Lattice chiral fermion without Hermiticity

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Nielsen-Ninomiya Theorem [Nielsen and Ninomiya 1981]

- prohibits a hermitian construction of d-dimensional fermion lattice action S_F:
  - 1. $D(x)$ is exponentially local, which implies that the operator is bounded by $\sim \exp(-|x|/c)$, where $c \propto a$;
  - 2. $\bar{D}(p) = i\gamma_\mu p_\mu + \mathcal{O}(ap^2)$ for $p \ll \pi/a$;
  - 3. $\bar{D}(p)$ is invertible for $p \neq 0$ (no massless doublers);
  - 4. $\gamma_5 D + D\gamma_5 = 0$ (continuum chiral symmetry),

  where $D(x)$ is a Dirac matrix satisfying

$$S_F = a^d \sum_{\text{all lattice points}} \bar{\psi}(D + m)\psi, \qquad (1)$$

and the $\bar{D}(p)$ is a Dirac matrix on a momentum space

Naive Non-Hermitian Lattice Fermion

- use forward finite difference operator (breaks Hermiticity) $D(n_1, n_2)_{\alpha_1,\alpha_2} \equiv (\gamma_1)_{\alpha_1,\alpha_2}(\delta_{n_1+1,n_2} - \delta_{n_1,n_2})/a$ [Stamatescu, Wu 1993]
- non-physical poles do not provide additional degrees of freedom when taking the continuum limit
- preserves chiral symmetry without square-root operators
- quenched averaging over all directions imposes the hypercubic symmetry

Two Flavors

- two fermion fields in 1d with a degenerate mass, the lattice action is

$$\begin{aligned} & S_{FD} \\ = \quad & a\sum_{n=0}^{N-1}\Big(\bar{\psi}_1(n)\big(D(n)+m\big)\psi_1(n) \\ & +\bar{\psi}_2(n)\big(-D^\dagger(n)+m\big)\psi_2(n)\Big). \end{aligned} \tag{2}$$

- a non-negative determinant after intergrating our the fermion fields:

$$\begin{aligned} & \det(D+m)\det(-D^\dagger+m) \\ = \quad & \det(D+m)\det\left(\gamma_5(-D^\dagger+m)\gamma_5\right) \\ = \quad & |\det(D+m)|^2. \end{aligned} \tag{3}$$

HMC

- introduce the pseudo-fermion field (bosonic field ϕ_f) to rewrite the partition function as in the following

$$\begin{aligned} &\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \ \exp(-S_{FD}) \\ \sim\ &\int \mathcal{D}\phi_{f,R}\mathcal{D}\phi_{f,I} \ \exp\Big(-\phi_f^\dagger\big((D+m)(D^\dagger+m)\big)^{-1}\phi_f\Big), \end{aligned} \tag{4}$$

where $\phi_f \equiv \phi_{f,R} + i\phi_{f,I}$.

- implement the Hybrid Monte Carlo algorithm to calculate:

$$\mathcal{O}_{jk}^{\alpha\beta} \equiv \frac{1}{2}\langle \phi_{f,j}^{\dagger\alpha}\phi_{f,k}^{\beta} + \phi_{f,k}^{\dagger\beta}\phi_{f,j}^{\alpha}\rangle = \big((D+m)(D+m)^\dagger\big)_{jk}^{\alpha\beta}, \tag{5}$$

where $j,k = 1,2,\cdots,N$; $\alpha,\beta = 1,2$.

Numerical Result

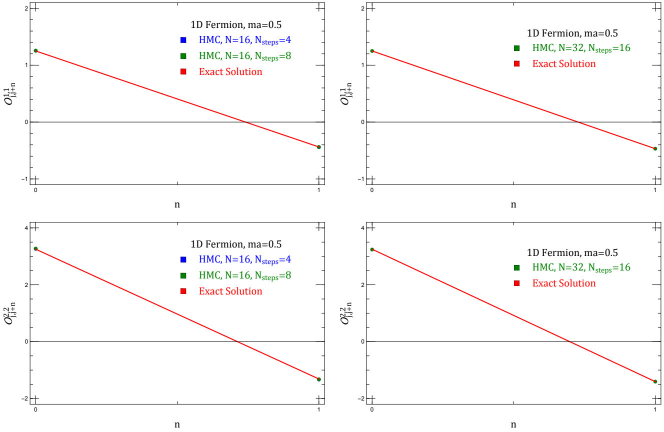


Figure: We use the HMC to get consistent results with the exact solution. The number of measurements is 2^{12} sweeps with thermalization 2^{6} sweeps and measure intervals 2^{5} sweeps. The error bars are less than 1%. The N_{steps} is the number of molecular dynamics steps.

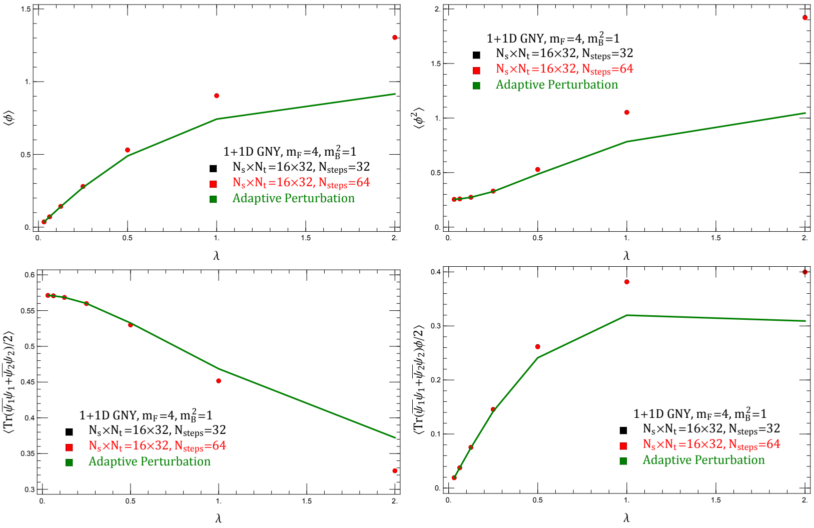
2D GNY Model

- GN model is UV finite, but it cannot be extended to four dimensions
- studying the Gross-Neveu-Yukawa (GNY) model

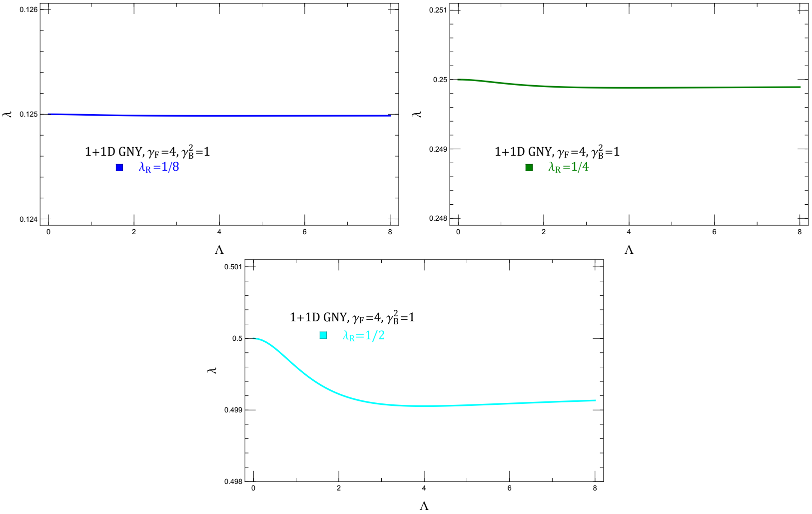
$$S_{\mathrm{GNY}}[\bar{\psi}, \psi, \phi] = \int d^2x \left(\bar{\psi}(\partial\!\!\!/ + m_{\mathrm{F}} + \phi)\psi - \phi\Box\phi + \frac{1}{2g^2}\phi^2 \right) \quad (6)$$

- GNY model introduces an additional scalar field ϕ to the GN model, primarily to ensure renormalizability in four dimensions
- kinetic term for ϕ may influence the UV behavior of the model, the expectation is that the essential property of asymptotic safety will still hold in the GNY model

Correlator


1+1D GNY, $m_F=4$, $m_B^2=1$
$N_s \times N_t = 16\times32$, $N_{steps}=32$
$N_s \times N_t = 16\times32$, $N_{steps}=64$
Adaptive Perturbation
$\langle\phi\rangle$
$\langle\phi^2\rangle$
$\langle \mathrm{Tr}(\overline{\psi_1}\psi_1+\overline{\psi_2}\psi_2)/2\rangle$
$\langle \mathrm{Tr}(\overline{\psi_1}\psi_1+\overline{\psi_2}\psi_2)\phi/2\rangle$
λ

Asymptotic Safety



Conclusion and Outlook

- non-Hermiticity allows doubler-free fermion formulation with successful lattice implementation via HMC
- 2D GNY model shows asymptotic safety
- extend to 4D simulation and applications to QCD-like theories

Thank you!