Probing Quantum Phases of Matter on Quantum Processors

Frank Pollmann
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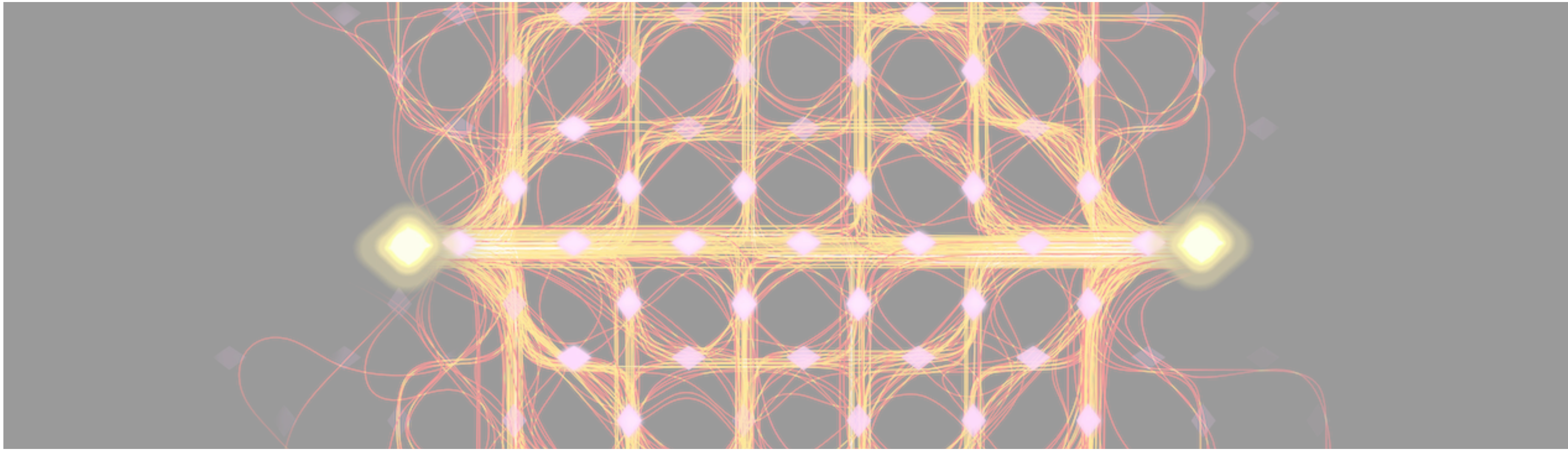


Bernhard Jobst **Melissa Will** **Yu-Jie Liu** Michael Knap Adam Smith

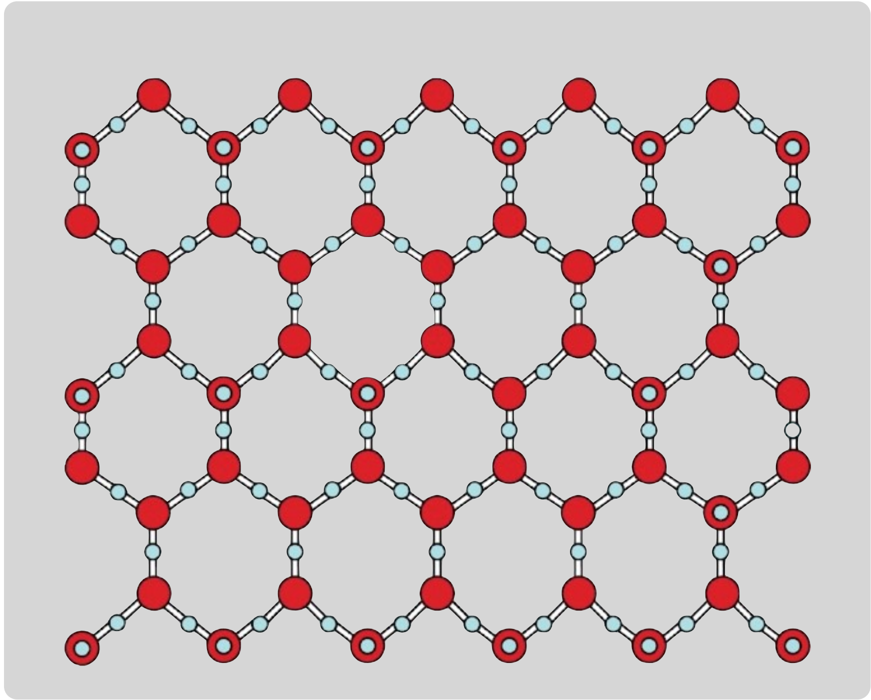
Kevin Satzinger Tyler Cochran Norhan M. Eassa Eliott Rosenberg Pedram Roushan

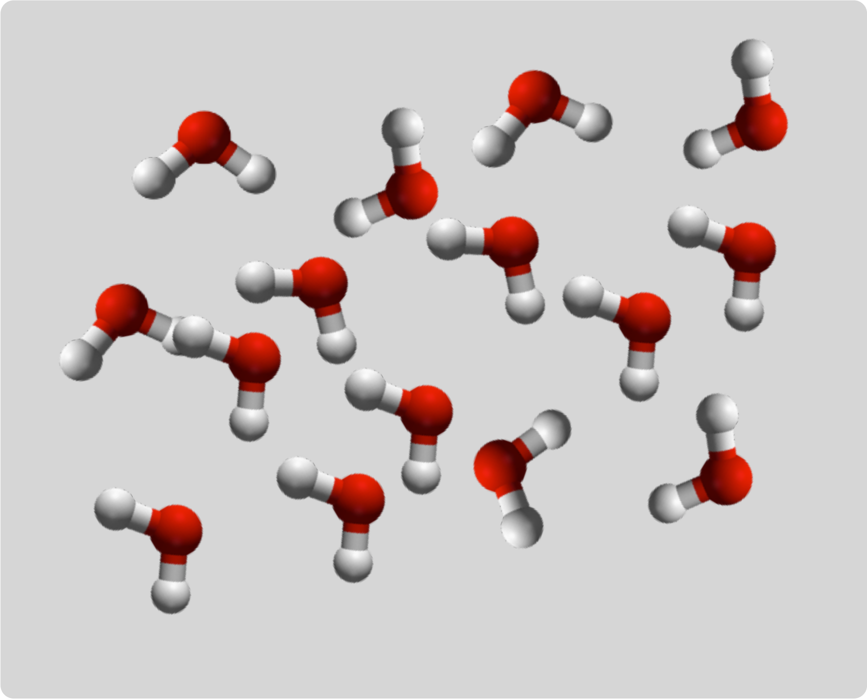
Quantum AI

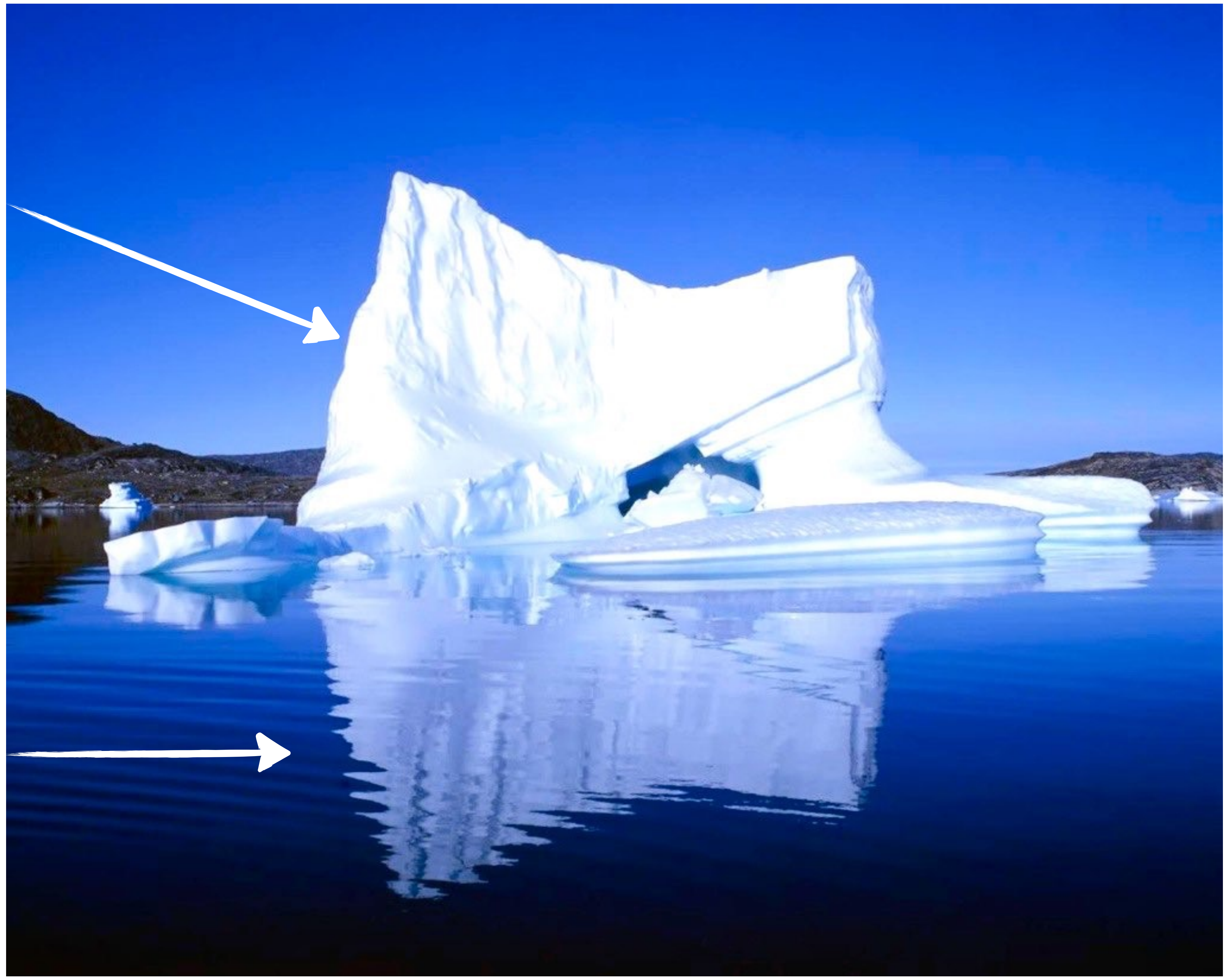
DFG TRR 360 FOR 5522 MCQST erc EQUAHUMO Efficient Quantum Algorithms for the Hubbard Model Munich Quantum Valley

SQAI-NCTS Workshop on Quantum Technologies and Machine Learning
Taipei Aug. 25, 2025

Phases of matter







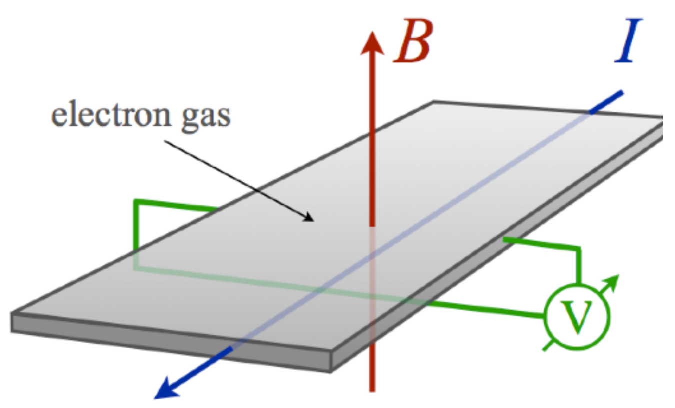
- Spontaneous symmetry breaking and local order parameter

Exotic quantum phases of matter

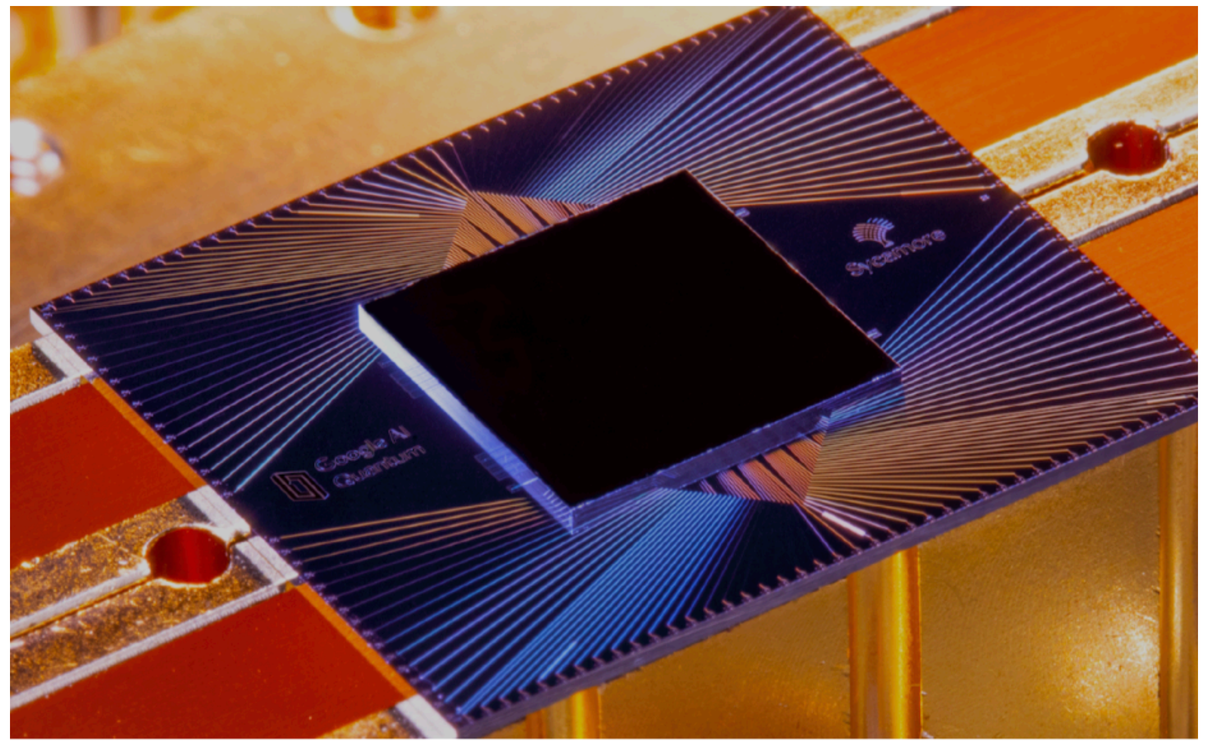
Topologically ordered phases

- Quantum Hall effects [Tsui '82, Laughlin '83]
- Quantum spin-liquids [Anderson '73]
- Floquet Topological Order [Kitagawa '10, Po et al. '17]
- …

Fascinating features: **Non-local entanglement, fractionalization, anyonic quasiparticles, …**



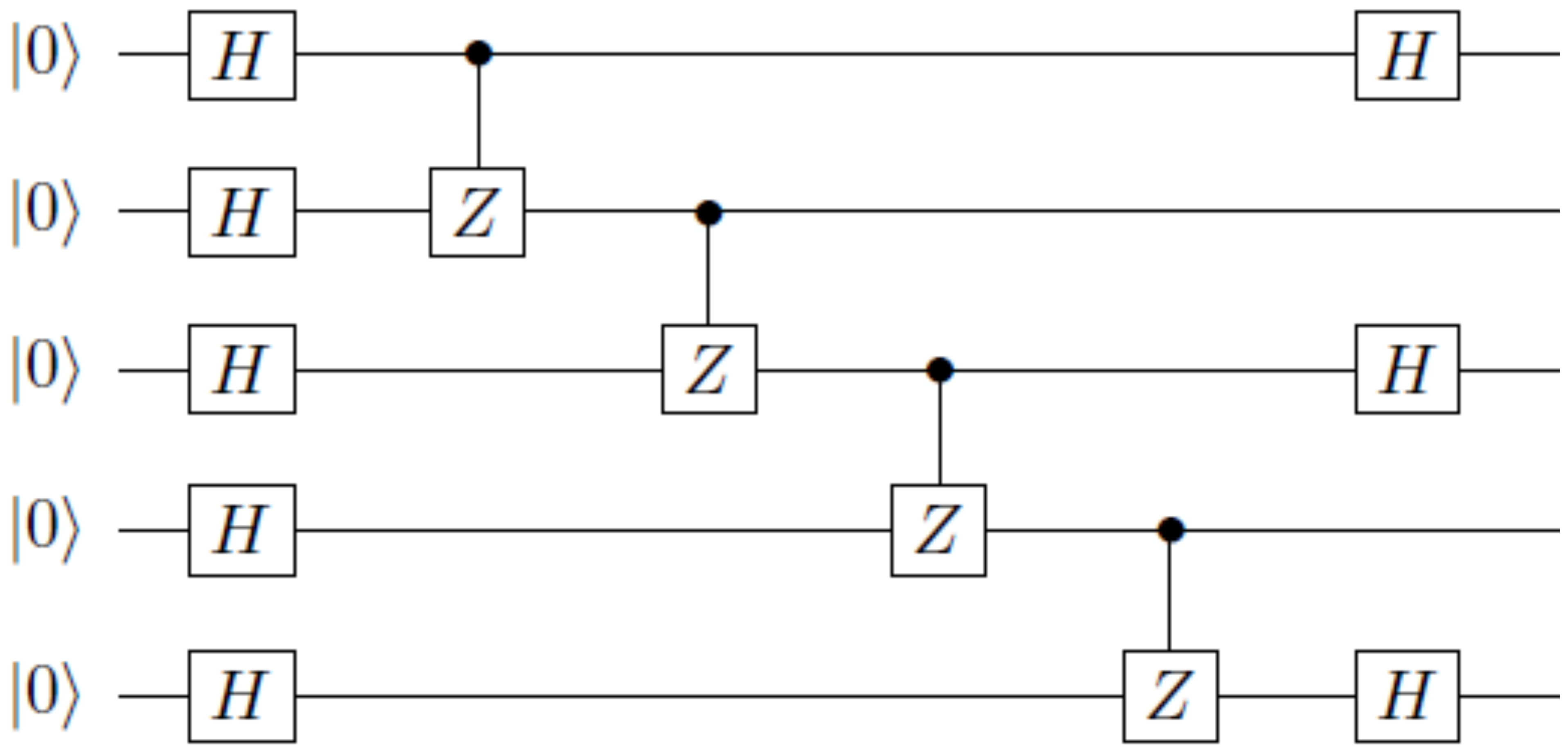
Quantum Computers as a tool to realize exotic states of matter

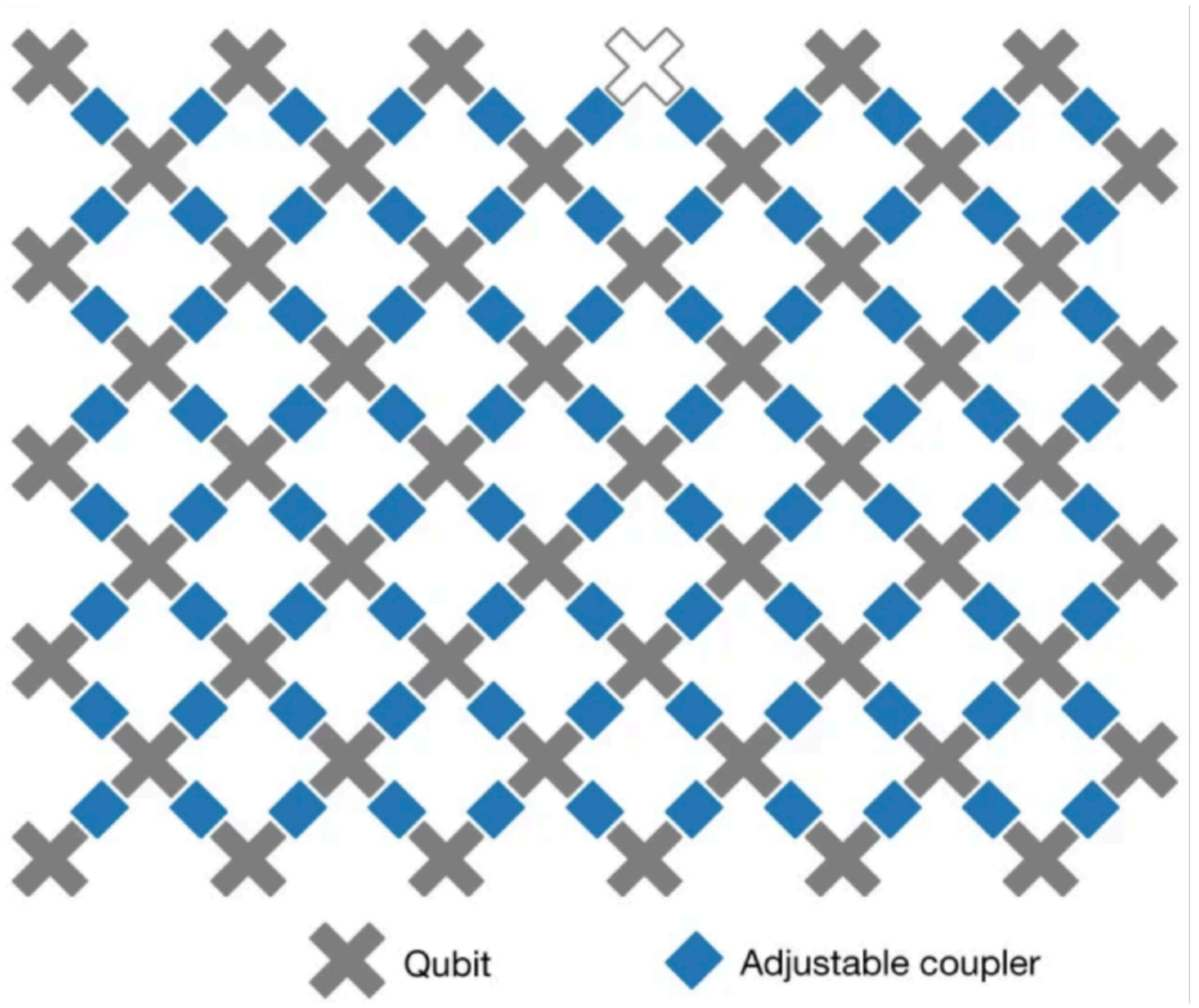


Quantum Processors

Quantum algorithms to generate entangled quantum many-body states

$|\psi\rangle =$





- "NISQ" devices: Limited circuit depth and connectivity, readout errors, …
- Identify problems that are hard on classical computers
  but doable on near term NISQ devices: Potential exponential speedup!

Outline

Realization of topological ordered ground states

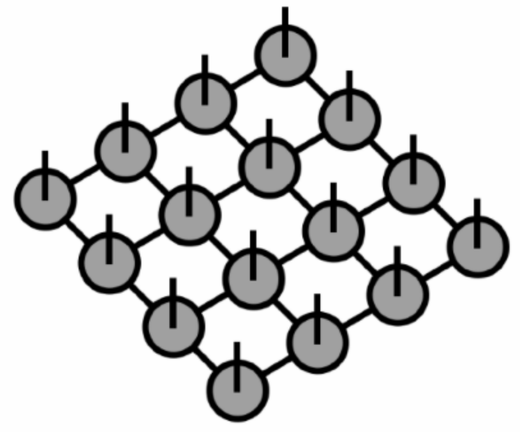
Topological entanglement

Anyonic quasi-particles

Dynamics of the confinement transition

Tuning away from the fixed point

Dynamics of quasi particles

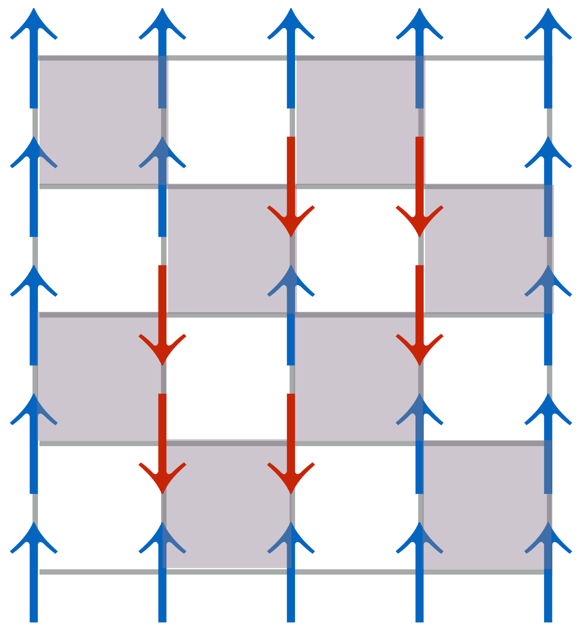


Floquet Topological Order

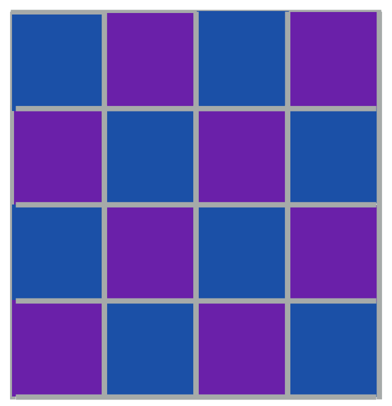
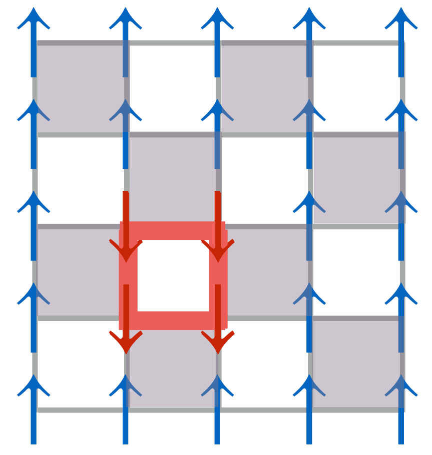
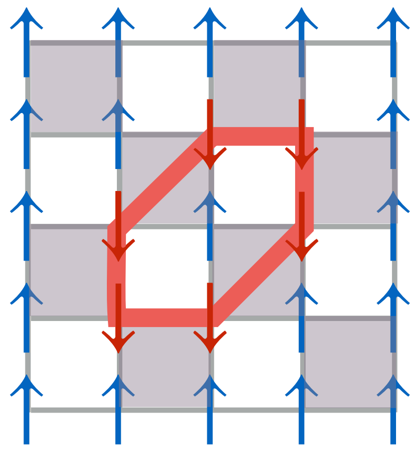
Probing non-equilbrium topological order

Toric code model [Kitaev '97]

$$\hat{H} = -\sum_{\{\blacksquare\}} \underbrace{\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z}_{A} - \sum_{\{\square\}} \underbrace{\hat{\sigma}^x\hat{\sigma}^x\hat{\sigma}^x\hat{\sigma}^x}_{B}$$



Ground state: All terms in H commute → Eigenvalue $+1$ on all plaquettes

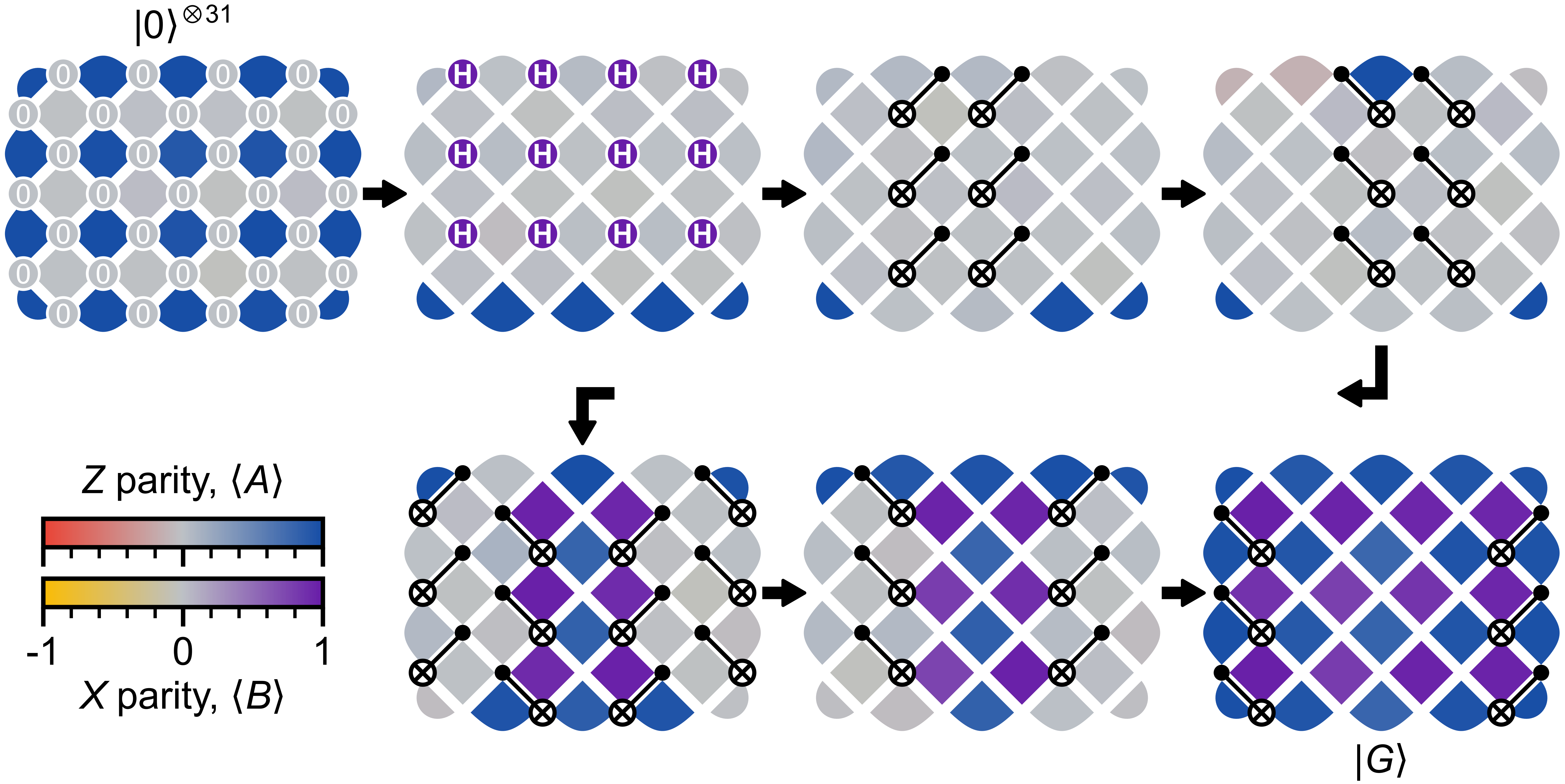
$|\psi_0\rangle = \Big|$  $\Big\rangle = \frac{1}{\mathcal{N}}\Big(\Big|$  $\Big\rangle + \dots + \Big|$  $\Big\rangle\Big)$

"Quantum Spin Liquid (QSL)": No symmetry breaking!

Hidden structure: Topological order and non-local entanglement! [Wen '90]

Realizing a QSL on a Quantum Processor

Generate the **Toric code** ground state using a quantum circuit

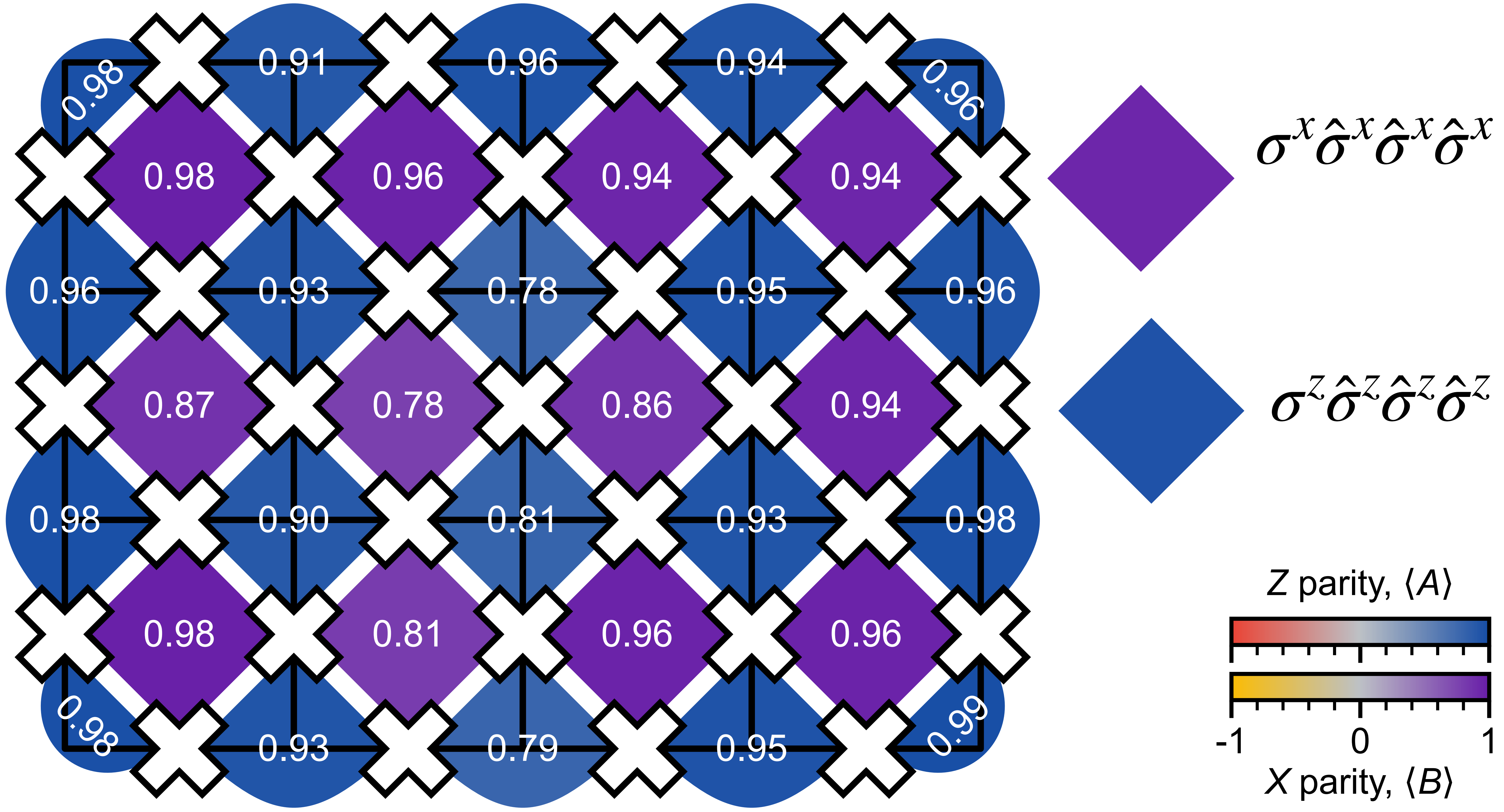


- **Linear depth to generate $|\psi_0\rangle$**

[Satzinger, Liu, Smith et al., Science **374**, 1237 (2021)]

Realizing a QSL on a Quantum Processor

Toric code ground state



31 qubits, average stabilizer fidelity 0.92 ±0.06

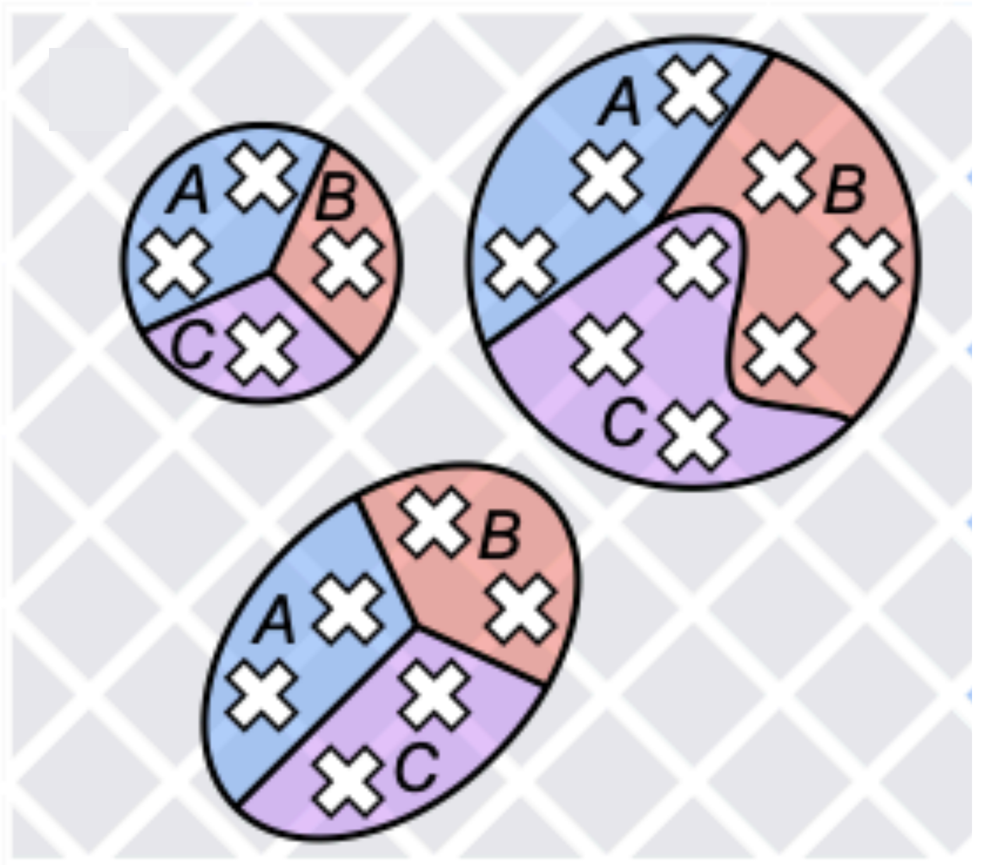
[Satzinger, Liu, Smith et al., Science **374**, 1237 (2021)]

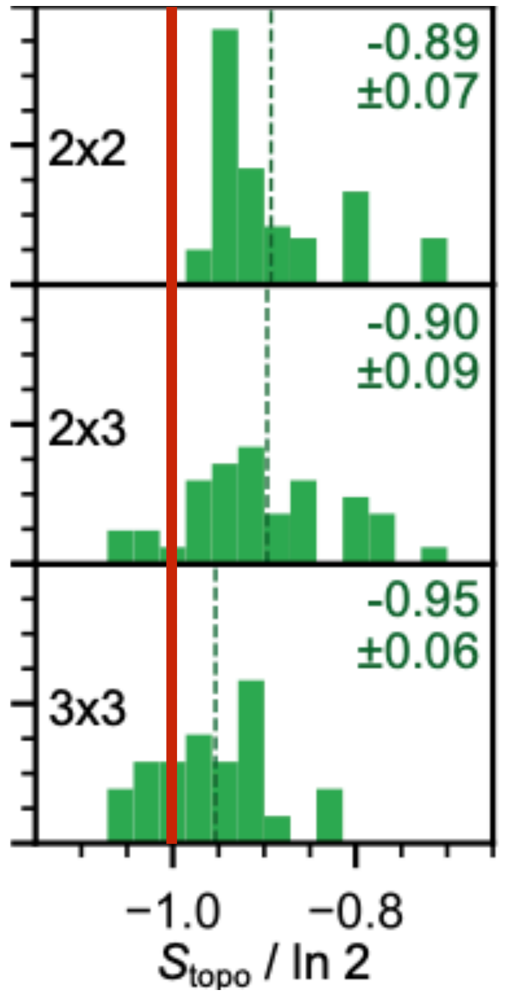
Realizing a QSL on a Quantum Processor

Topological entanglement entropy [Kitaev and Preskill '06, Levin and Wen '06]

$$S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

Toric code has $\mathbb{Z}_2$ topological order: $S_{\text{topo}} = -\ln 2$

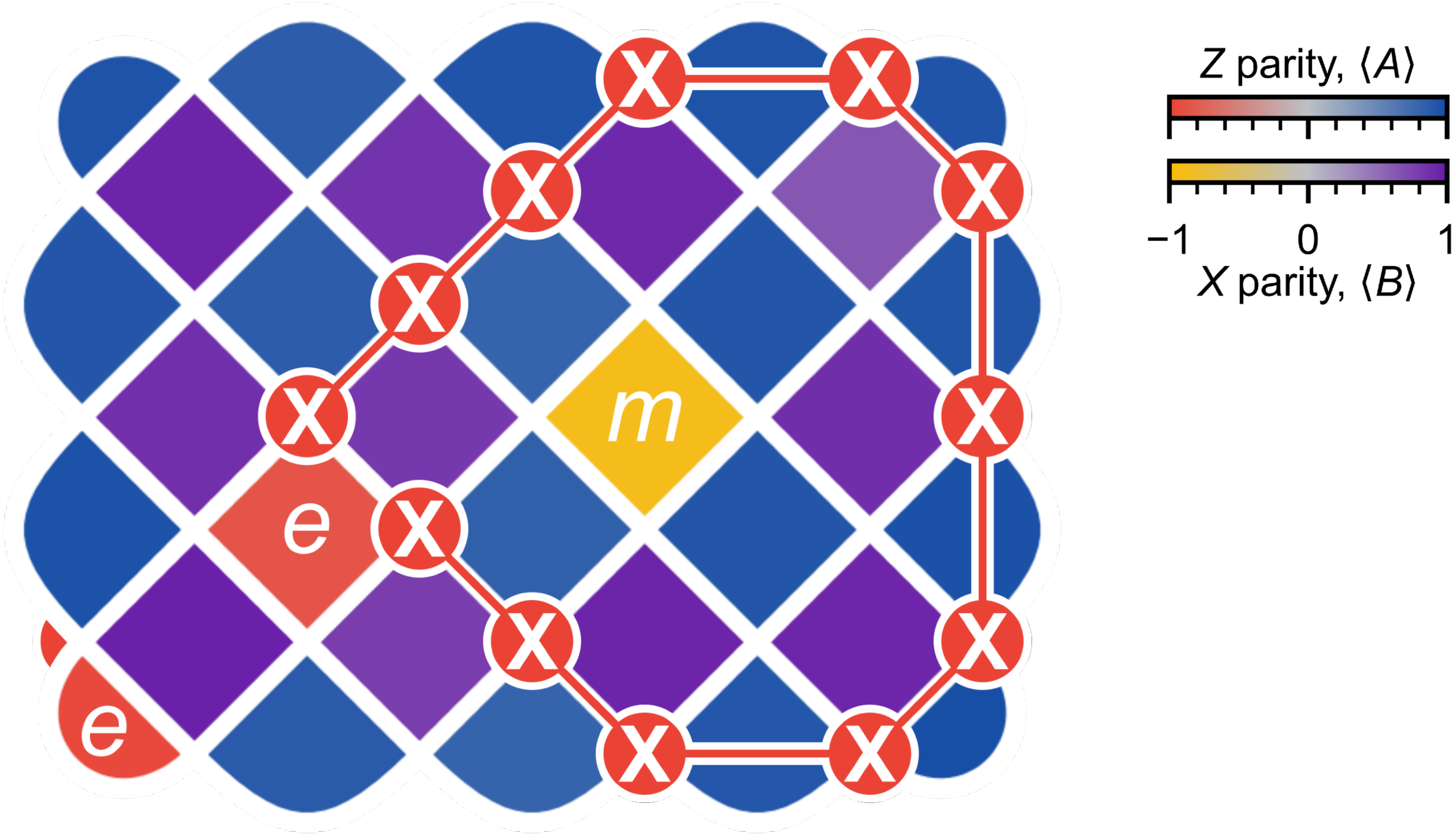




- Quantum matter with non-local entanglement!

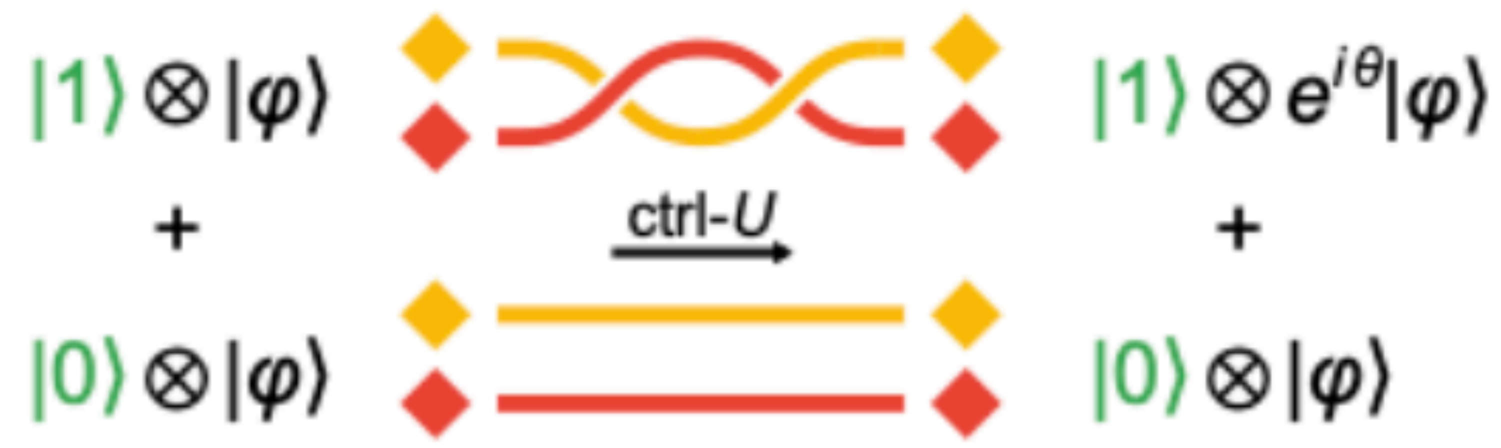
[Satzinger, Liu, Smith et al., Science **374**, 1237 (2021)]

Anyonic Statistics on a Quantum Processor

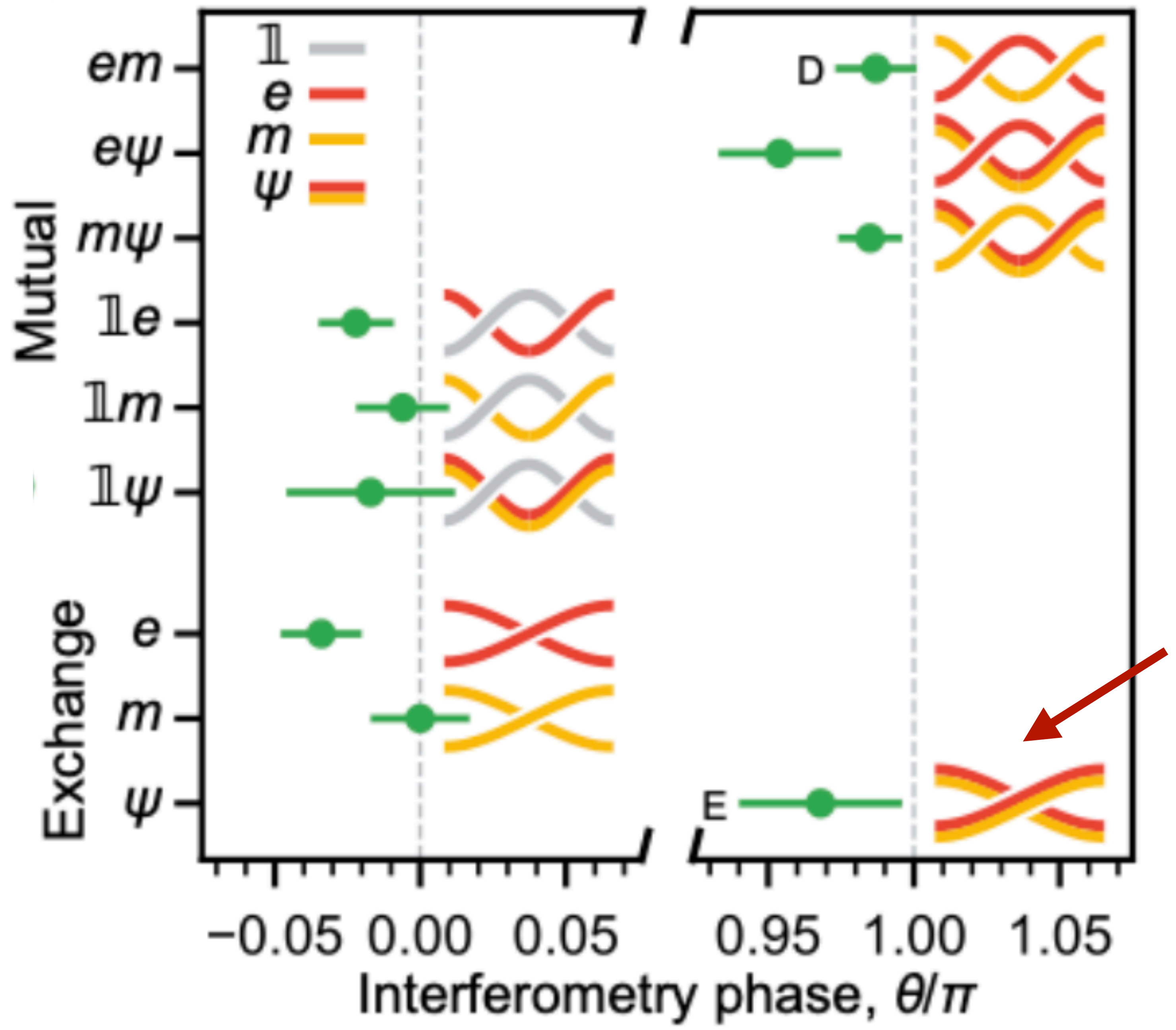


[Satzinger, Liu, Smith et al., Science **374**, 1237 (2021)]

Anyonic Statistics on a Quantum Processor



[Jiang et al. '08]



Emergent Fermion!

[Satzinger, Liu, Smith et al., Science **374**, 1237 (2021)]

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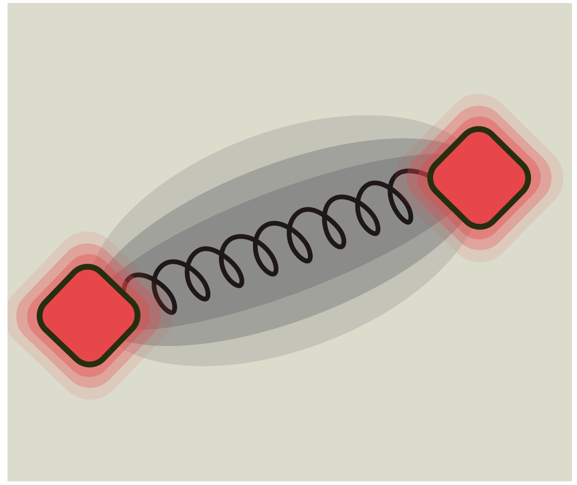
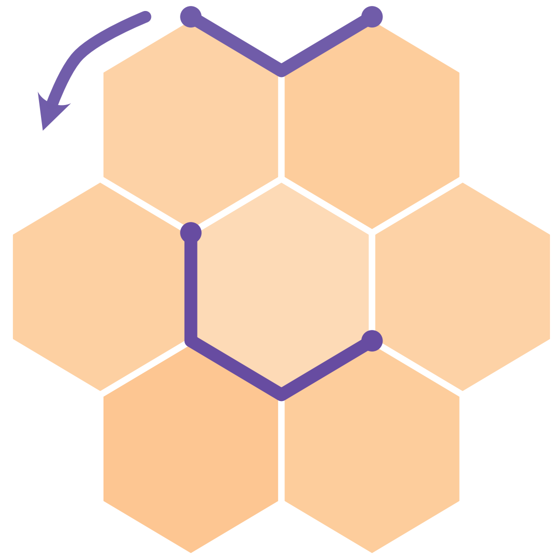


Floquet Topological Order

Probing non-equilbrium topological order

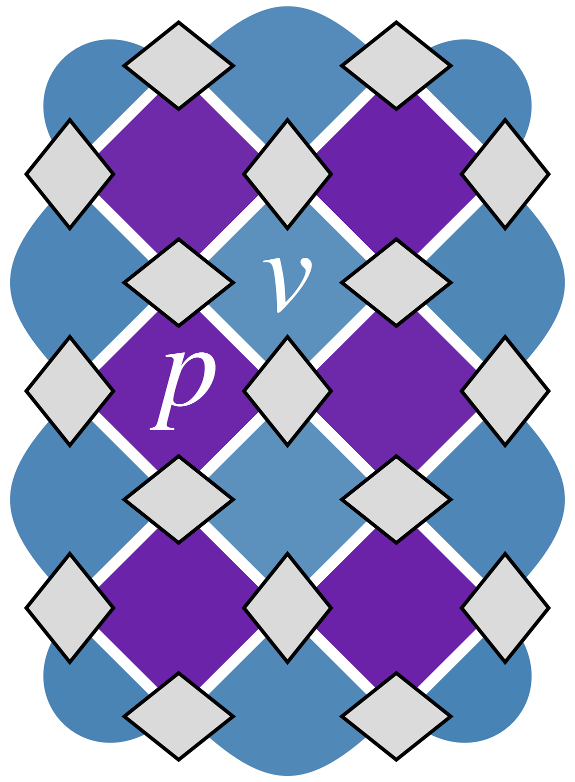


Beyond the fixed point: Confinement transition

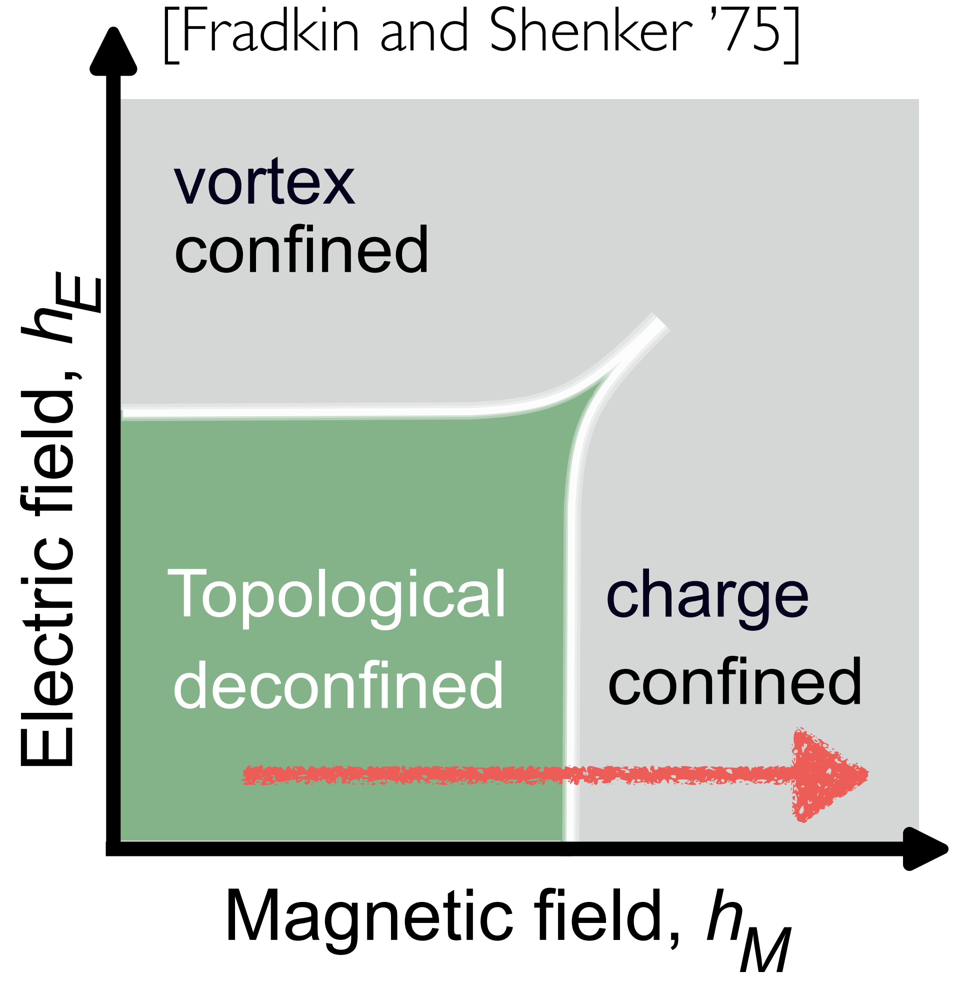
Toric code in a field

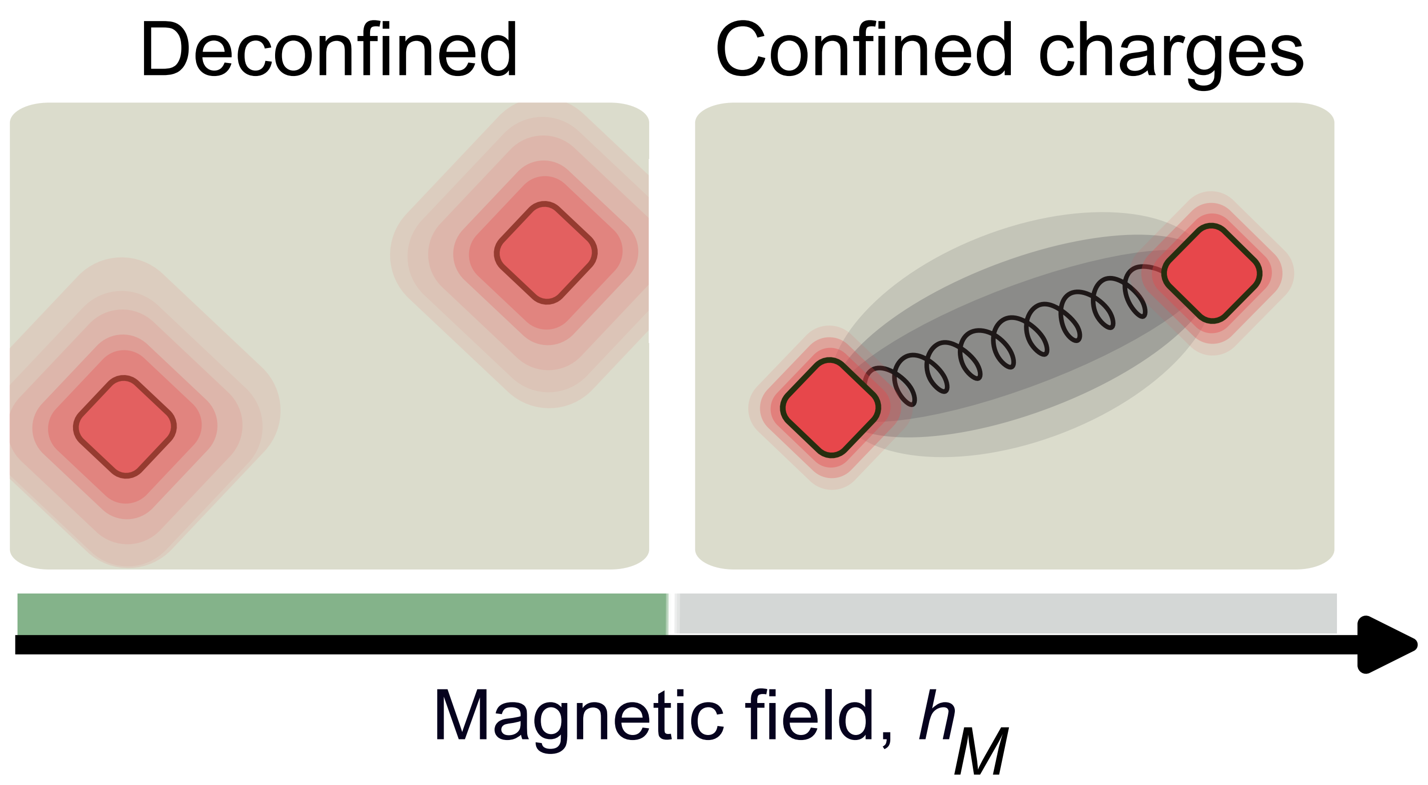
$$H = -J\sum_v A_v - J\sum_p B_p - h_M\sum_l Z_l - h_E\sum_l X_l$$

with $A_v = \prod_{i\in v} Z_i$ and $B_p = \prod_{i\in p} X_i$



[Fradkin and Shenker '75]





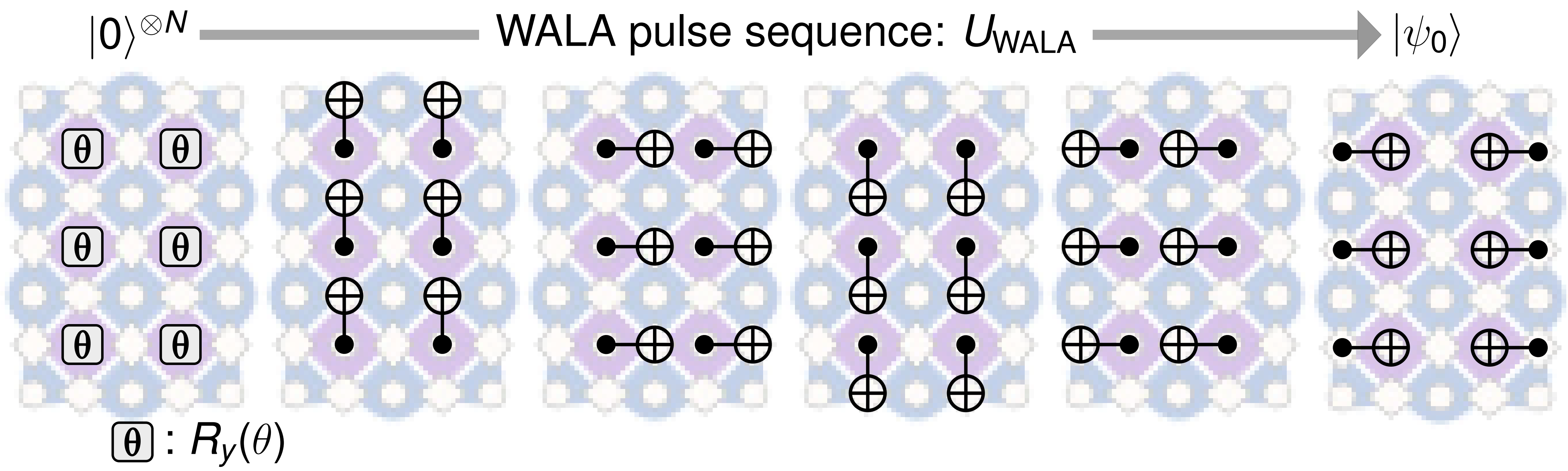
- Condensation of magnetic fluxes and confinement of electric charges

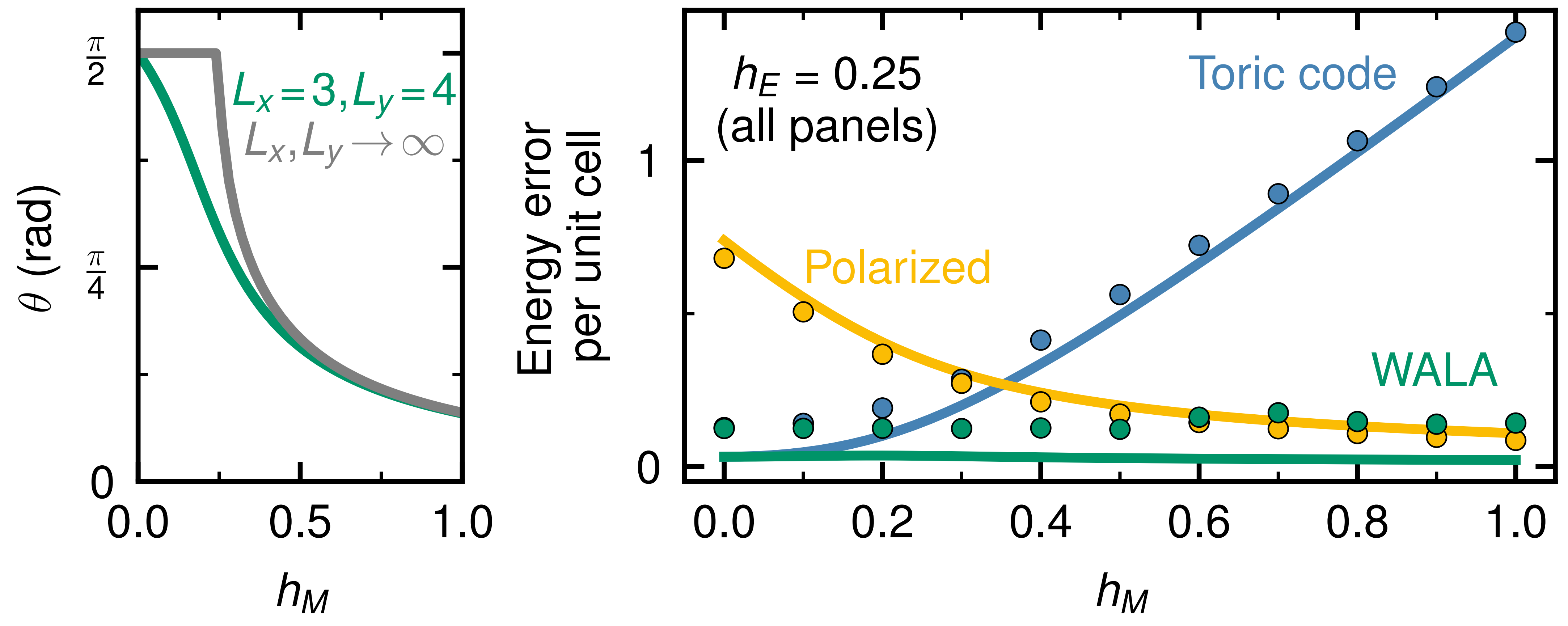
Beyond the fixed point: Confinement transition

Weight Adjustable Loop Ansatz (WALA):

$$|\psi\rangle = \prod_p \left(\cos\left(\tfrac{\theta}{2}\right)\mathbb{1} + \sin\left(\tfrac{\theta}{2}\right) B_p\right)|0\rangle$$

[Dusuel and Vidal '15, Sun et. al '23]

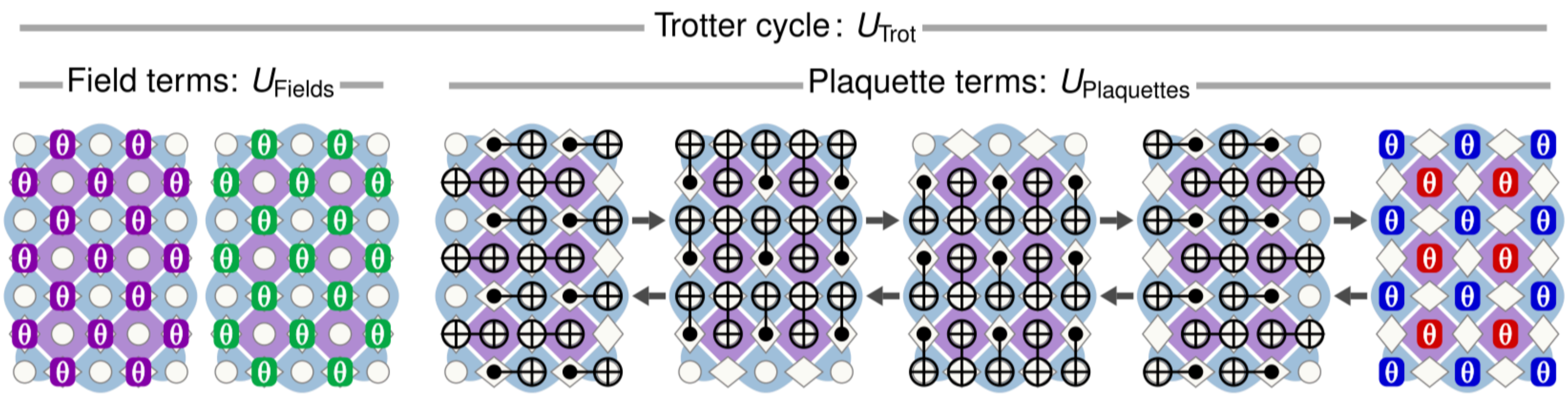




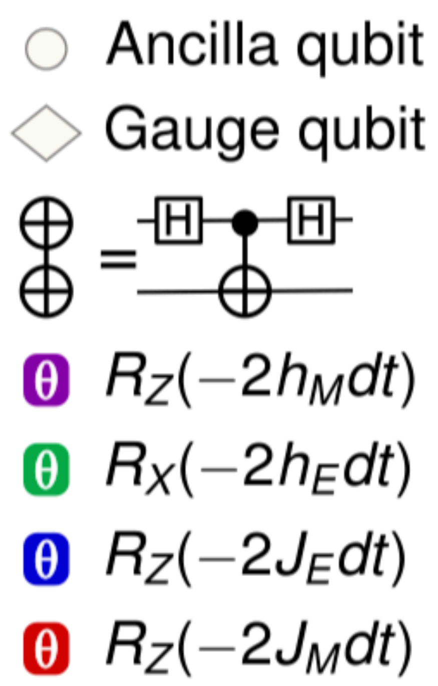
- Variational WALA state is simple to prepare and has low energy density

[Cochran, Jobst, et al., Smith, FP, Knap, Roushan, Nature **642**, 315 (2025)]

Suzuki-Trotter evolution circuit

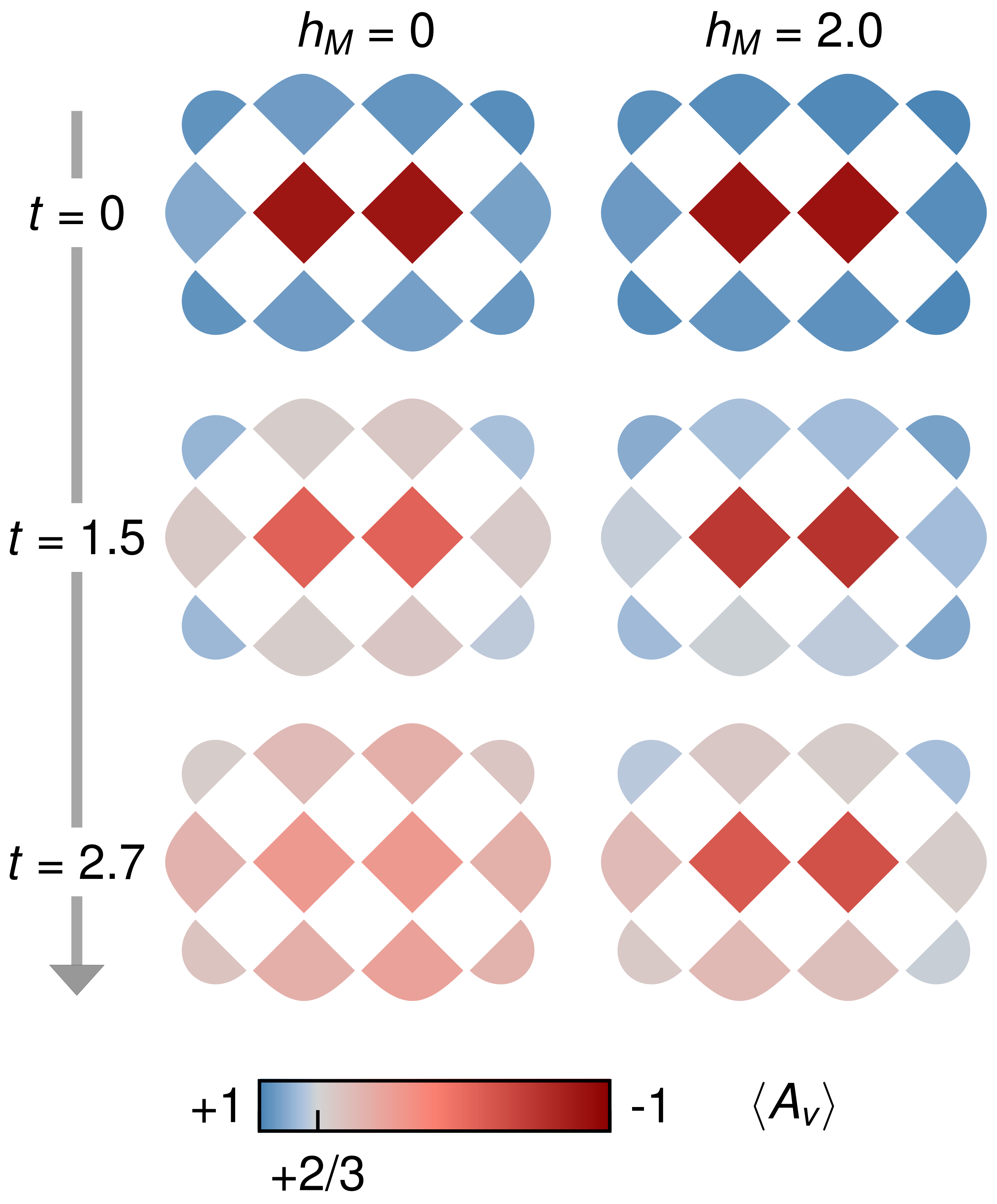


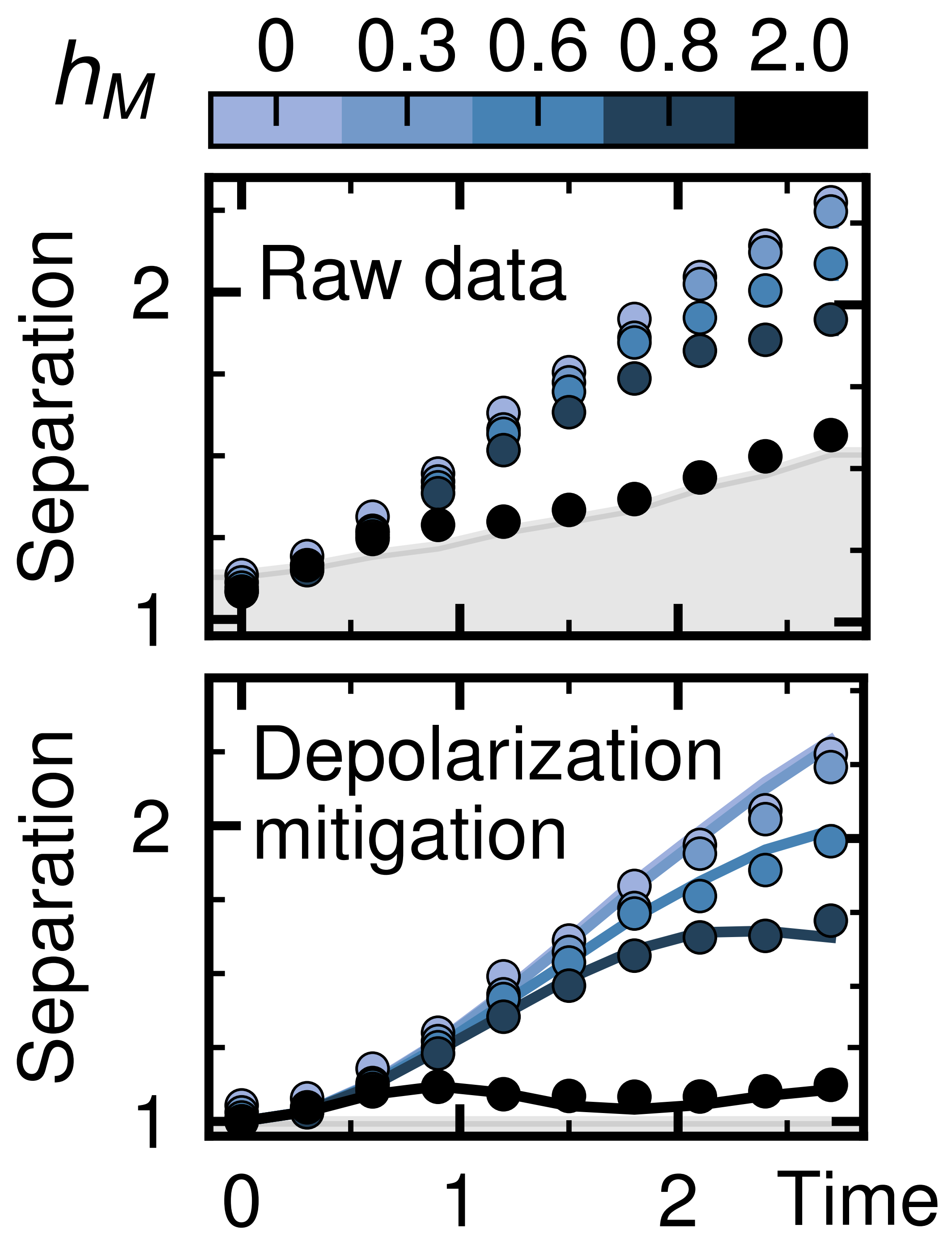
- Efficient Suzuki-Trotter using ancilla qubits (116 CZ gates per time step for the grid of 35 qubits)
- Post-select the measured data on the ancilla $|0\rangle$ state to mitigate decoherence



[Cochran, Jobst, et al., Smith, FP, Knap, Roushan, Nature **642**, 315 (2025)]

Confinement dynamics of electric charges





- Qualitatively distinct dynamical signatures as the magnetic field is tuned

[Cochran, Jobst, et al., Smith, FP, Knap, Roushan, Nature **642**, 315 (2025)]

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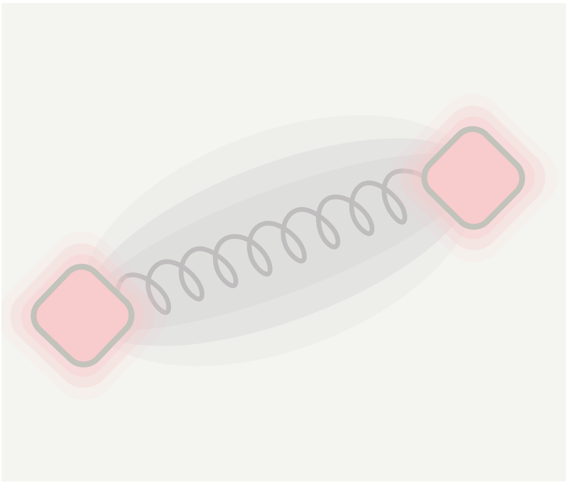
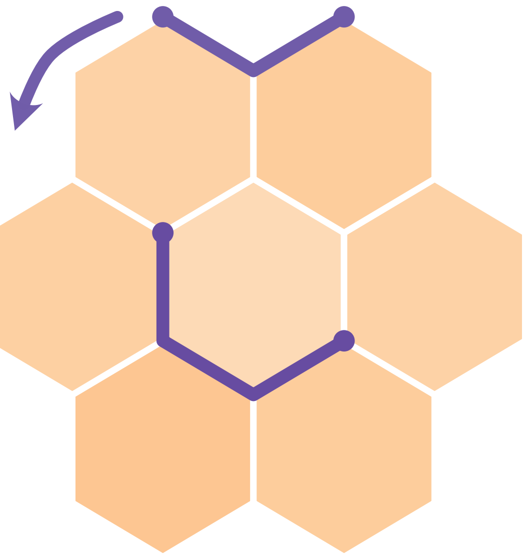


Floquet Topological Order

Probing non-equilbrium topological order

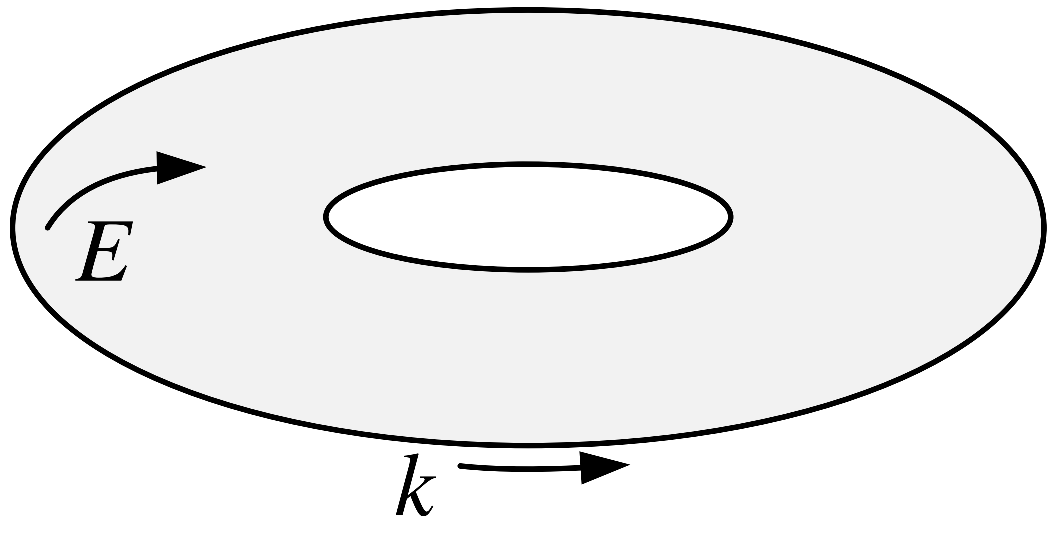


Non-equilibrium phases of matter in Floquet systems

Periodically modulated Hamiltonian: $H(t) = H(t+T)$

$$U(t) = \mathcal{T} \exp\left(-i\int_0^t d\bar{t}\, H(\bar{t})\right) \quad \rightarrow \quad \text{Floquet operator } U(T) = e^{iTH_F}$$

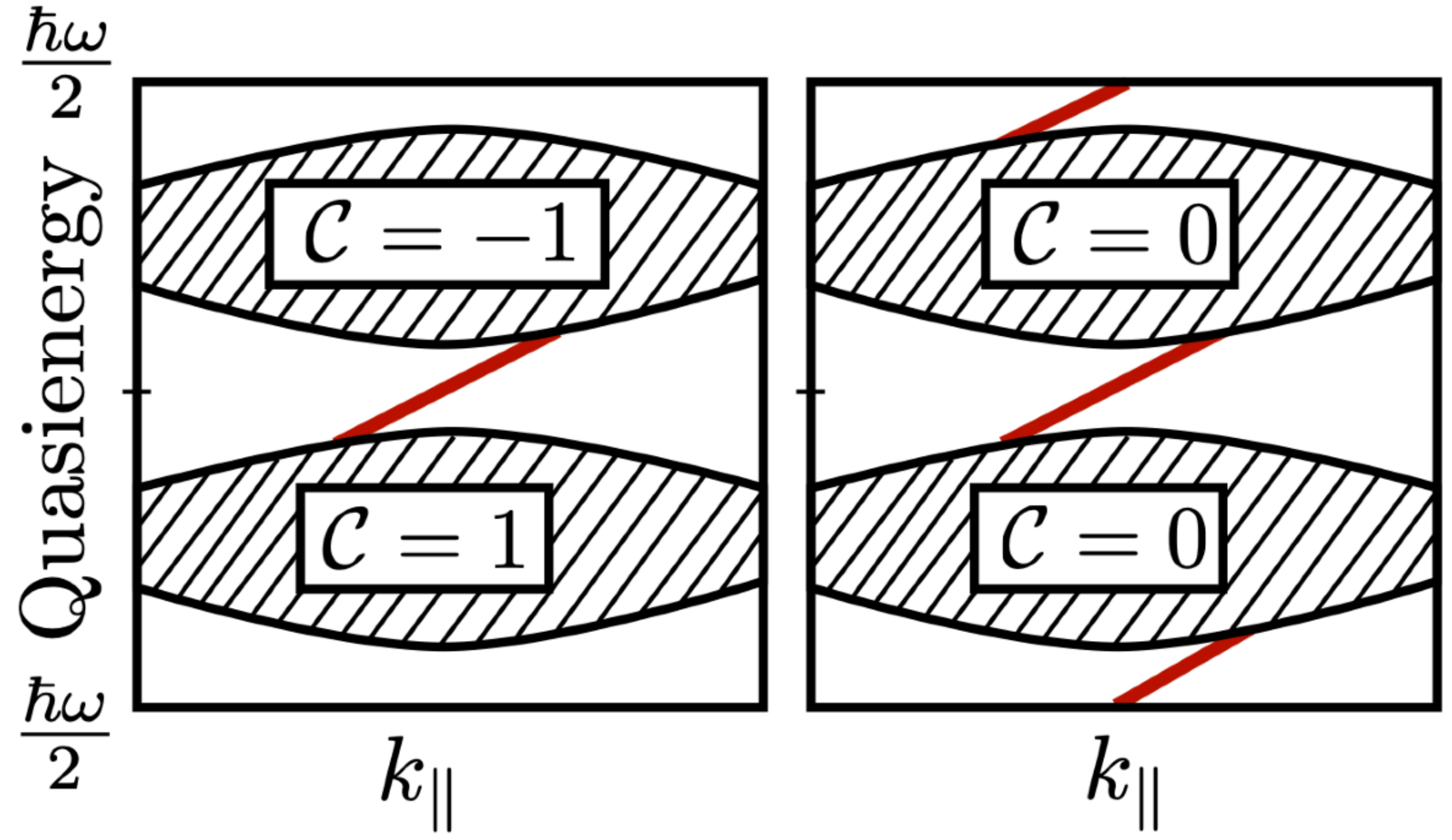
Eigenvalues of determined only up to modulo $2\pi/T$



- **Topological phenomena beyond equilibrium:**

  Protected chiral edge modes with $c = 0$

  [Kitagawa et al '10, Rudner et al. '13]



Periodically driven Kitaev model: Floquet Topological Order (FTO) with chiral Majorana edge modes [Po et al.'17]

$U_T = U_Z(JT)\ U_Y(JT)\ U_X(JT)$ with

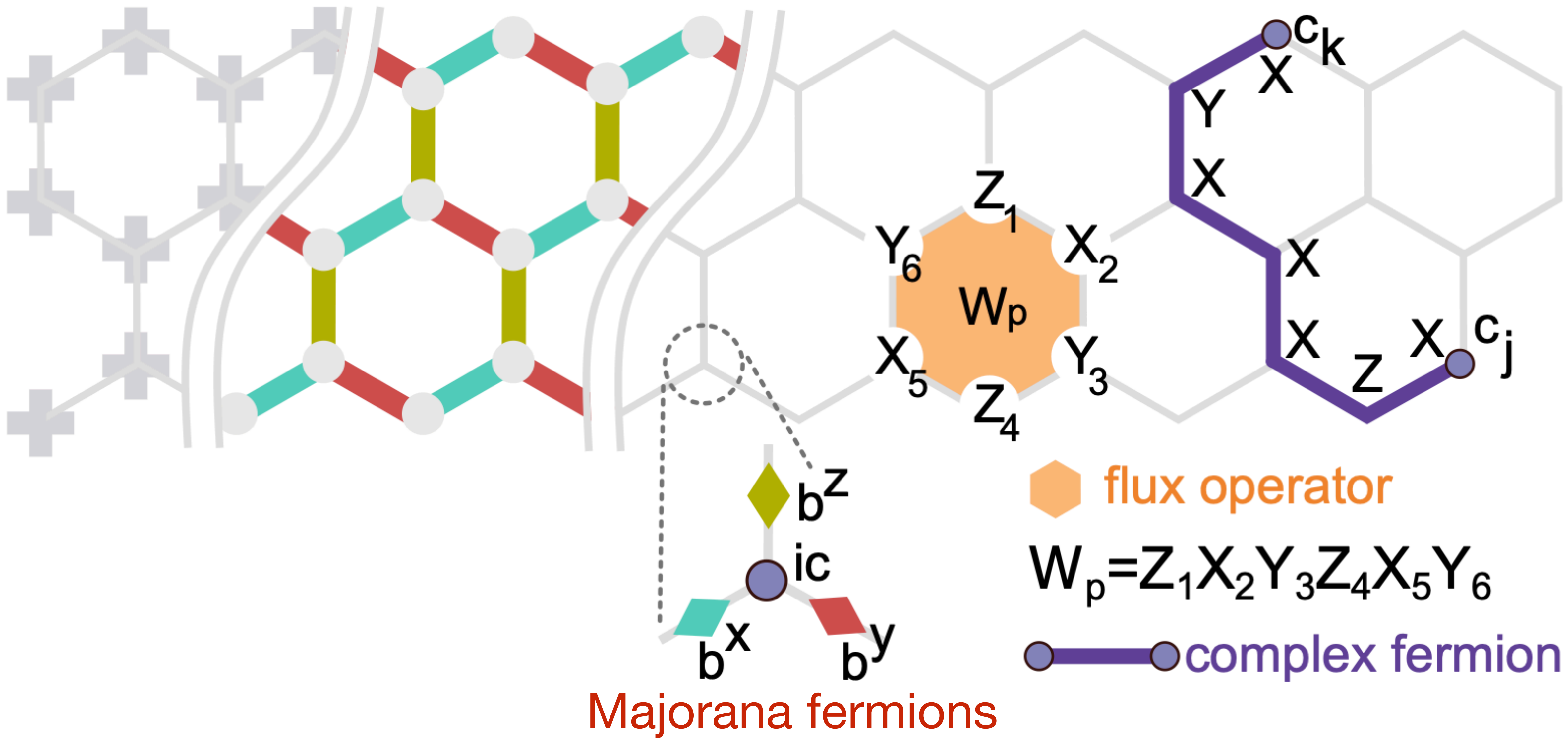
$$U_\alpha(JT) = \exp\Big(- i\frac{\pi}{4} JT \sum_{\langle j,k\rangle_\alpha} \alpha_j \alpha_k \Big),\ \alpha = X, Y, Z$$

$$= \exp\{-JT\frac{\pi}{4}\sum_{\langle j,k\rangle_\alpha} u_{jk} c_j c_k\}$$

Qubit implementation

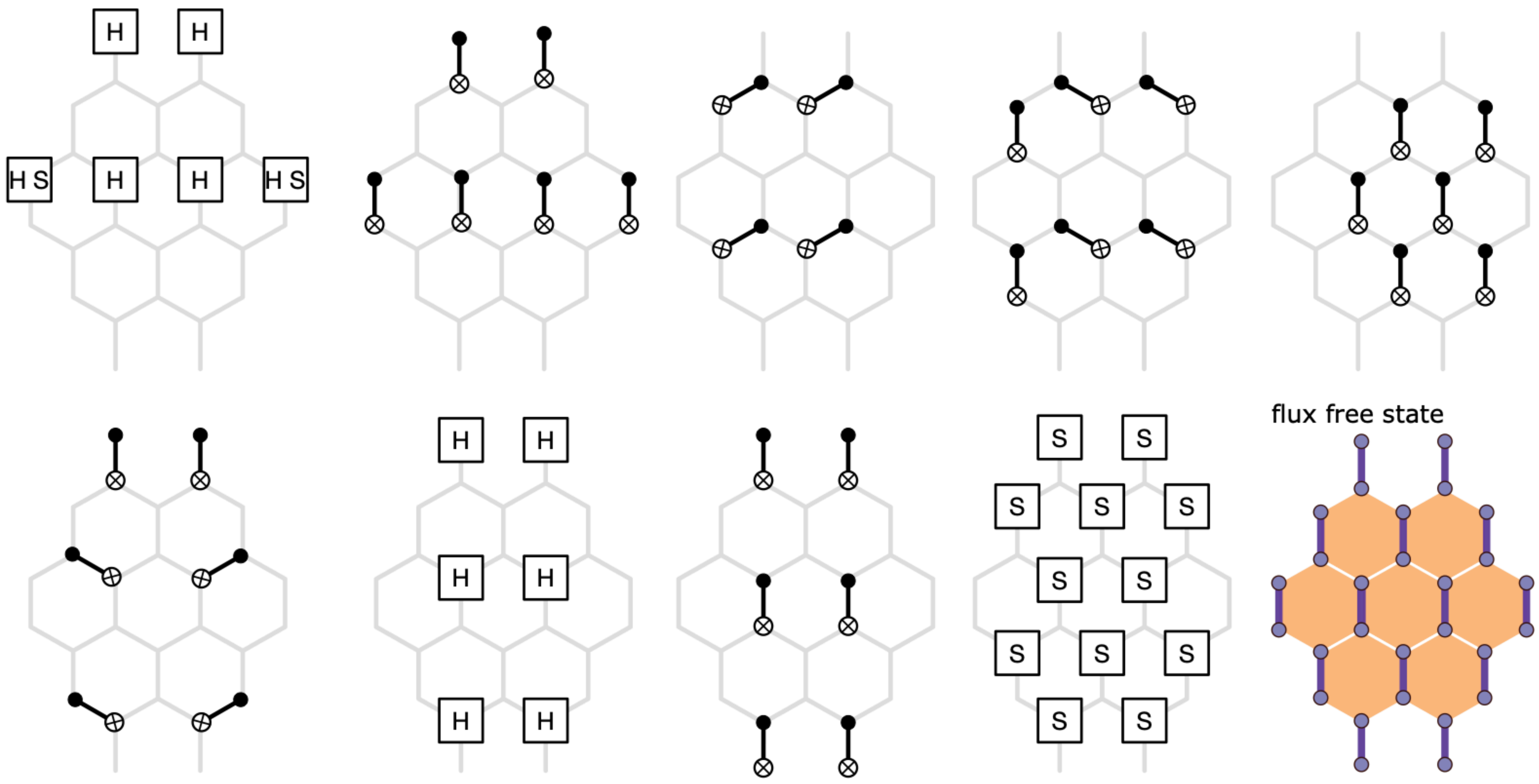
Floquet drive

Flux operator and a complex fermion



Preparing the flux free state

Flux-free state preparation in two-site gates (c.f. toric code state)

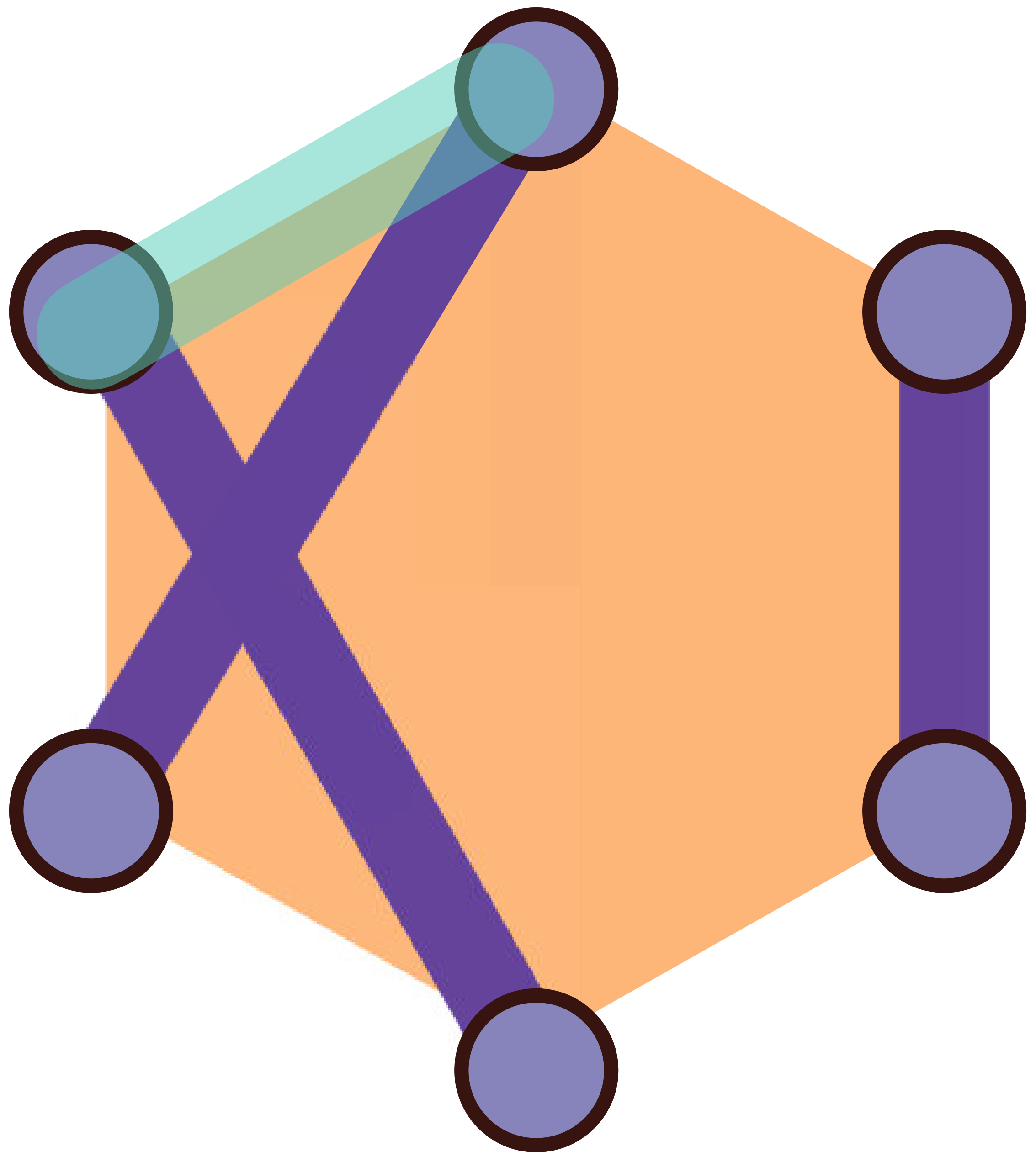


Non-equilibrium Floquet Topological Order

Periodically driven Kitaev model: Floquet Topological Order (FTO) with chiral Majorana edge modes [Po et al. '17]

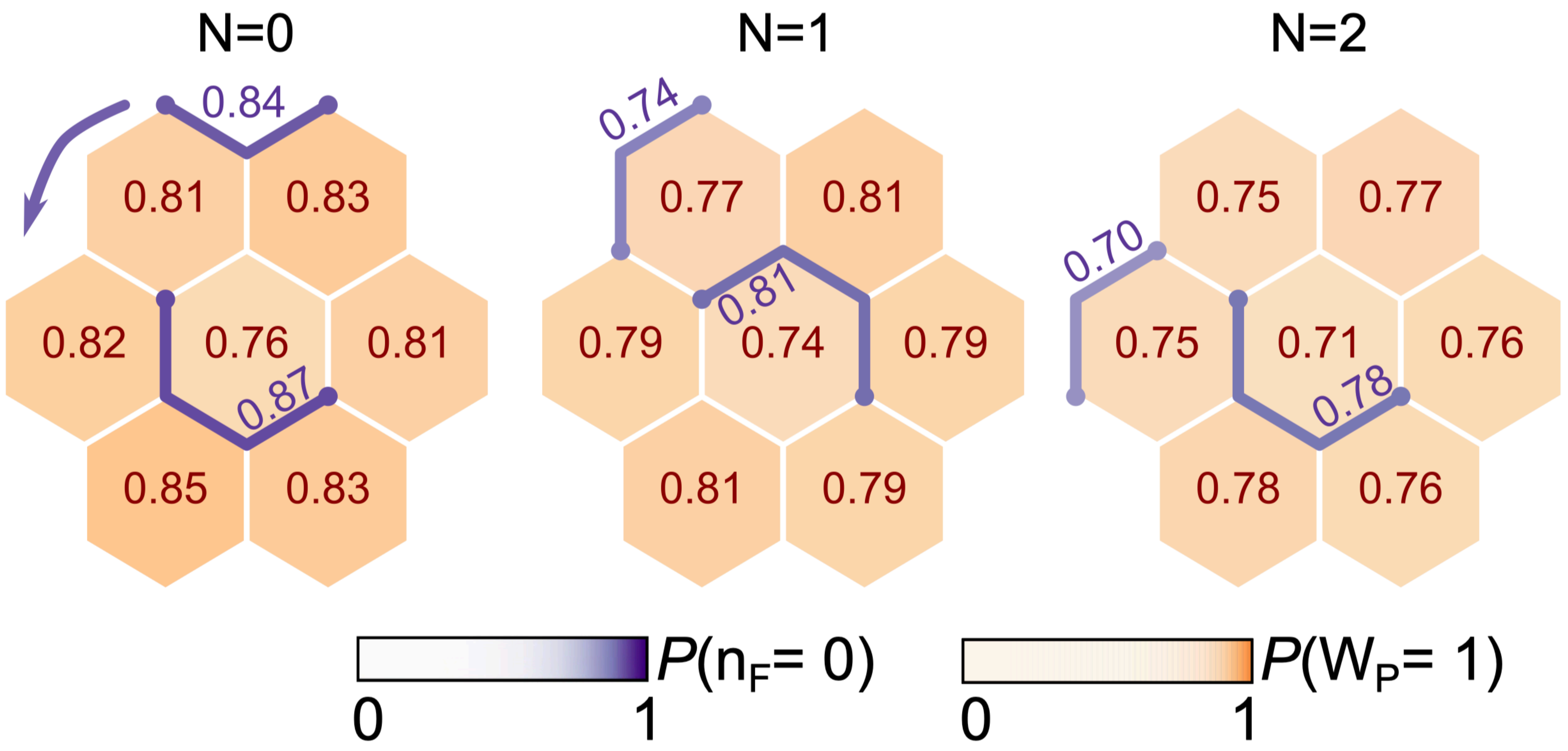
$U_T = U_Z(JT)\ U_Y(JT)\ U_X(JT)$ with $U_\alpha(JT) = \exp\{-JT\frac{\pi}{4}\sum_{\langle j,k\rangle_\alpha} u_{jk}c_jc_k\}$

$JT = 1$: Majorana swap:



Floquet Topological Order

Probing the dynamics of the FTO model ($JT = 1$) on the QPU

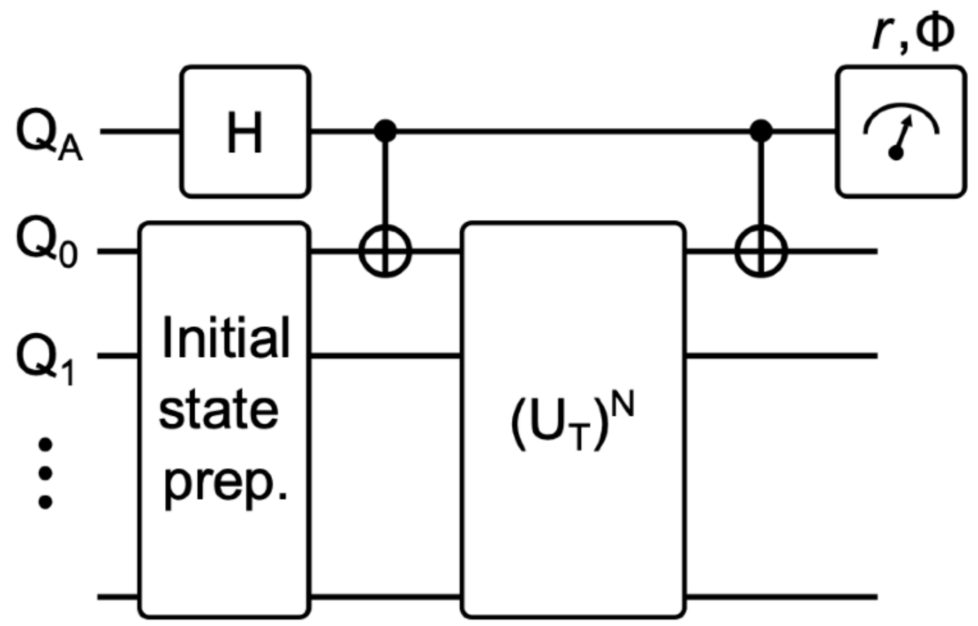


Localized bulk and **chiral edge modes!**

[Will, Cochran, Rosenberg, Jobst, Eassa, Smith, Roushan, Knap, FP, arXiv:2501.18461]

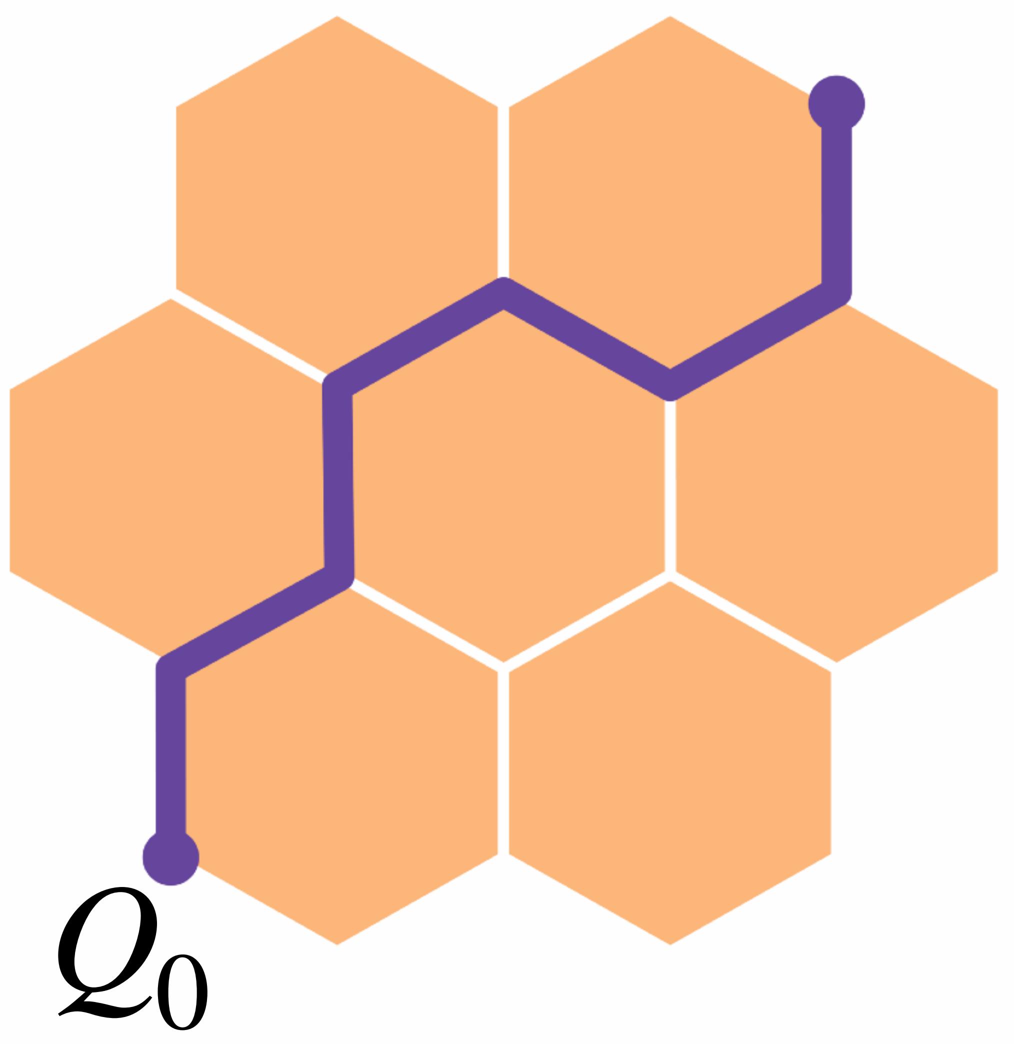
Braiding Majoranas at the edge

Dynamical Majorana edge mode interferometry ($JT = 1$)



N=0

$$\langle \Psi_{n_F=1}(N) | X_{Q_0} | \Psi_{n_F=0}(N) \rangle = re^{i\Phi}$$

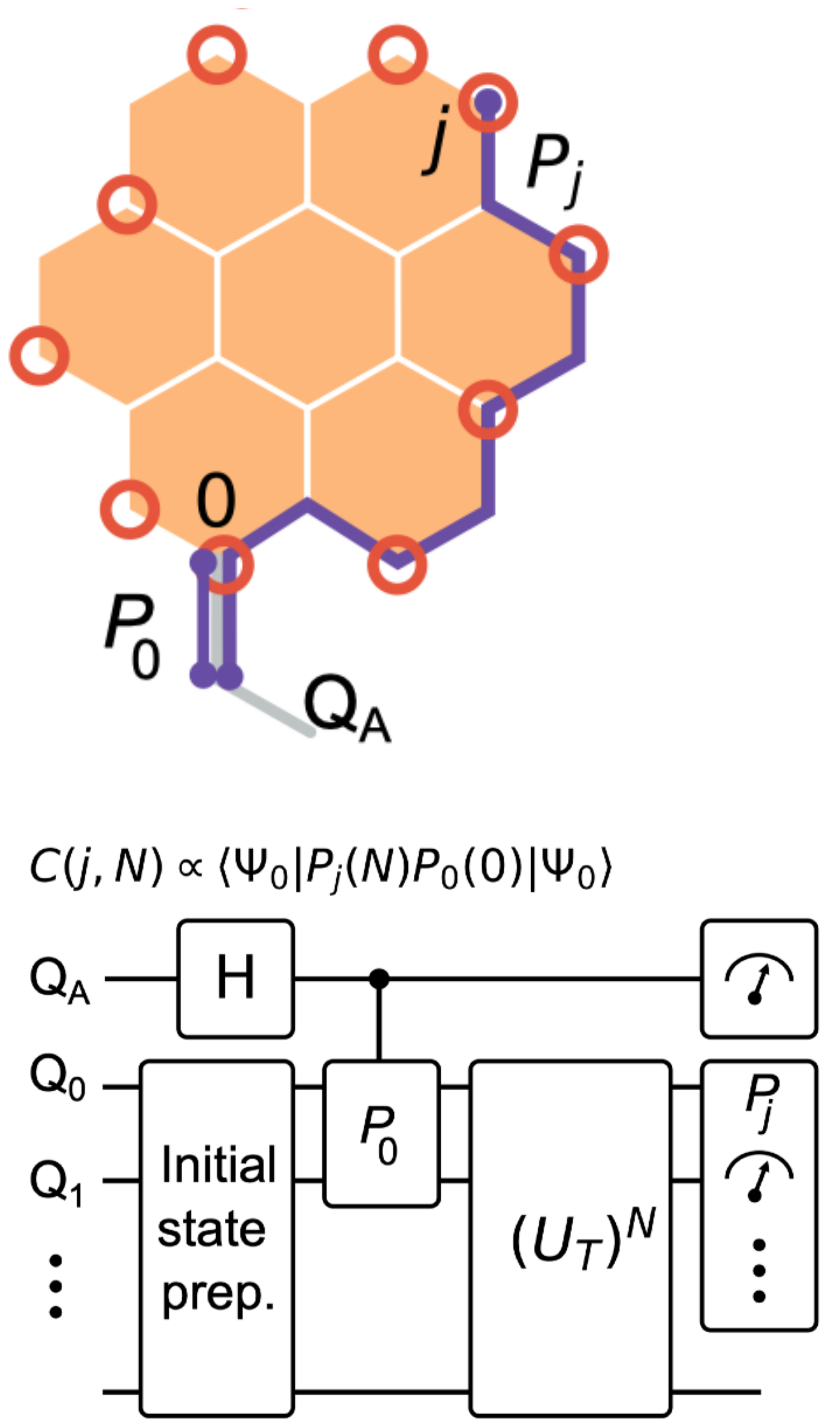


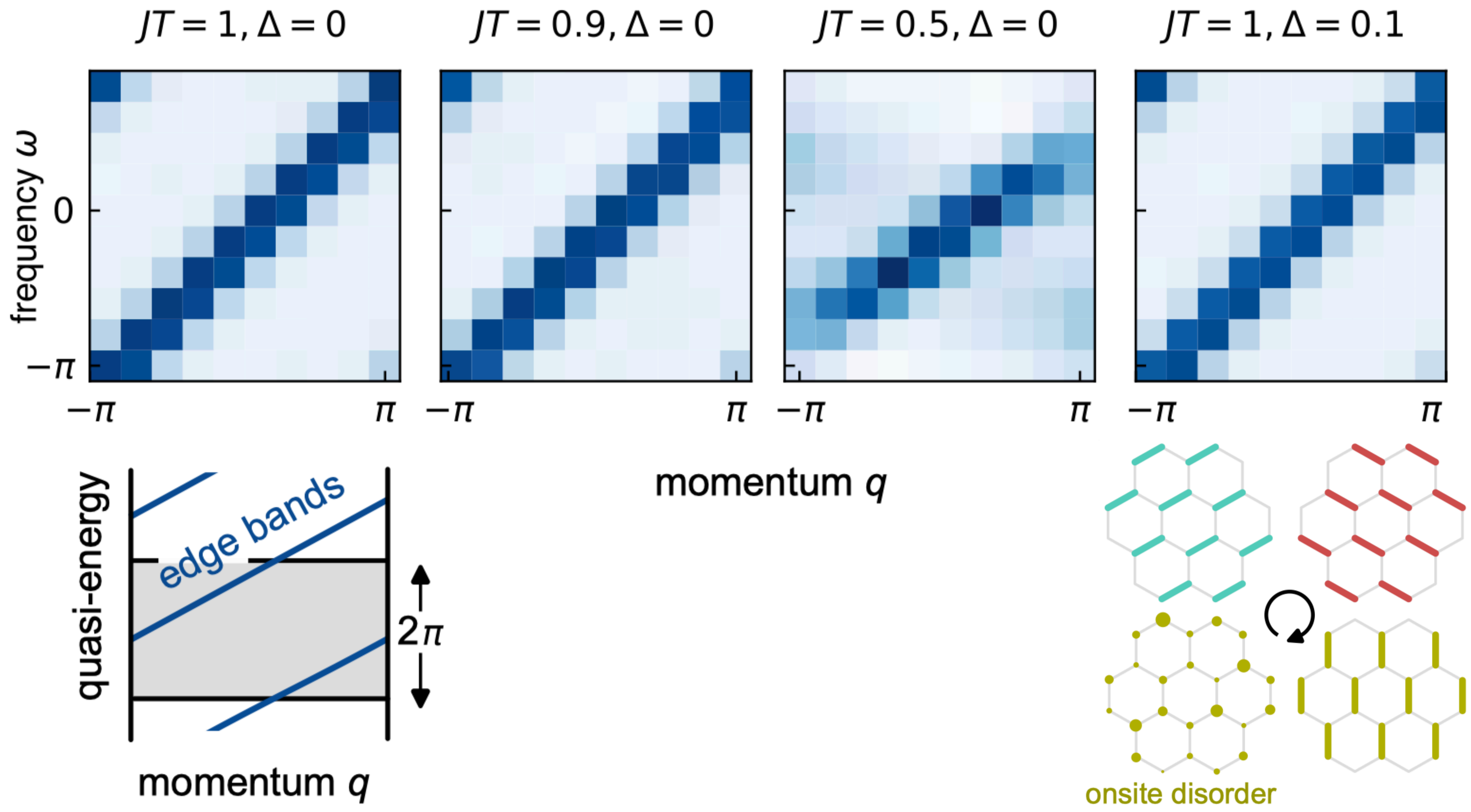
- Relative phase of $e^{i\pi/2}$ for exchange of the Majorana modes in $|0\rangle$ and $|1\rangle$

[Will, Cochran, Rosenberg, Jobst, Eassa, Smith, Roushan, Knap, FP, arXiv:2501.18461]

Chiral Edge Modes

Majorana edge mode spectrum



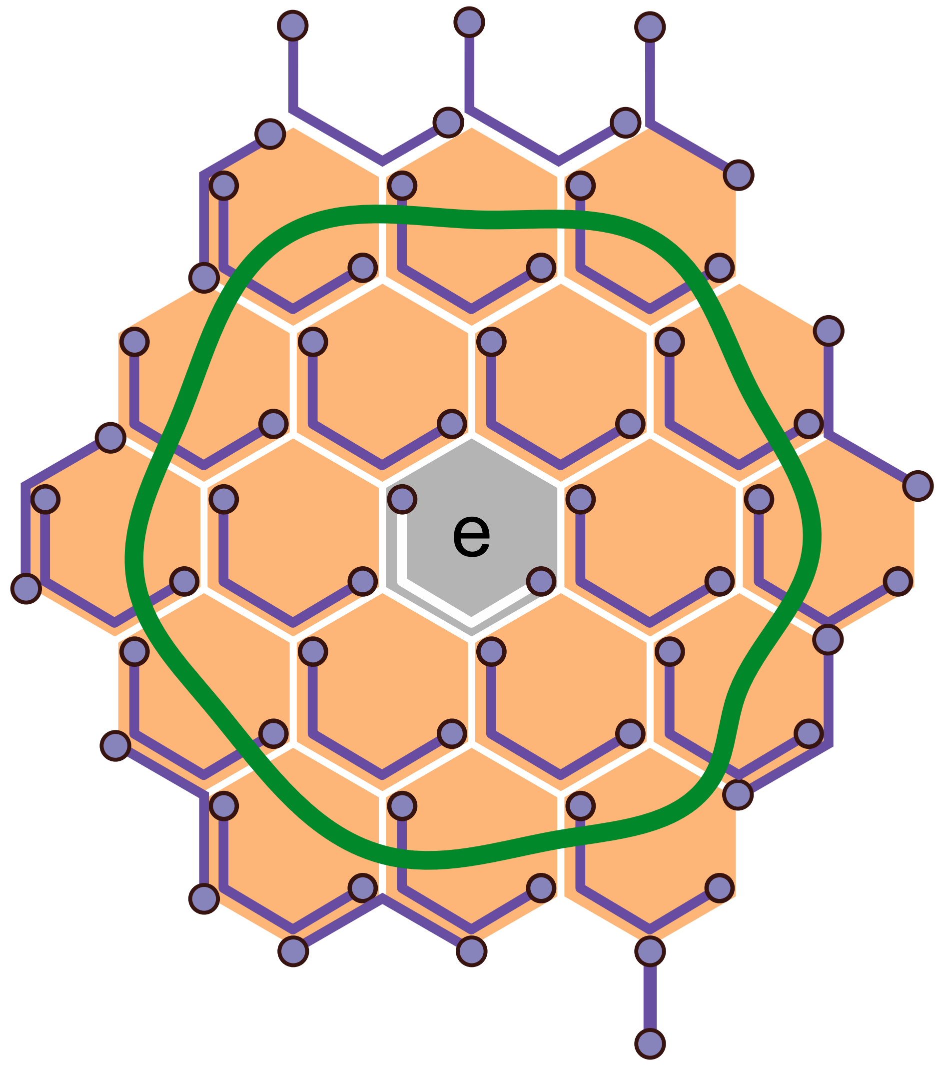


Robust chiral edge mode in FTO phase

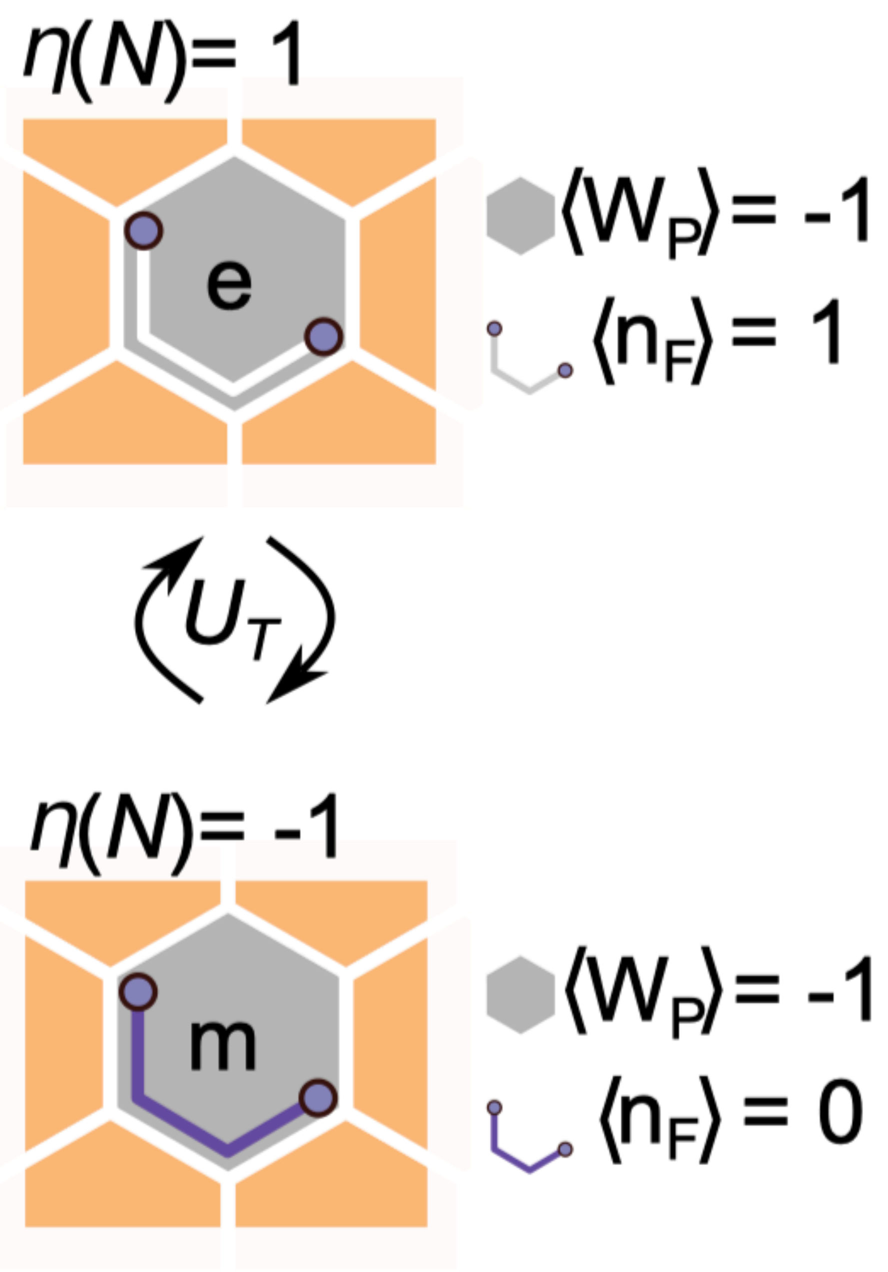
[Will, Cochran, Rosenberg, Jobst, Eassa, Smith, Roushan, Knap, FP, arXiv:2501.18461]

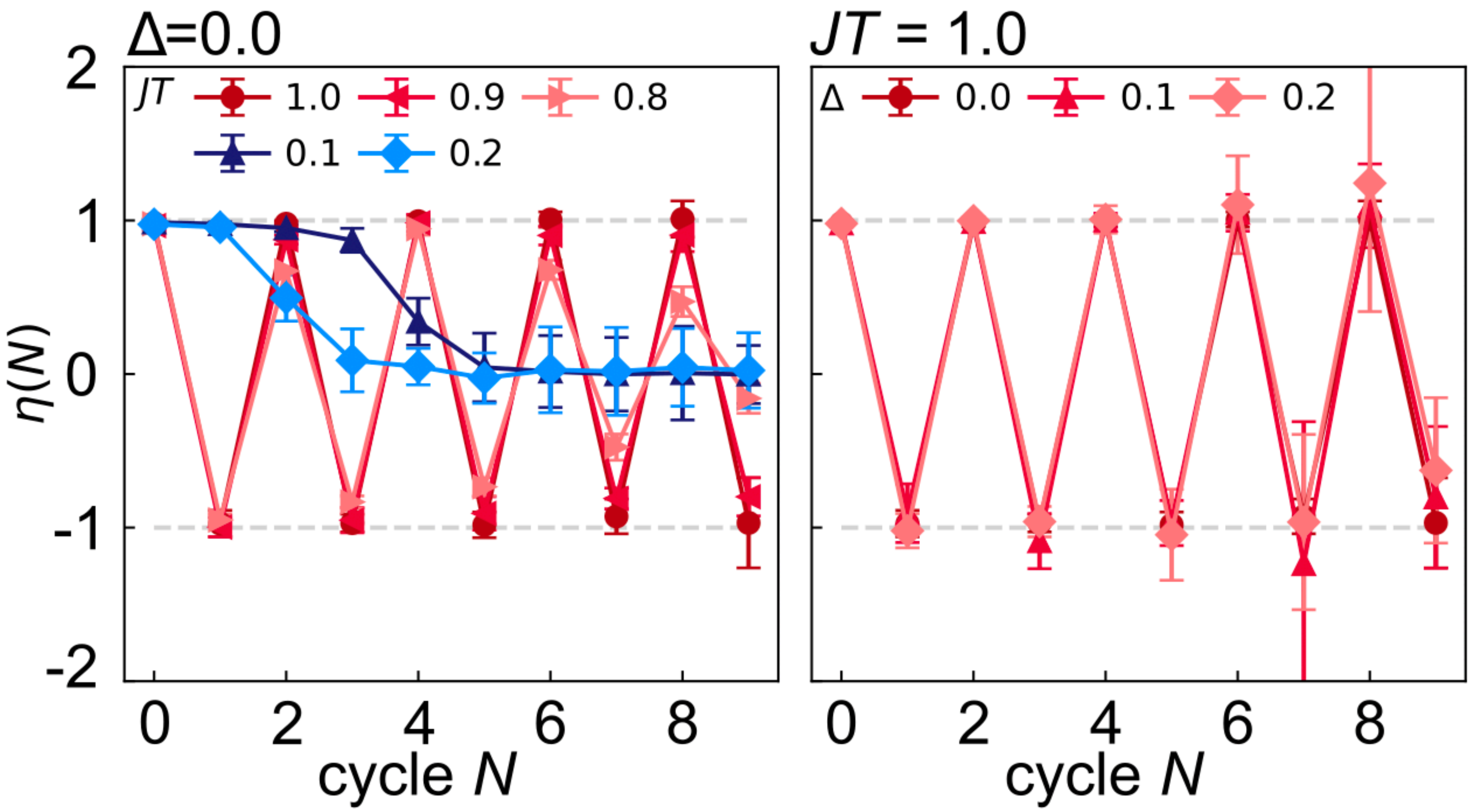
Bulk topological invariant

Anyon transmutation as order parameter on systems with 58 qubits



$$\eta(N) = \frac{\langle\Psi_e|O(N)|\Psi_e\rangle}{\langle\Psi_0|O(N)|\Psi_0\rangle}$$



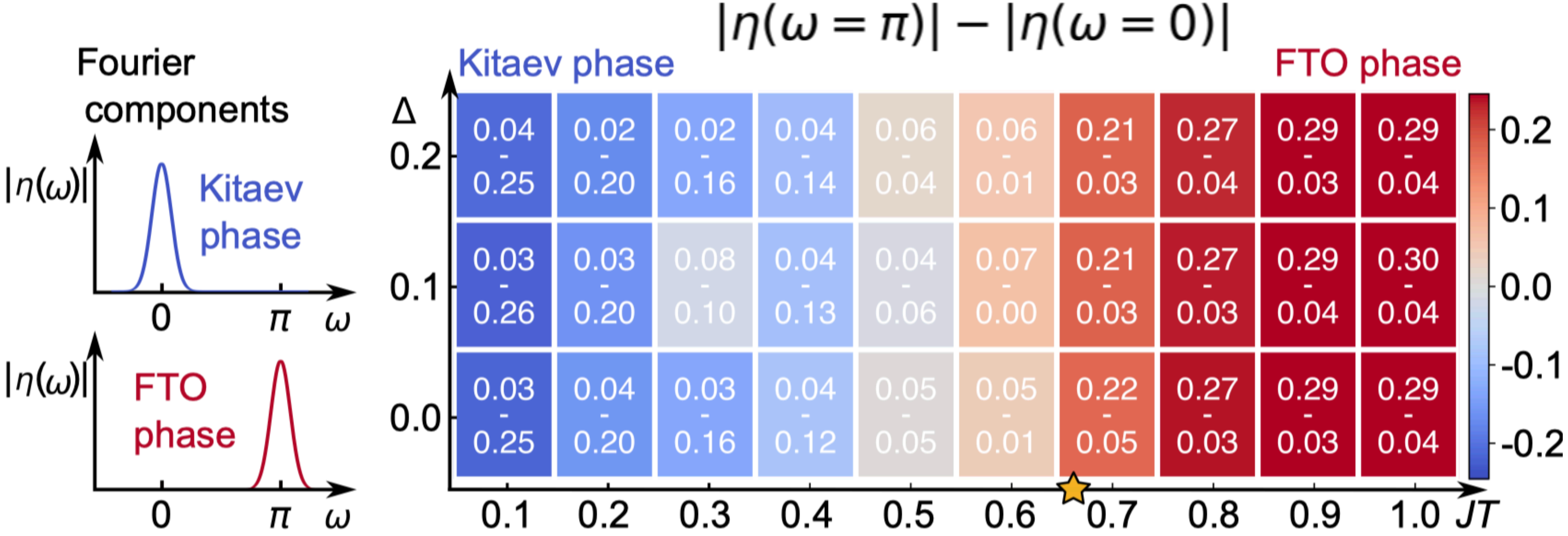


- Anyon transmutation robust for a range of parameters!

[Will, Cochran, Rosenberg, Jobst, Eassa, Smith, Roushan, Knap, FP, arXiv:2501.18461]

Non equilibrium phase diagram

Anyon transmutation as order parameter
on systems with 58 qubits



[Will, Cochran, Rosenberg, Jobst, Eassa, Smith, Roushan, Knap, FP, arXiv:2501.18461]

Conclusions

Realization of topological ordered ground states

Topological entanglement

Anyonic quasi-particles

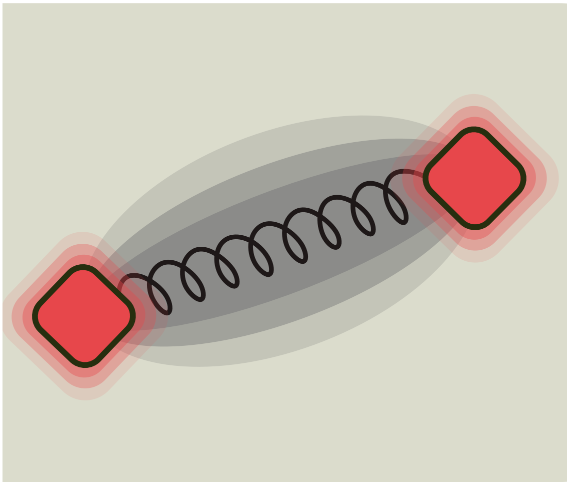
[Satzinger, Liu, Smith, et al., Knap, FP, Roushan, Science **374**, 1237 (2021)]

Dynamics of the confinement transition

Dynamics of electric charges

String dynamics

[Cochran, Jobst, et al., Smith, FP, Knap, Roushan, Nature **642**, 315 (2025)]



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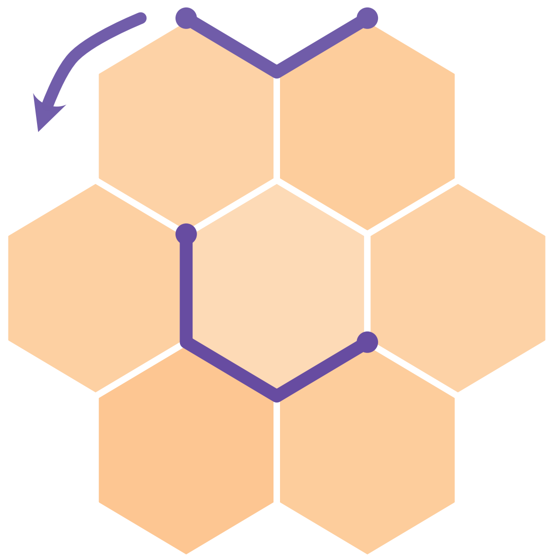

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Cumulative distribution functions of errors