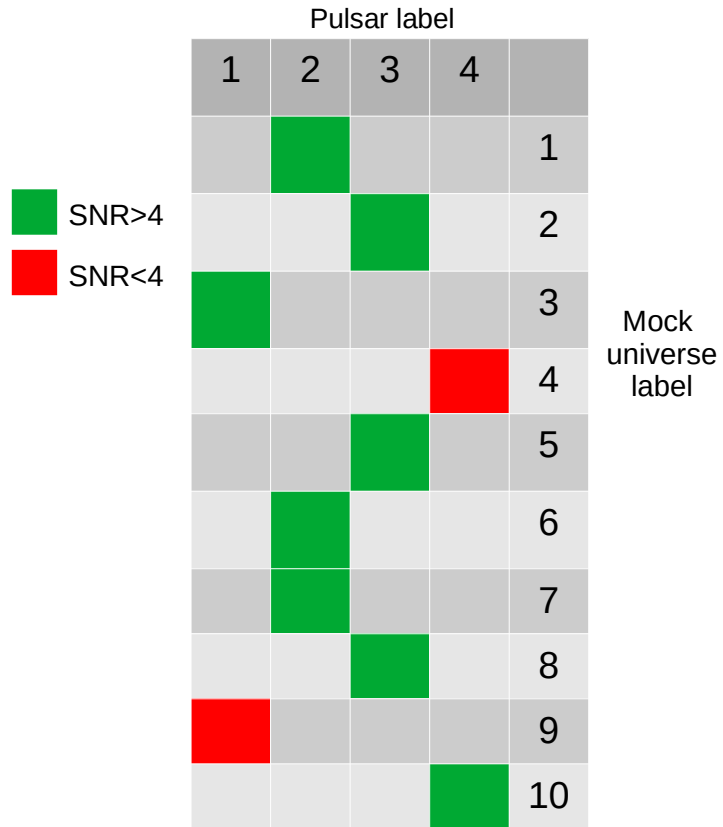
- multiple images are formed, but not resolved as separate
- change the magnification
- microlensing event is registered if magnification is >1.34

Number of expected microlensing events:

- $N_* T_{\text{obs}}$ (rate per source star) (transit time)
- rate per source star $\sim 1/M$



Pulsar timing: sensitivity limits^[6,7]



Sensitivity criterion: 90% of mock universes have max SNR > 4

^[6]Lee, Vincent SH, et al. (2021)

^[7]Ramani, Harikrishnan, Tanner Trickle, and Kathryn M. Zurek. (2020)

Generic quartic potential

$$V_{eff}(\phi, T) = D (T^2 - T_0^2) \phi^2 - (AT + C) \phi^3 + \frac{\lambda}{4} \phi^4$$

cf. Ref. [10]

$$-B = -DT_0^2 \phi_0^2 - C \phi_0^3 + \frac{\lambda}{4} \phi_0^4$$

$$0 = -2DT_0^2 \phi_0 - 3C \phi_0 + \lambda \phi_0^2,$$

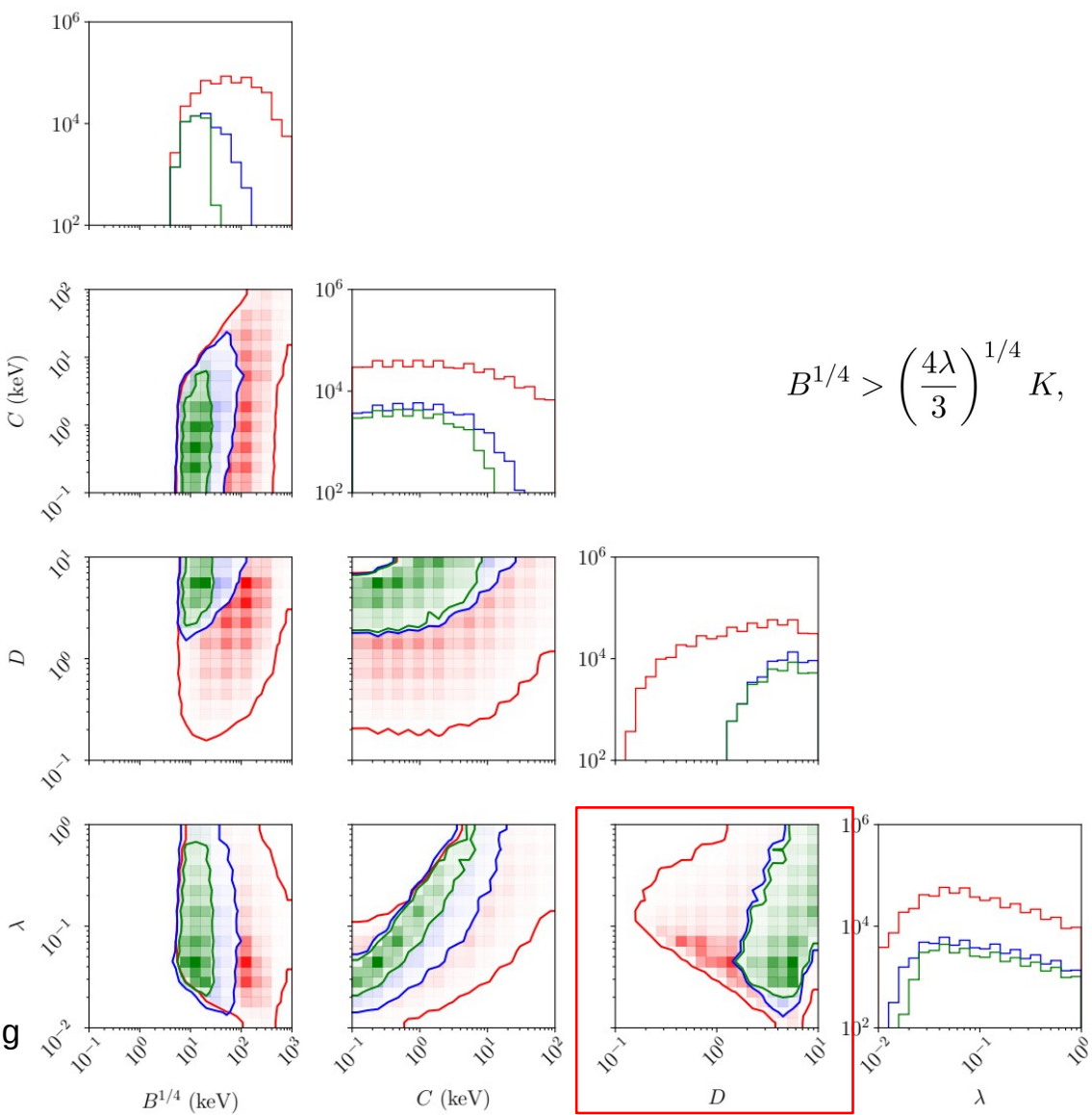
$$\{B^{1/4}, C, D, \lambda\} \longrightarrow \{\eta_\chi, T_*, \alpha_{tr}, T_c, \xi, \beta/H_*, v_w\} \longrightarrow$$

Effective potential
parameters

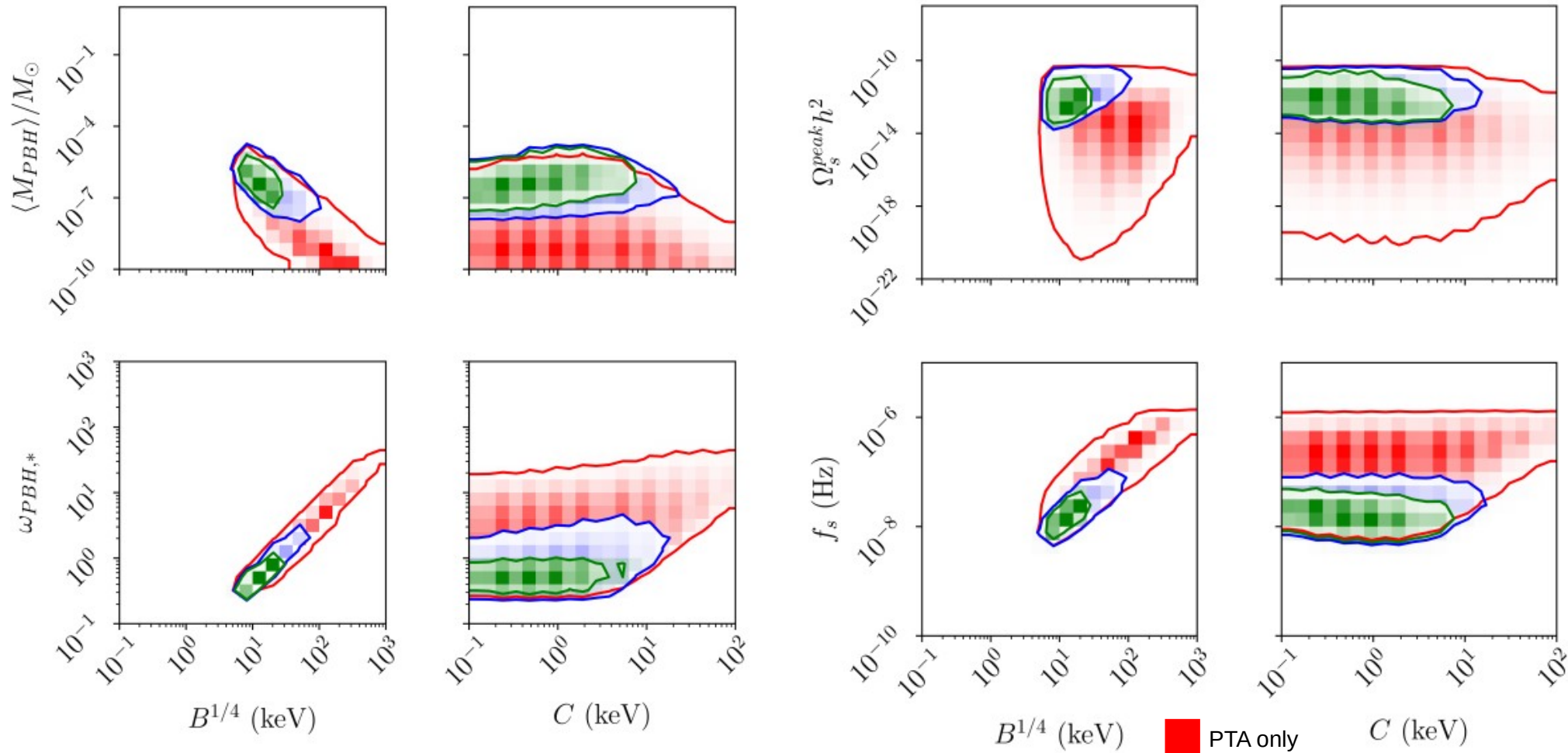
FOPT parameters

Observables:
-Ave. PBH mass
-PBH fraction
-Peak GW
abundance
-Peak GW
frequency

^[12]Marfatia, Danny, and Po-Yan Tseng. "Correlated signals of first-order phase transitions and primordial black hole evaporation." *Journal of High Energy Physics* 2022.8 (2022): 1-14.



Plot from: JTA, Po-yan Tseng
 JHEP 08 (2023) 117



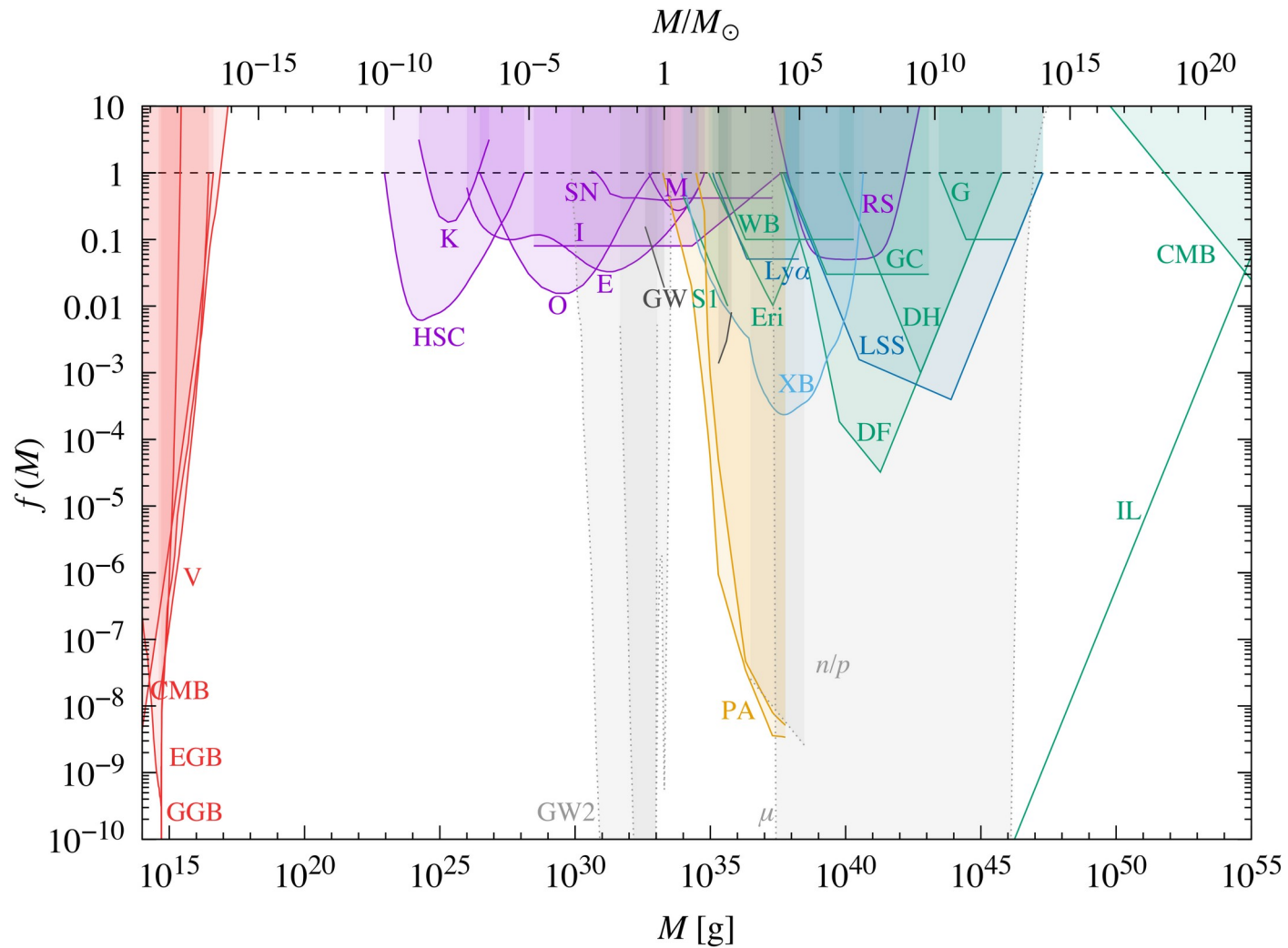


Figure from: B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Constraints on primordial black holes, Rept. Prog. Phys. 84(11), 116902 (2021)