Using the magnetic equation of state to determine the curvature of the chiral phase transition line of (2+1)-flavor QCD

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The Future is Flavorful
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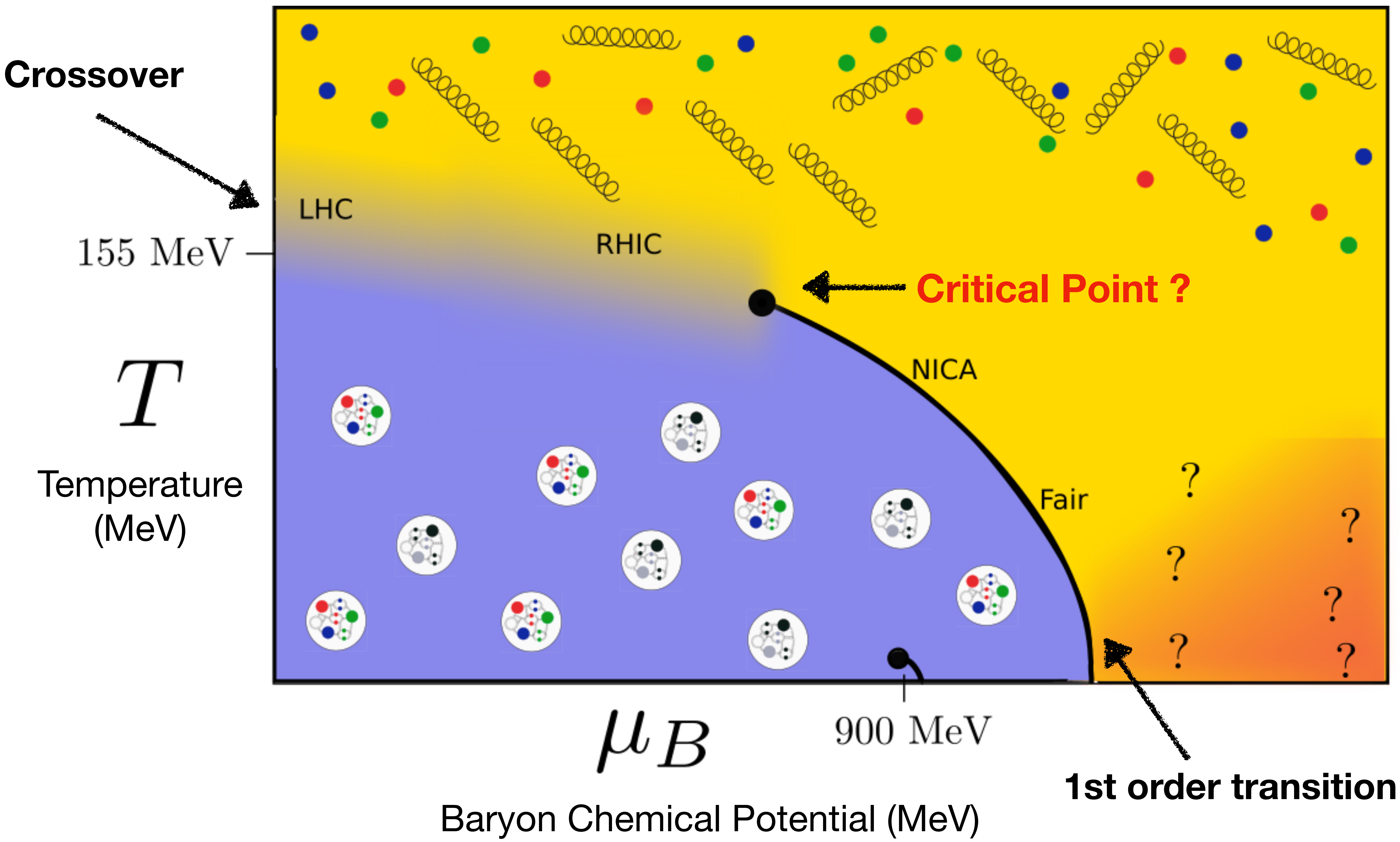
Outline

- Introduction and Motivation
- Calculational Setup
- Chiral Observables
- Universal Critical Behavior
- Magnetic Equation of State
- Pseudo-critical temperatures
- Curvature Coefficients
- Conclusions

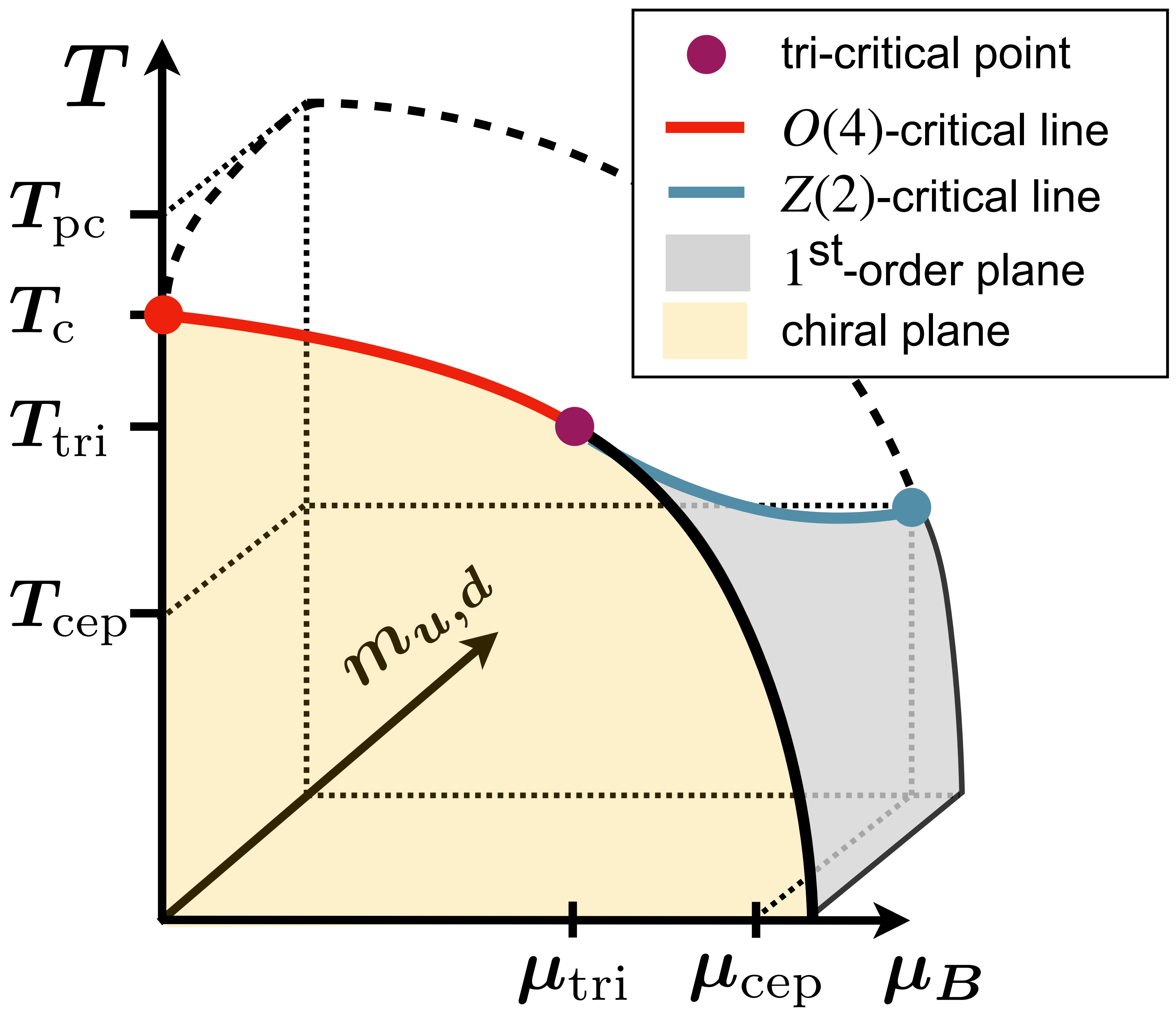
- Heavy-ion collision experiments, studying QCD thermodynamic, actively searching for the critical endpoint
- Lattice studies suffer from sign problem at non-zero μ
- Lattice studies established crossover between low-temperature hadronic phase and high-temperature quark-gluon plasma

Guenther, 2010.15503



QCD phase diagram

Phase diagram in the quark mass space



- The exact chiral $SU(2)_L \times SU(2)_R$ symmetry is expected to undergo a 2nd order PT at T_c in the $3d$ $O(4)$ universality class
- Lattice determination $T_c \approx 132$ MeV consistent with a continuous transition *[Ding et al., Phys. Rev. Lett. (2019), Kotov et al, Phys. Lett. B (2021)]*
- However, the universality class is not yet confirmed
- Expected order of temperatures $T_{cep} < T_{tri} < T_c < T_{pc}$ puts an upper limit on T_{cep}
- Important to determine the dependence of T_c on chemical potentials

Calculational Setup

QCD Partition function in 4d discrete Euclidean spacetime

$$Z(T, \vec{\mu}, V, m_f) = \int \mathcal{D}U e^{-S_G(\beta,U)} \prod_{f=u,d,s} (\det \mathcal{M}_f(m_f, \mu_f))^{1/4}$$

$\mathcal{M}_f$: Highly Improved Staggered Quark Dirac Matrix for quark flavor f

$m_l \equiv m_u = m_d \to 0; m_s$ fixed to physical value

Quark chemical potential $\vec{\mu} \equiv (\mu_u, \mu_d, \mu_s)$

Simulation details

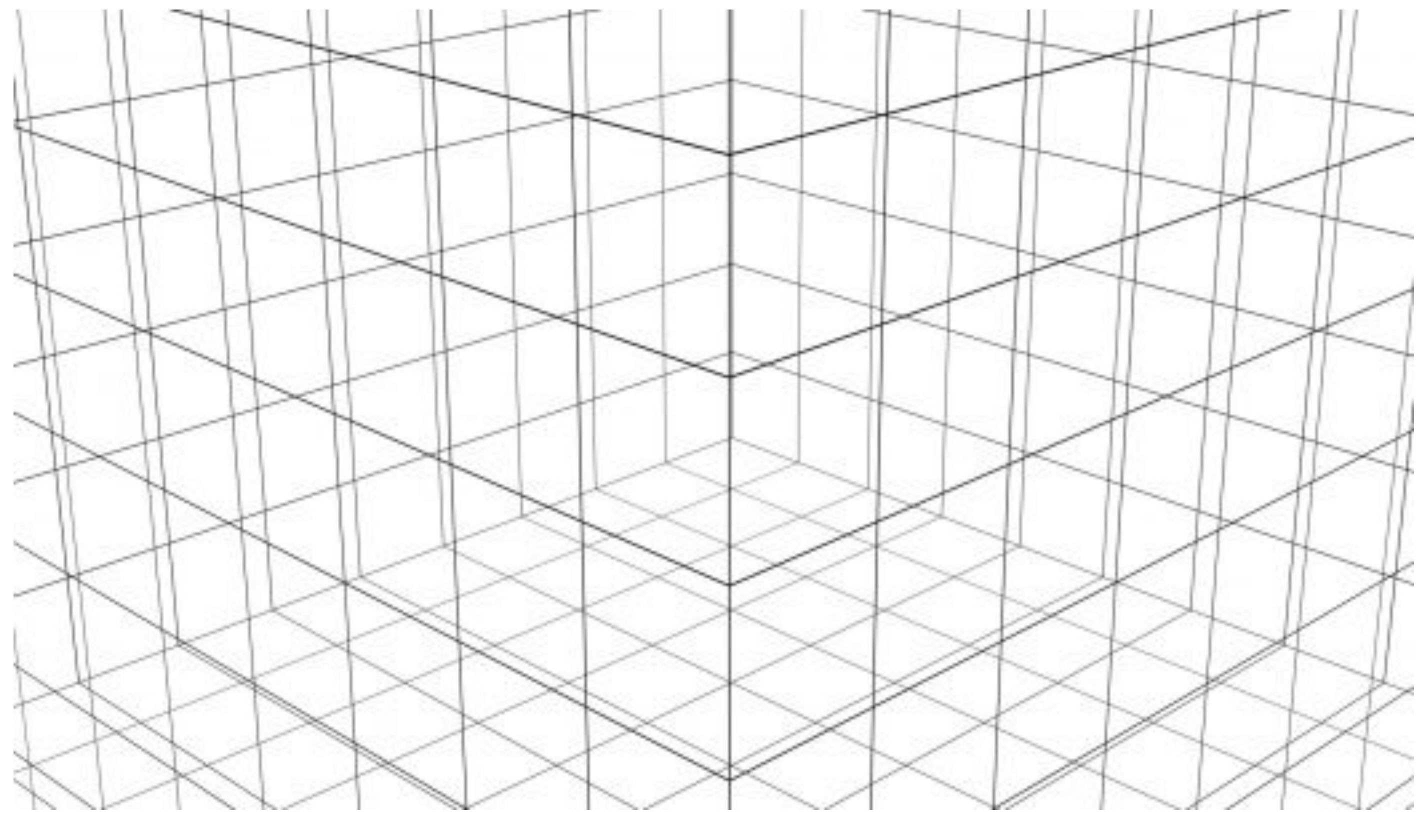
$T \simeq 135 - 175$ MeV

Symmetry breaking parameter $H = m_l/m_s$

$H = 1/20, 1/27, 1/40, 1/80, 1/160$

(Pion masses : 160 ~ 55 MeV)

Image : https://www.olcf.ornl.gov/tag/lattice-qcd/



$N_\sigma^3 N_\tau$ lattice points

Lattice spacing a controlled by gauge coupling β

Volume $V = (aN_\sigma)^3$

Temperature $T = 1/(aN_\tau)$

Fixed temporal extent $N_\tau = 8$; continuum limit not done in this work

Package `SIMULATeQCD` used in this study

Mazur et al (HotQCD collaboration) Comp Phys Comm 2024

Chiral Observables

Chiral condensate

$$\tilde{\Sigma}_f = \frac{T}{V}\frac{\partial \ln Z}{\partial m_f}$$

$$\tilde{\Sigma}_\ell = \tilde{\Sigma}_u + \tilde{\Sigma}_d$$

$$M_\ell = \frac{m_s}{f_K^4}\tilde{\Sigma}_\ell$$

$$M_{\rm sub} = M_\ell - 2H\frac{m_s}{f_K^4}\tilde{\Sigma}_s$$

dimensionless, renormalized

$$M = M_\ell - H\chi_\ell$$

kaon decay constant $f_K = 155.7/\sqrt{2}$ MeV

Chiral susceptibility

$$\tilde{\chi}_{m,f} = \frac{\partial \tilde{\Sigma}_f}{\partial m_f}$$

$$\tilde{\chi}_{m,\ell} = \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\tilde{\Sigma}_\ell$$

$$\chi_\ell = \frac{m_s^2}{f_K^4}\tilde{\chi}_{m,\ell}$$

$$\chi_m^{M_{\rm sub}} = \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)M_{\rm sub}$$

Mixed susceptibilities

$$\chi_{t(T)}^{M_\ell} = -T_c\frac{\partial M_\ell}{\partial T}$$

$$\chi_{t(f,f)}^{M_\ell} = -\frac{\partial^2 M_\ell}{\partial \hat{\mu}_f^2}$$

$$\hat{\mu} = \mu/T$$

$$\mu_\ell = \frac{1}{3}\mu_B$$

$$\chi_{t(\ell,s)}^{M_\ell} = -\frac{\partial^2 M_\ell}{\partial \hat{\mu}_\ell \partial \hat{\mu}_s}$$

$$\mu_s = \frac{1}{3}\mu_B - \mu_S$$

$$\chi_{t(T)}^{M} = -T_c\frac{\partial M}{\partial T}$$

derivative with temperature-like variables

Universal Critical Behavior

In the vicinity of the critical point, the free energy density

$$f = -(T/V)\,\ln Z = f_s(T, \overrightarrow{\mu}, \overrightarrow{m}) + f_{\text{sub-lead}}(T, \overrightarrow{\mu}, \overrightarrow{m})$$

with $\overrightarrow{m} = (m_\ell, m_s)$ **and** $\overrightarrow{\mu} = (\mu_\ell, \mu_s)$ **or** (μ_B, μ_S)

Energy-like : $t = \dfrac{\bar{t}}{t_0} = \dfrac{1}{t_0}\left(\dfrac{T - T_c}{T_c} + \kappa_2^\ell \hat{\mu}_\ell^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{\ell s}\hat{\mu}_\ell \hat{\mu}_s\right)$

Magnetization-like : $h = \dfrac{H}{h_0} = \dfrac{1}{h_0}\dfrac{m_\ell}{m_s}$

- derivatives of f_s -> divergences
- sub-leading : regular + corrections-to-scaling
- isospin or electric charge chemical potentials not included

Universal scaling functions

$$f_s(T, \overrightarrow{\mu}, \overrightarrow{m}) = h_0 h^{1+1/\delta} f_f(z)$$

$$z = z_0 z_b\ ,\ z_b = \bar{t}/H^{1/\beta\delta}\ ,\ z_0 = h_0^{1/\beta\delta}/t_0$$

$$f_G(z) = -\left(1 + \frac{1}{\delta}\right) f_f(z) + \frac{z}{\beta\delta} f_f'(z)$$

$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f_G'(z)\right)$$

Universal Critical Behavior

Staggered quarks at finite lattice spacing leads to $O(2)$ symmetry

Critical exponents in $3d\ O(2)$ universality class

$\beta = 0.3490(30)$, $\delta = 4.7798(5)$

$$M_{\ell/\mathrm{sub}} = h^{1/\delta} f_G(z) + \text{sub-leading}$$

$$M = h^{1/\delta}\left(f_G(z) - f_\chi(z)\right) + \text{ sub-leading}$$

$$\chi_X^{M_\ell} = -\, t_0^{-1} h^{(\beta-1)/\beta\delta} f'_G(z) + \text{ sub-leading, } X = t(T), t(f,f), t(\ell, s) \longrightarrow \text{maxima at } z_t$$

$$\chi_{t(T)}^{M} = -\, t_0^{-1} h^{(\beta-1)/\beta\delta} (f'_G(z) - f'_\chi(z)) + \text{ sub-leading} \longrightarrow \text{maxima at } z_{t,M}$$

$$\chi_m^{M_{\mathrm{sub}}} = \frac{h^{1/\delta-1}}{h_0} f_\chi(z) + \text{ sub-leading} \longrightarrow \text{maxima at } z_m$$

$z_m = 1.6675(68)$

$z_t = 0.7991(96)$

$z_{t,M} = 0.629(10)$

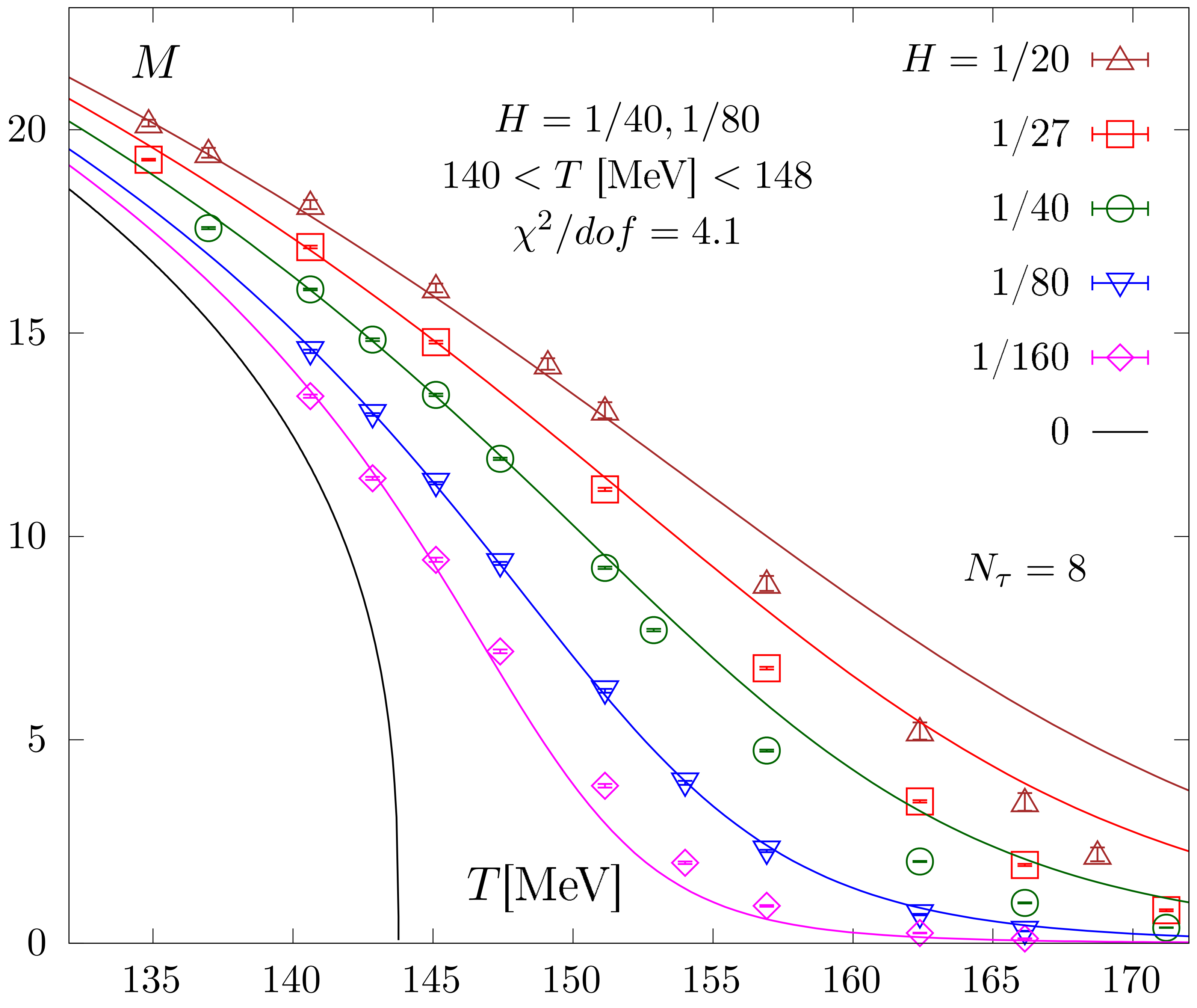
Pseudocritical temperatures :

$$T_{pc,x}(\hat\mu_\ell, \hat\mu_s, H) = T_c \left(1 + (\kappa_2^\ell \hat\mu_\ell^2 + \kappa_2^s \hat\mu_s^2 + 2\kappa_{11}^{\ell s} \hat\mu_\ell \hat\mu_s) + \frac{z_x}{z_0} H^{1/\beta\delta} \right) , \quad x = t, m, (t, M)$$

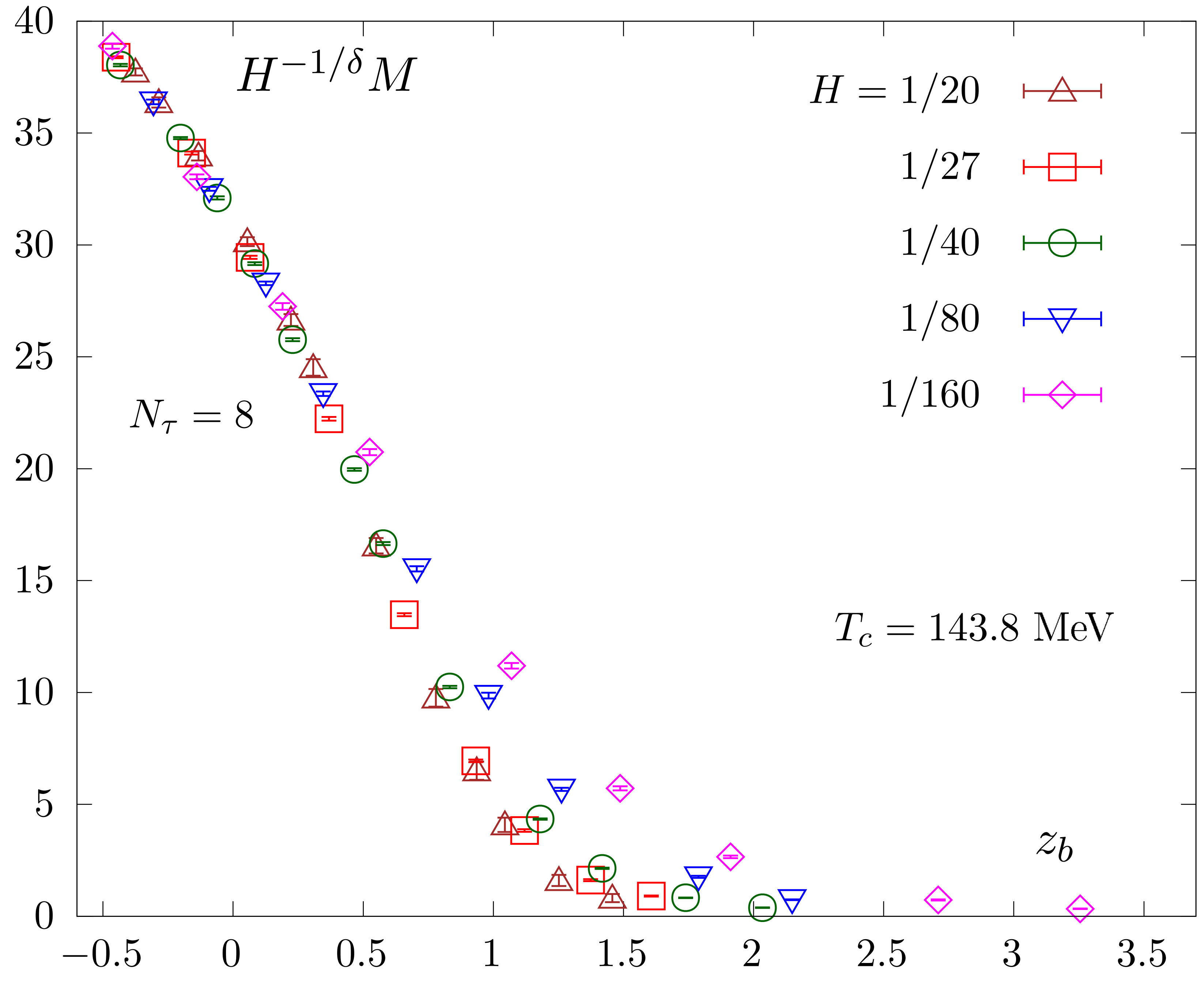
curvature coefficients

Magnetic Equation of State

$$M = h_0^{-1/\delta} H^{1/\delta}(f_G(z) - f_\chi(z))$$



$T_c = 143.7(2)$ MeV
$z_0 = 1.42(6)$
$h_0^{-1/\delta} = 39.2(4)$

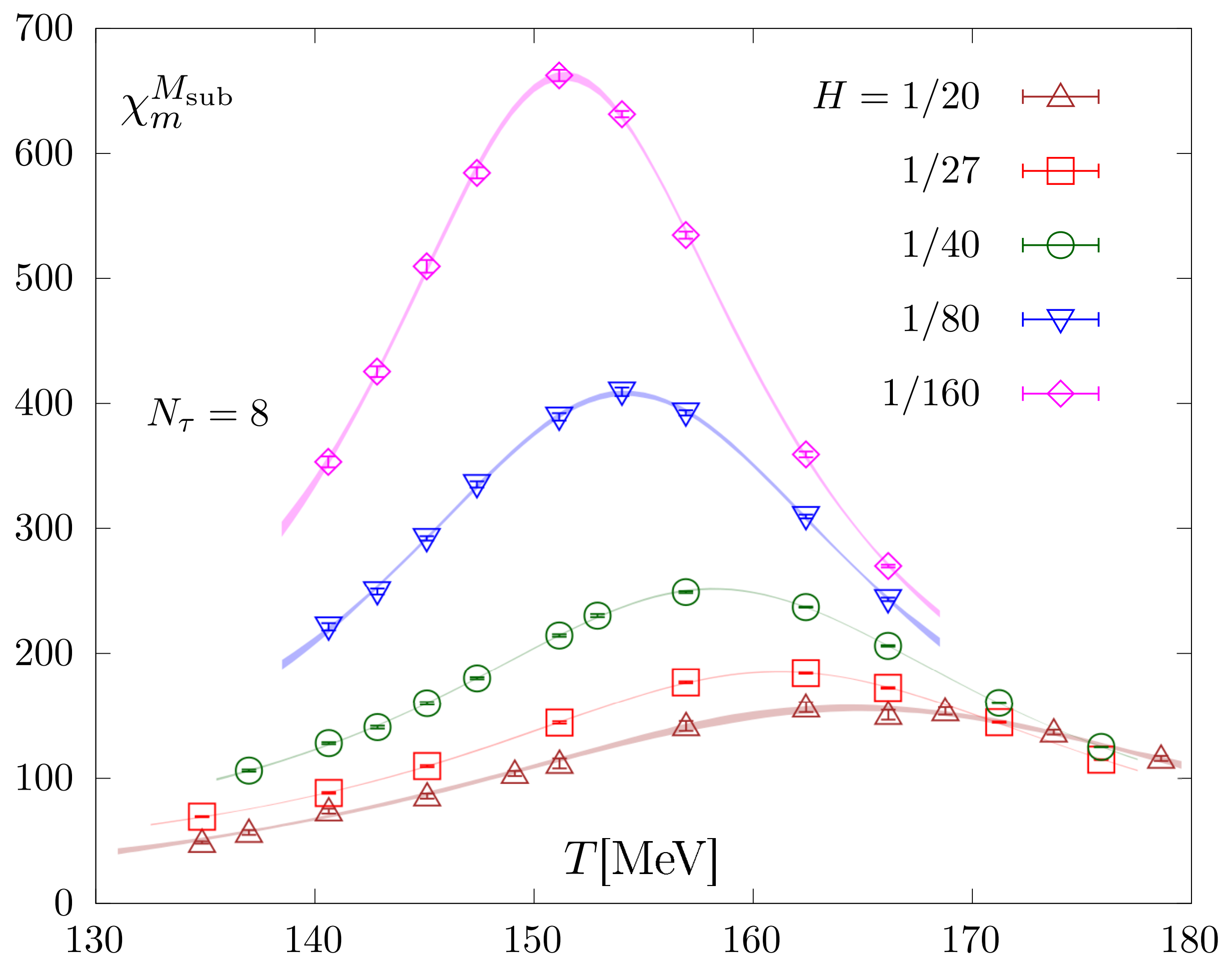


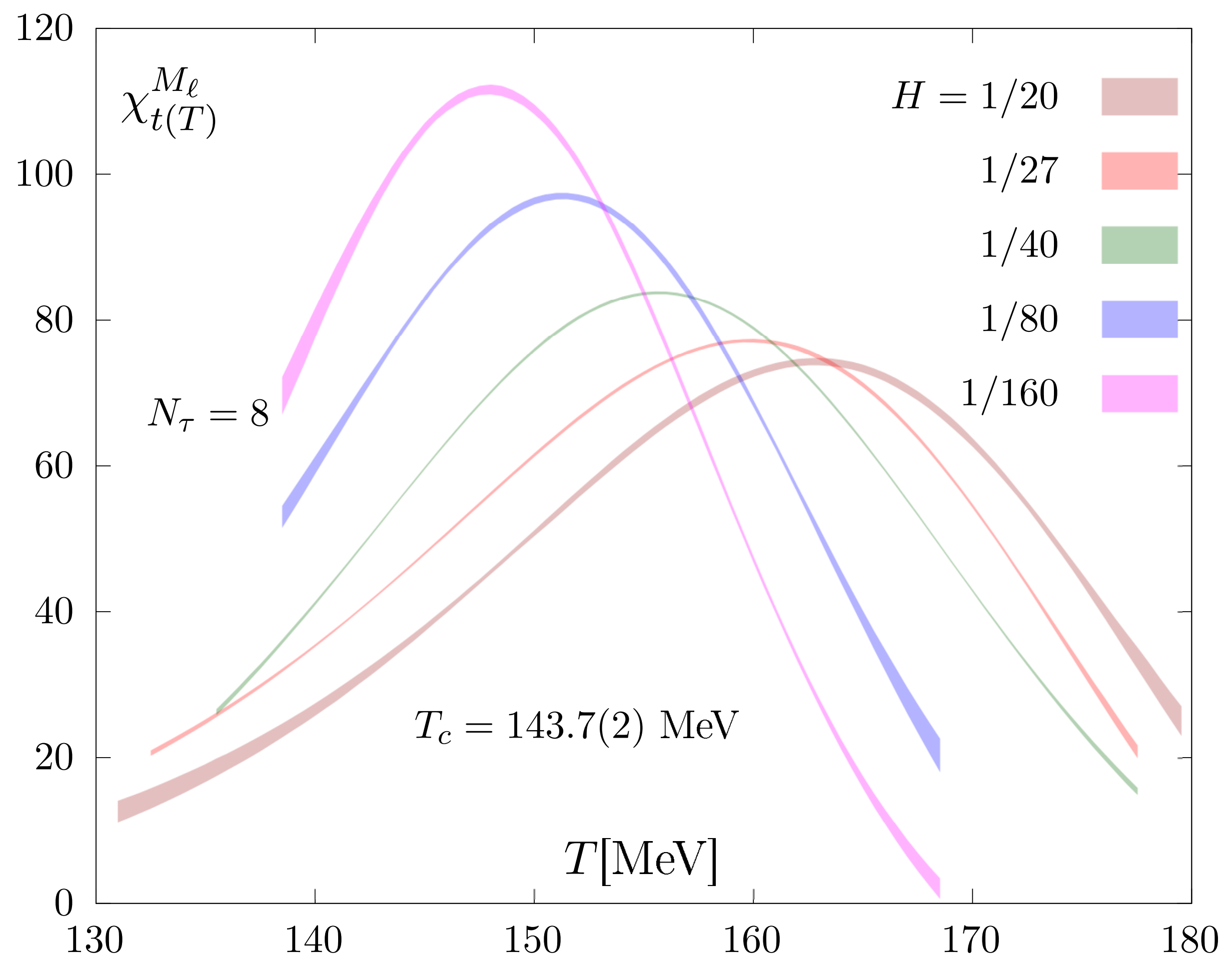
Fit results with $3d$ $O(2)$ universality class scaling functions determined using the Schofield parametrization in Karsch, Neumann, and MS, Phys. Rev. D 108, 014505 (2023)

corrections to universal scaling behavior arising from regular or sub-leading universal terms remain smaller than 10% for $(T - T_c)/T_c \lesssim 0.06$

Pseudo-critical temperatures

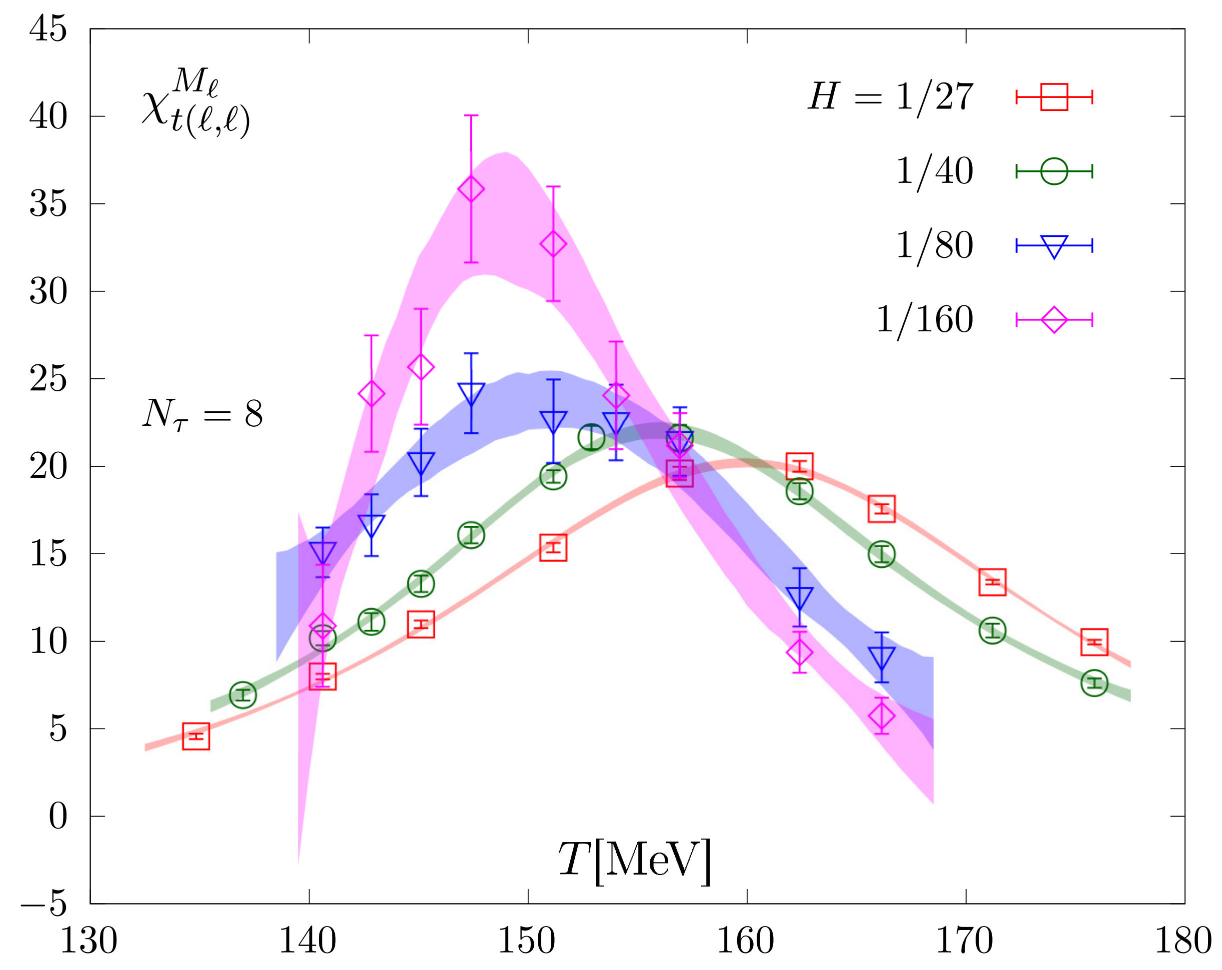
$T_{pc,m}$

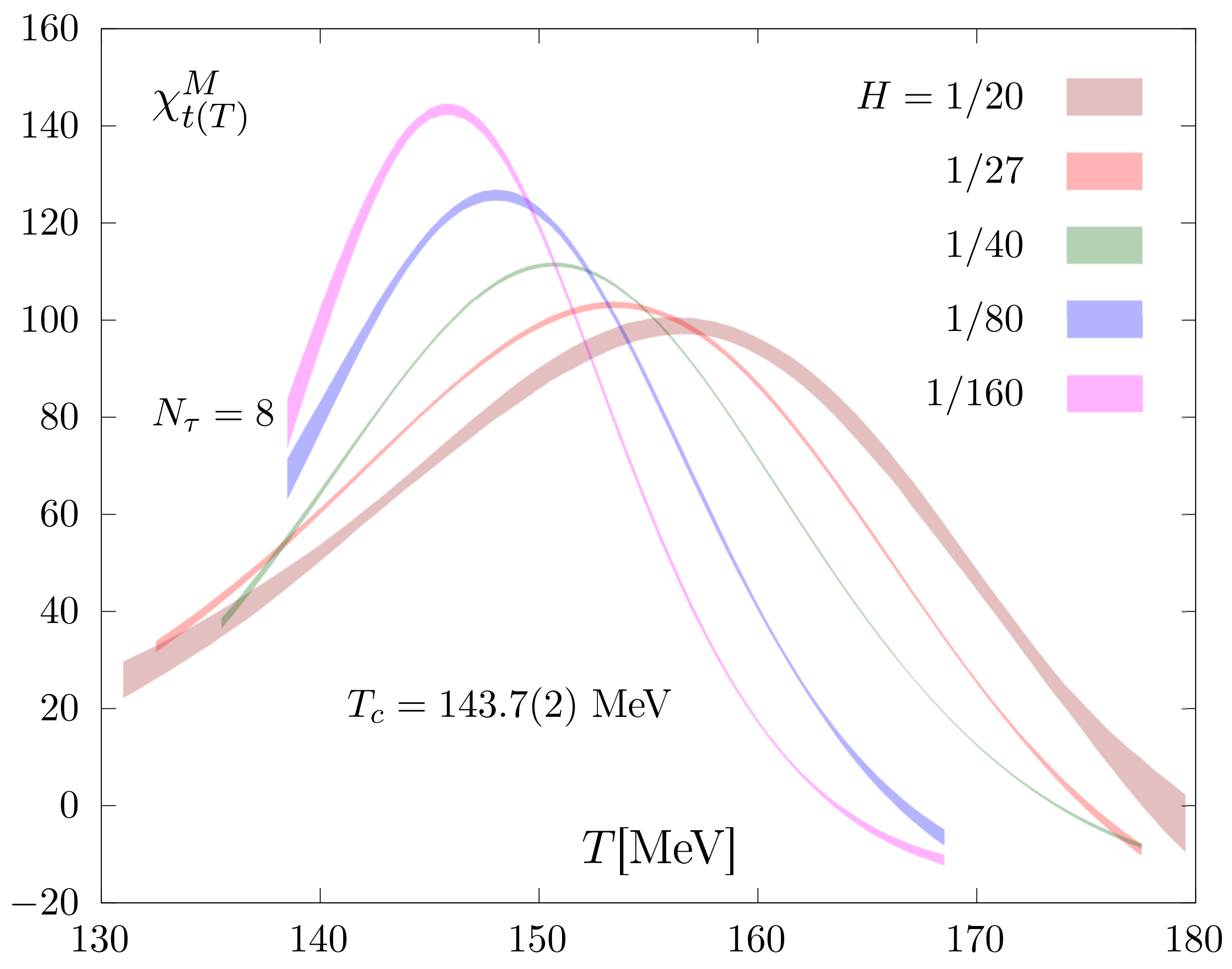




$T_{pc,t}$

$T_{pc,t}$

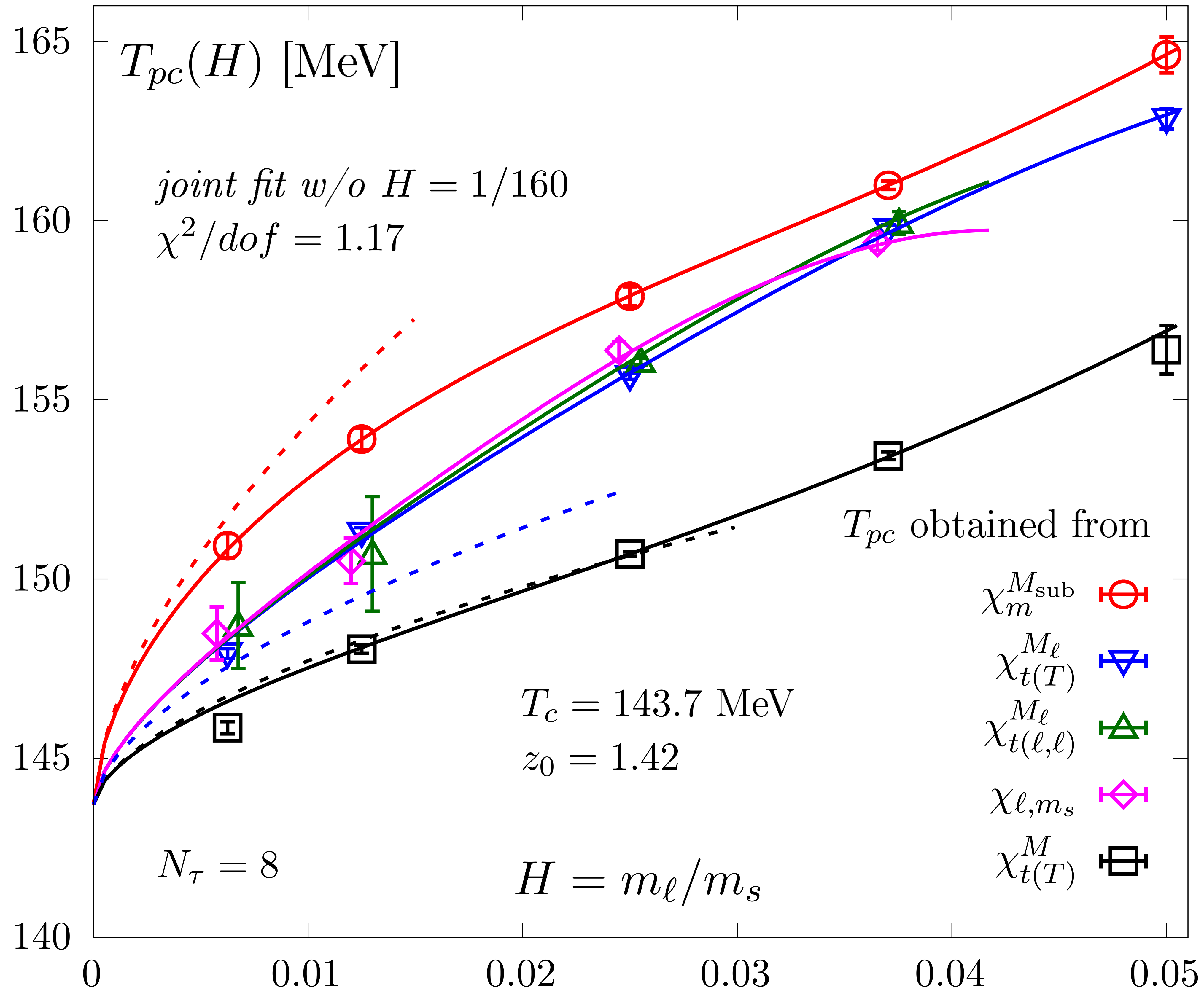




$T_{pc,(t,M)}$

Pseudo-critical temperatures

$$T_{pc,m}(H) > T_{pc,t}(H) > T_{pc,(t,M)}(H)$$



$$M_\ell = h_0^{-1/\delta} H^{1/\delta}\left(f_G(z) + cH^{\omega\nu_c} f_{G,cts}(z) + \mathcal{O}(H^{2\omega\nu_c})\right) + H\sum_{n=0}^{n_{max}} a_n t^n + \mathcal{O}(H^3)$$

$$T_{pc,x}(H) = T_c\Big(1 + t_{1,x}H^{1/\beta\delta} + t_{c,x}H^{1/\beta\delta+\omega\nu_c} + t_{2,x}H^{1+(3-\beta)/\beta\delta} + t_{3,x}H^{1+(4-\beta)/\beta\delta}\Big)$$

$$x = t,\ (t, M)$$

Fit ansatz includes regular + corrections-to-scaling

$$T_{pc,m}(H) = T_c\Big(1 + t_{1,m}H^{1/\beta\delta} + t_{c,m}H^{1/\beta\delta+\omega\nu_c} + t_{2,m}H^{1+(2-\beta)/\beta\delta} + t_{3,m}H^{1+(3-\beta)/\beta\delta}\Big)$$

$$t_{1,x} \equiv \frac{z_x}{z_0},\ x = m,\ t,\ (t, M)$$

Curvature coefficients

Estimators

$$\mathscr{K}_2^f(T,H) = \frac{1}{2T_c}\left(\frac{\partial^2 M_\ell/\partial\hat{\mu}_f^2}{\partial M_\ell/\partial T}\right)_{(T,\vec{\mu}=0)} \quad , \ f=\ell,s$$

$$\mathscr{K}_{11}^{\ell s}(T,H) = \frac{1}{2T_c}\left(\frac{\partial^2 M_\ell/\partial\hat{\mu}_\ell\partial\hat{\mu}_s}{\partial M_\ell/\partial T}\right)_{(T,\vec{\mu}=0)}$$

$$\kappa_2^f \equiv \mathscr{K}_2^f(T_c,0)$$

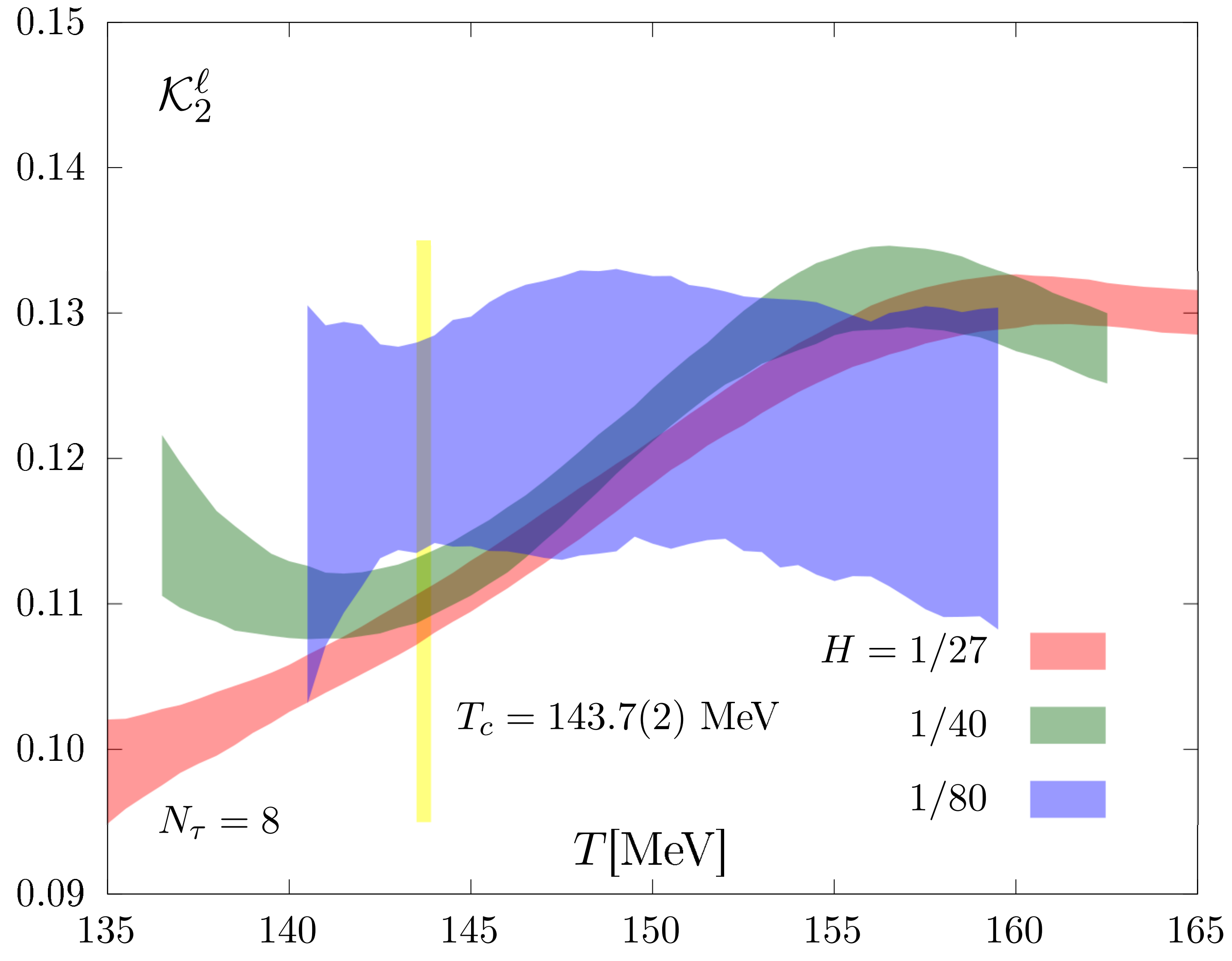
$$\kappa_{11}^{\ell s} \equiv \mathscr{K}_{11}^{\ell s}(T_c,0)$$

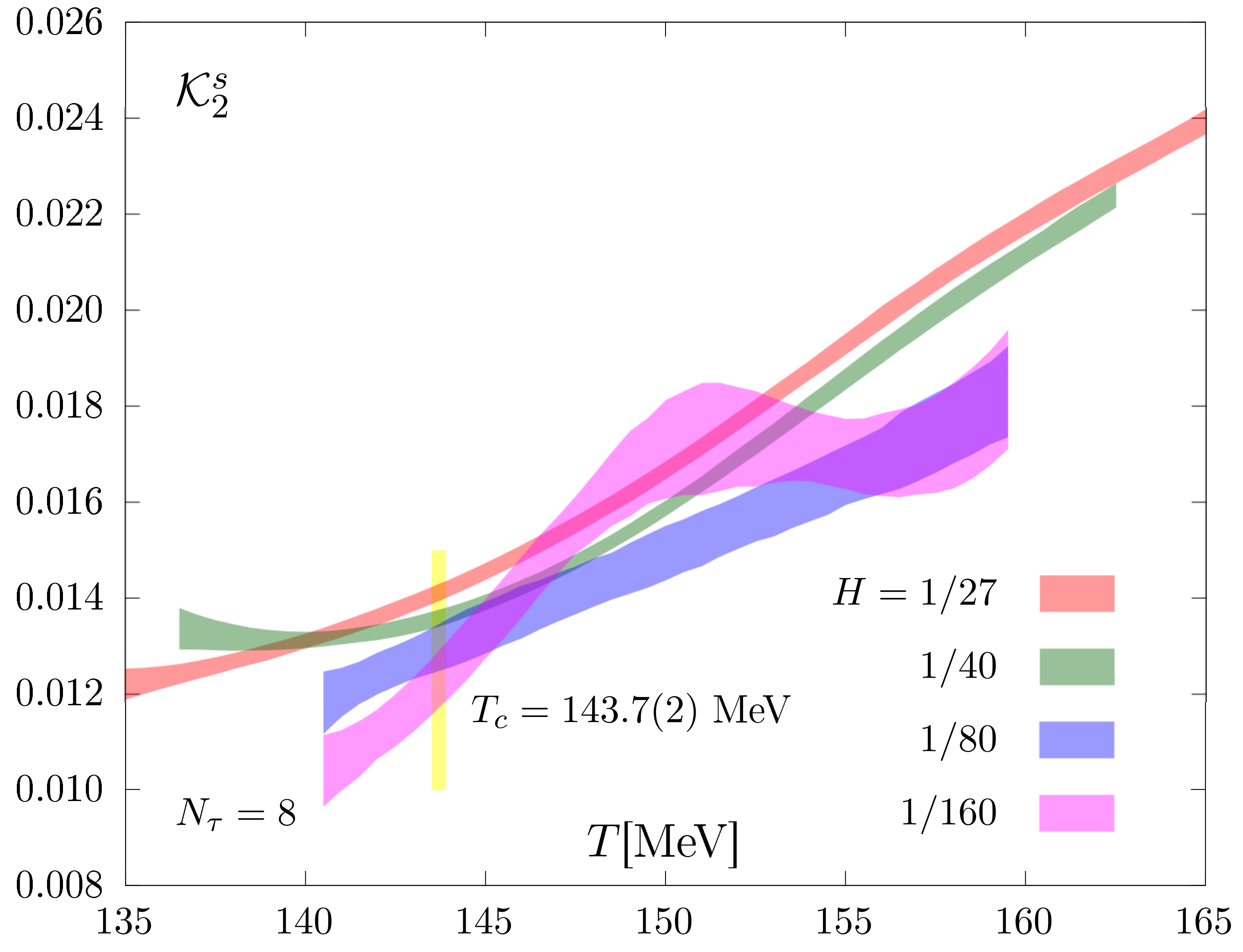
$$T_c(\hat{\mu}_\ell,\hat{\mu}_s) = T_c\left(1+(\kappa_2^\ell\hat{\mu}_\ell^2+\kappa_2^s\hat{\mu}_s^2+2\kappa_{11}^{\ell s}\hat{\mu}_\ell\hat{\mu}_s)\right)$$

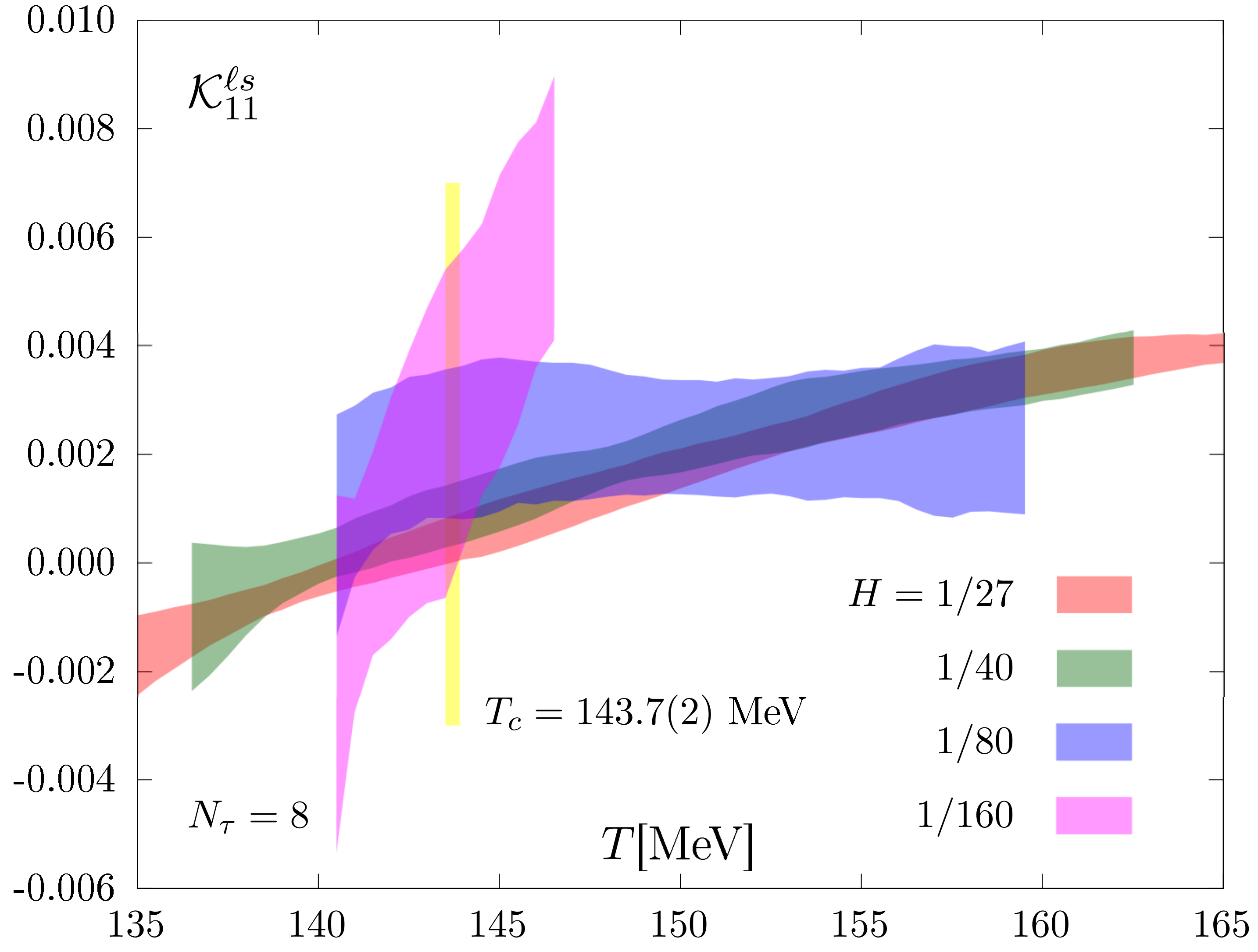
dependence of chiral phase transition temperature on the chemical potentials

$$\begin{aligned}\mathscr{K}_2^f(T,H) &= \frac{\kappa_2^f h^{(\beta-1)/\beta\delta}f'_G(z)/t_0 + Ha_f}{h^{(\beta-1)/\beta\delta}f'_G(z)/t_0+Ha_t} \\ &= \frac{\kappa_2^f+\tilde{a}_f H^{1+(1-\beta)/\beta\delta}/f'_G(z)}{1+\tilde{a}_t H^{1+(1-\beta)/\beta\delta}/f'_G(z)} \\ &= \kappa_2^f + A_f H^{1+(1-\beta)/\beta\delta} + B_f H^{1-1/\delta}t\end{aligned}$$

Curvature coefficients



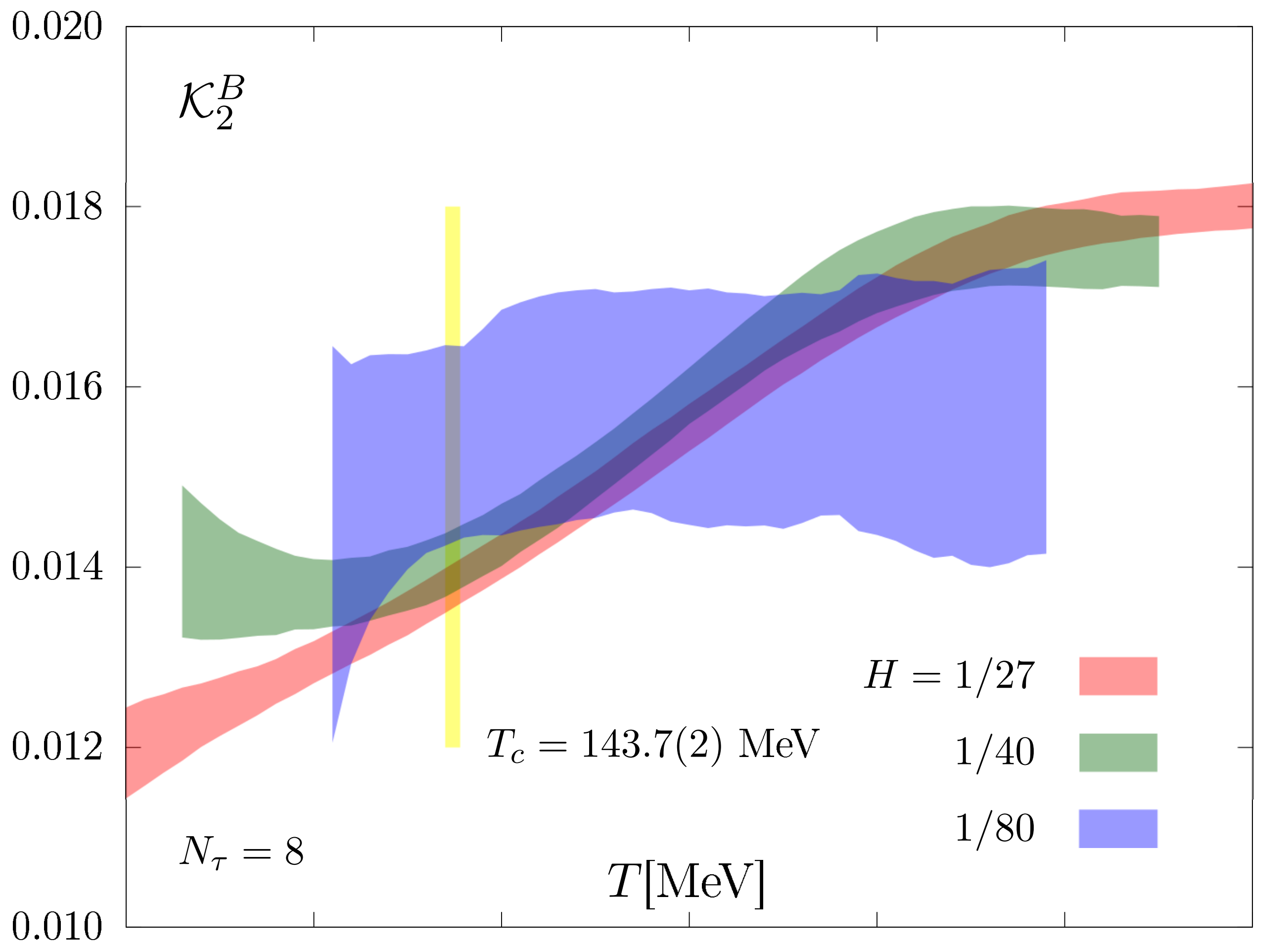


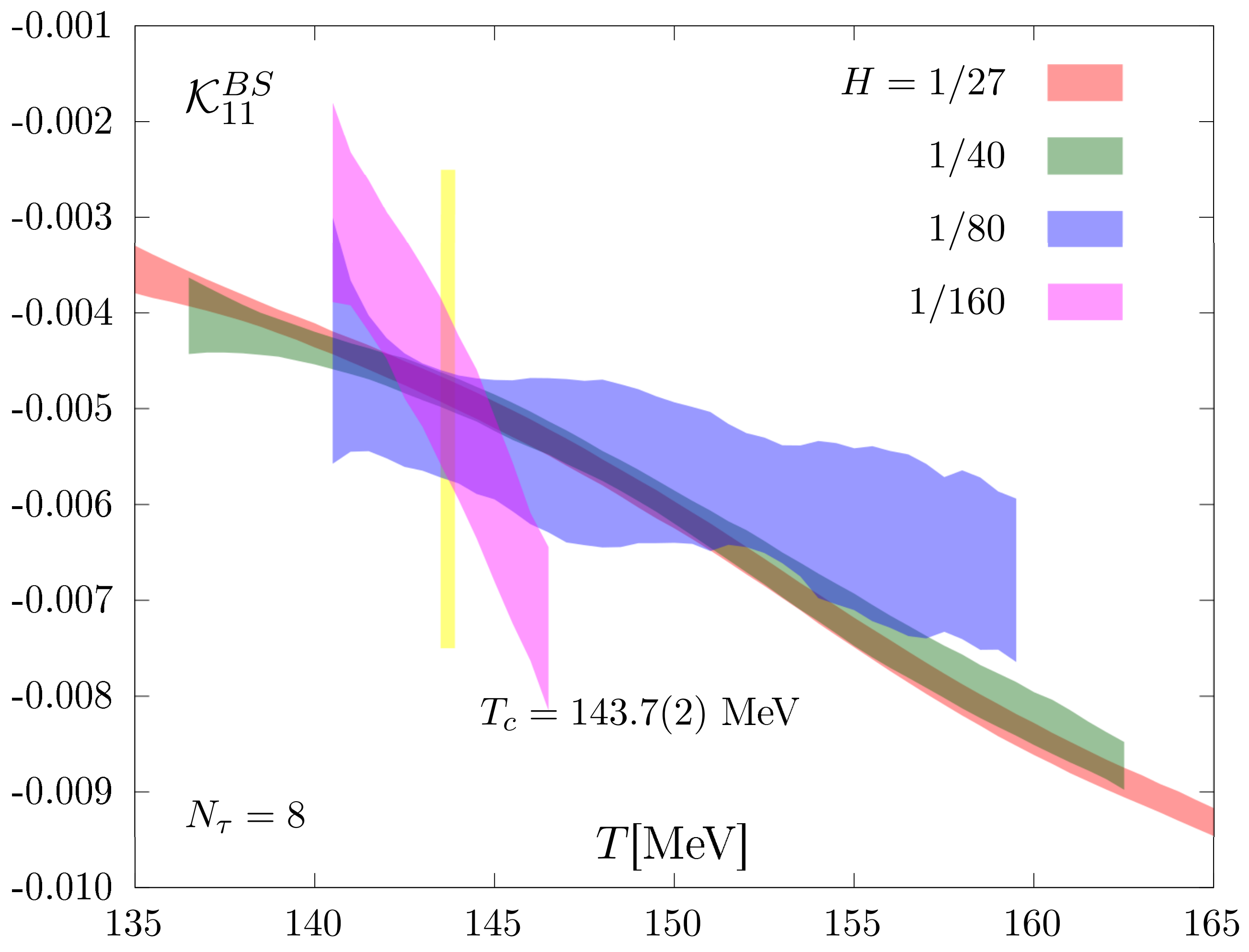


$\kappa_2^\ell = 0.122(7)$ $\kappa_2^s = 0.0124(5)$ $\kappa_{11}^{\ell s} = 0.003(2)$

Curvature coefficients

$\kappa_2^B = 0.015(1)$





$\kappa_{11}^{BS} = -0.0050(7)$

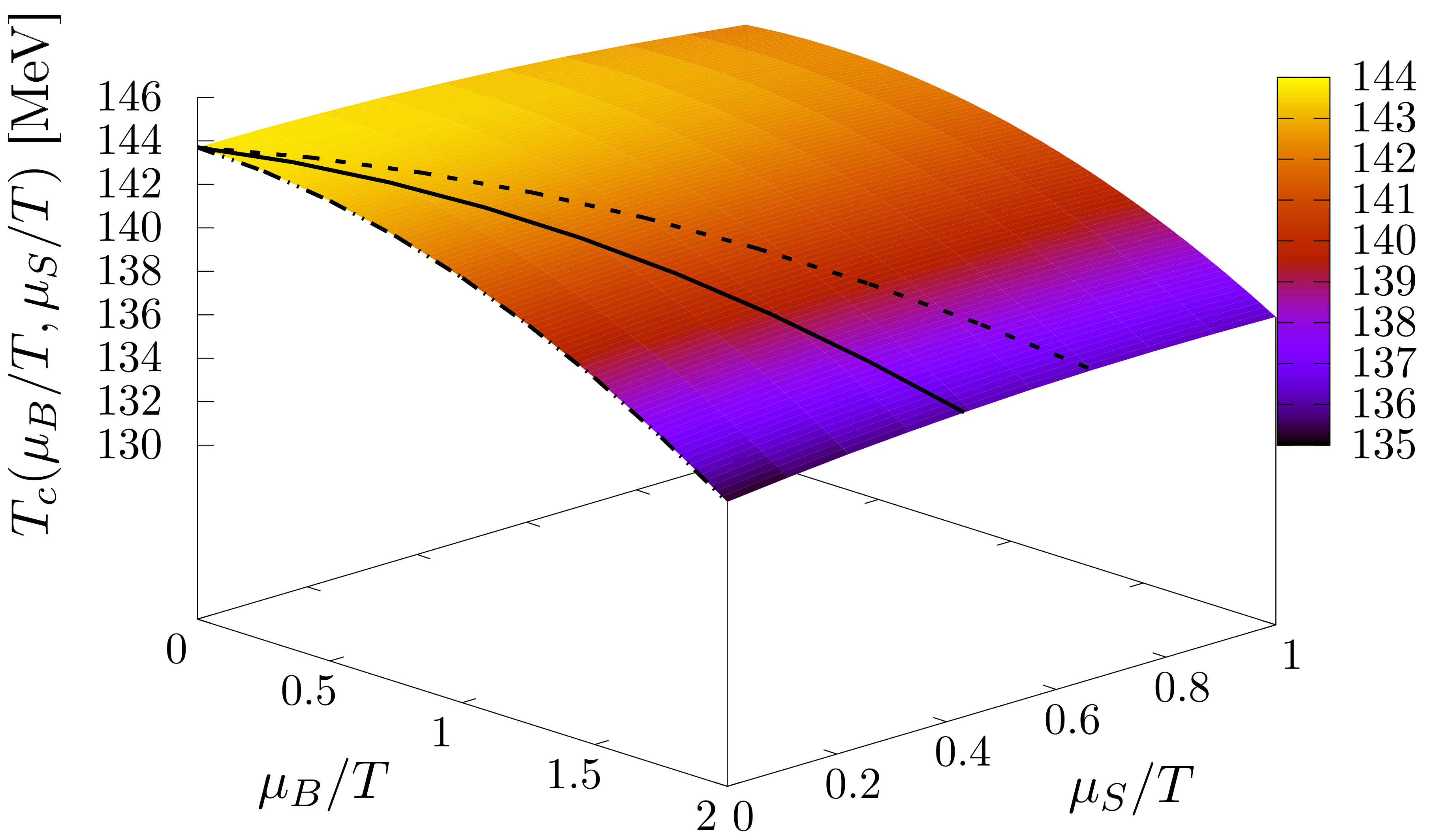
$$\kappa_2^{B,\mu_S=0} \equiv \kappa_2^B = 0.015(1)$$

$$\kappa_2^{B,n_S=0} = \kappa_2^B + s_1^2(T_c,0)\kappa_2^S + 2s_1(T_c,0)\kappa_{11}^{BS}$$
$$= 0.893(35)\ \kappa_2^B$$

strangeness neutral medium

$$\kappa_2^{B,\mu_s=0} = \frac{1}{9}\kappa_2^{\ell} = 0.968(23)\ \kappa_2^{B,n_S=0}$$

vanishing strange quark



$T_c(\hat{\mu}_B, \hat{\mu}_S)$

Consistent with values obtained with physical quark masses *[Borsanyi et al, PRL 2020]*

Conclusions

- systematic analysis of the m_ℓ dependence of pseudo-critical temperatures in (2+1)-flavor QCD with m_s tuned to its physical value

- determination of curvature coefficients defining the dependence of T_c on the various chemical potentials, for several values of the light to strange quark mass ratio H, with extrapolation to the chiral limit.

- results seem consistent with a 2nd order transition belonging to the $3d\ O(4)$ universality class in the chiral limit

Thank you for your attention!

Image : https://www.olcf.ornl.gov/tag/lattice-qcd/