Lifshitz Josephson Junction

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Abstract

Scaling symmetry of space time plays important role in many physical phenomena. For example, the CMB spectrum of cosmology and critical phenomena in condensed matter physics are governed by scaling symmetry. In this project, we consider anisotropic scaling symmetry in non-relativistic system and study its effect on quantum mechanics. As an example, we consider an anisotropic Josephson junction and show that its efficiency can be greatly enhanced by tuning the degree of anisotropy. Such anisotropic Josephson junction can be realized with a special form of lattice, which is realizable with the help of modern computational materials science.

1. Introduction

Lifshitz scaling has the form

$$x \to \lambda\, x, \qquad t \to \lambda^z t, \qquad z \in \mathbb{R}, \tag{1}$$

which gives anisotropic scaling between the space and time coordinates. Theories that exhibit symmetry under this scaling transformation has been of interest to holography duality [1] and quantum gravity [2].

In this project, we consider Lifshitz symmetry more fundamentally in quantum mechanics, which can be encoded in terms of a fractional quantum mechanics.

2. Fractional Quantum Mechanics

Consider a Hamiltonian of the form

$$\hat{H} = -\frac{\hbar^z}{M^{z-2}}\frac{\Delta_z}{2m} + V(\hat{r})\,, \tag{2}$$

where M is some momentum scale that characterizes the Lifshitz symmetry, and Δ_z is a fractional derivative of order z, defined by the operation

$$\Delta_z\, e^{kx} = (k^2)^{z/2}\, e^{kx}. \tag{3}$$

One can also try to construct a discretized fractional derivative in order to study fractional quantum mechanics on lattice.

Acting Δ_z on a wave function $\psi(\boldsymbol{r}, t)$ gives

$$\begin{aligned} \hbar^z \Delta_z \psi(\boldsymbol{r}, t) &= \frac{\hbar^z}{(2\pi\hbar)^D}\int d^D p \left(\Delta_z\, e^{i\frac{\boldsymbol{p}\cdot\boldsymbol{r}}{\hbar}}\right)\varphi(\boldsymbol{p}, t) \\ &= \frac{(-1)^{z/2}}{(2\pi\hbar)^D}\int d^D p\, |\boldsymbol{p}|^z e^{i\frac{\boldsymbol{p}\cdot\boldsymbol{r}}{\hbar}}\varphi(\boldsymbol{p}, t), \end{aligned} \tag{4}$$

where $\varphi(\boldsymbol{p}, t)$ is the fourier transform of the wave function.

3. Probability Current Density

Using (2), the time derivative of the probability density $\rho(\boldsymbol{r}, t) \equiv \psi^*(\boldsymbol{r}, t)\psi(\boldsymbol{r}, t)$ is

$$\frac{\partial \rho(\boldsymbol{r}, t)}{\partial t} = -\frac{1}{m\hbar}\left(\frac{1}{M^{z-2}}\right) Im[\psi^*(\boldsymbol{r}, t)\hbar^z \Delta_z \psi(\boldsymbol{r}, t)] \tag{6}$$

$$= -\nabla \cdot \boldsymbol{J}. \tag{7}$$

The probability current density that satisfies (6) and (7) has the form

$$\boldsymbol{J} = -\frac{1}{2m}\left(\frac{(-1)^{\frac{z}{2}}}{M^{z-2}}\right)\frac{1}{(2\pi\hbar)^{2D}}\iint d^D p\, d^D q\, e^{i\frac{(\boldsymbol{p}-\boldsymbol{q})\cdot\boldsymbol{r}}{\hbar}} \varphi^*(\boldsymbol{q}, t)\varphi(\boldsymbol{p}, t)\frac{|\boldsymbol{q}|^z - |\boldsymbol{p}|^z}{|\boldsymbol{q}|^2 - |\boldsymbol{p}|^2}(\boldsymbol{q} + \boldsymbol{p}) \tag{8}$$

In the past, the continuity equation has been written as $\frac{\partial \rho(\boldsymbol{r},t)}{\partial t} + \nabla \cdot \boldsymbol{J} = \boldsymbol{K}$, where $\boldsymbol{K}$ is interpreted [3] as an additional source term, thus violates the fundamental unitarity property of quantum mechanics. Here, with (7) and (8), we point out that this result in the literature is wrong and misleading.

4. Josephson Junction

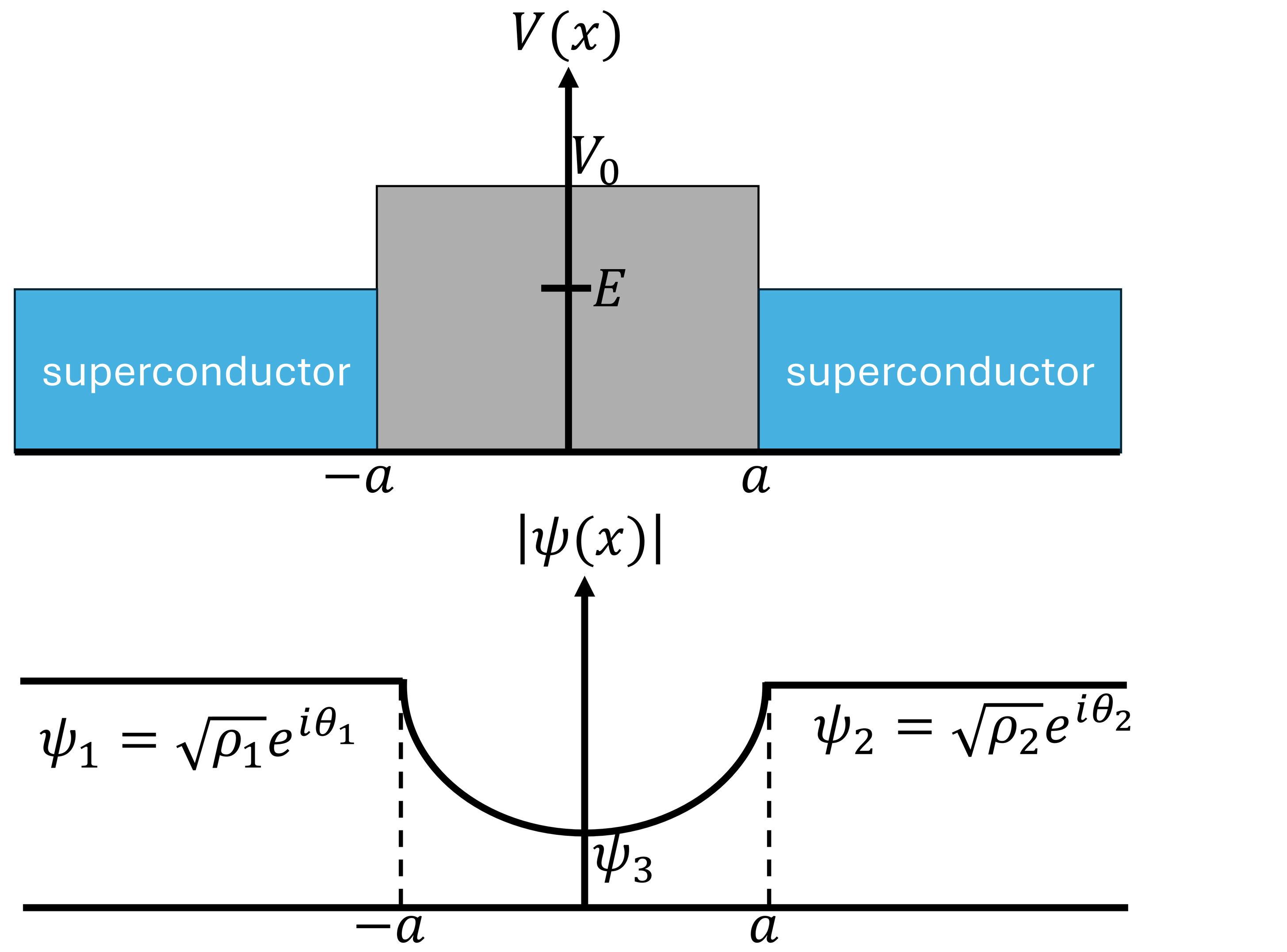

Figure 1. Configuration of the system (top) and the wave function in each region (bottom).

In the barrier region,

$$\psi_3(x) = C_1 \cosh\left(\frac{x}{\zeta}\right) + C_2 \sinh\left(\frac{x}{\zeta}\right) \tag{9}$$

In terms of isotropic penetration depth $\zeta_0 = \hbar/\sqrt{2m(V_0 - E)}$ and $\lambda_M = \hbar/M$, the penetration depth is

$$\zeta = \zeta_0 \left(\frac{\zeta_0}{\lambda_M}\right)^{\left(\frac{2}{z}\right)-1}. \tag{10}$$

From (8) and the boundary condition between the barrier and the superconductors, $\boldsymbol{J} = \boldsymbol{J_c}\sin(\theta_2 - \theta_1)$, where the critical current density $\boldsymbol{J_c}$ in terms of isotropic critical current $\boldsymbol{J}_{c0}$ is

$$\frac{\boldsymbol{J_c}}{\boldsymbol{J}_{c0}} = \frac{z}{2}\left(\frac{\zeta_0}{\lambda_M}\right)^{\frac{2}{z}-1}\frac{\sinh\left(\frac{2a}{\zeta_0}\right)}{\sinh\left(\frac{2a}{\zeta_0}\left(\frac{\lambda_M}{\zeta_0}\right)^{\frac{2}{z}-1}\right)}. \tag{11}$$

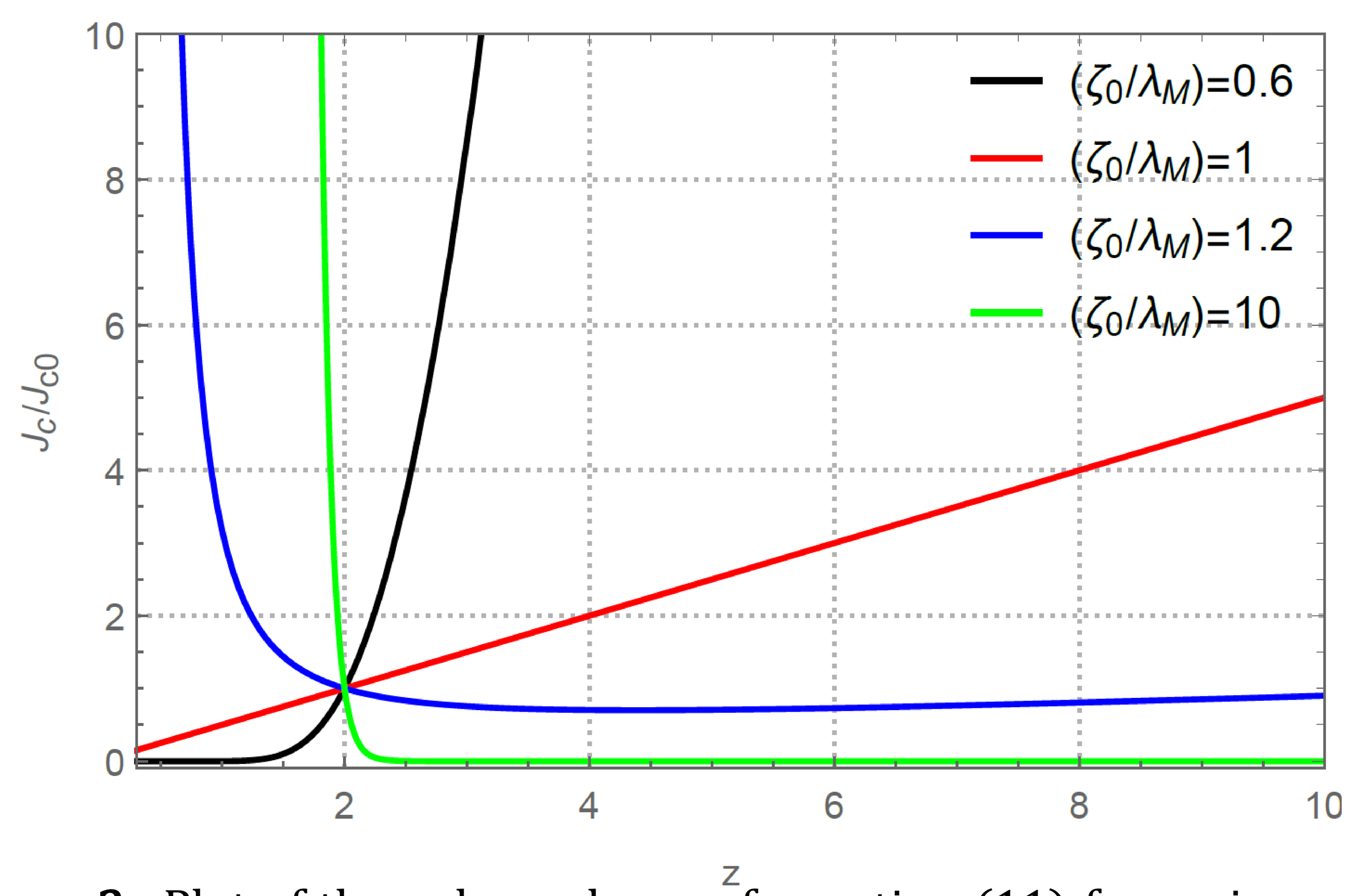

Figure 2. Plot of the z dependence of equation (11) for various values of ζ_0/λ_M with $2a/\zeta_0 = 10$. One can see that the critical current can be enhanced with small penetration depth and high anisotropy index z (black line).

Conclusion

We have demonstrated that Lifshitz symmetry can have interesting effects in quantum mechanics. The enhancement of the tunneling current in the Josephson Junction is predicted. This may have interesting real world applications.

References

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