Flavor Equilibration of Supernova Neutrinos: Exploring the Dynamics of Slow Modes

Based on: Padilla-Gay et al. (2025), arXiv:2505.11588

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2025 NCTS, The Future is Whispering

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Outline

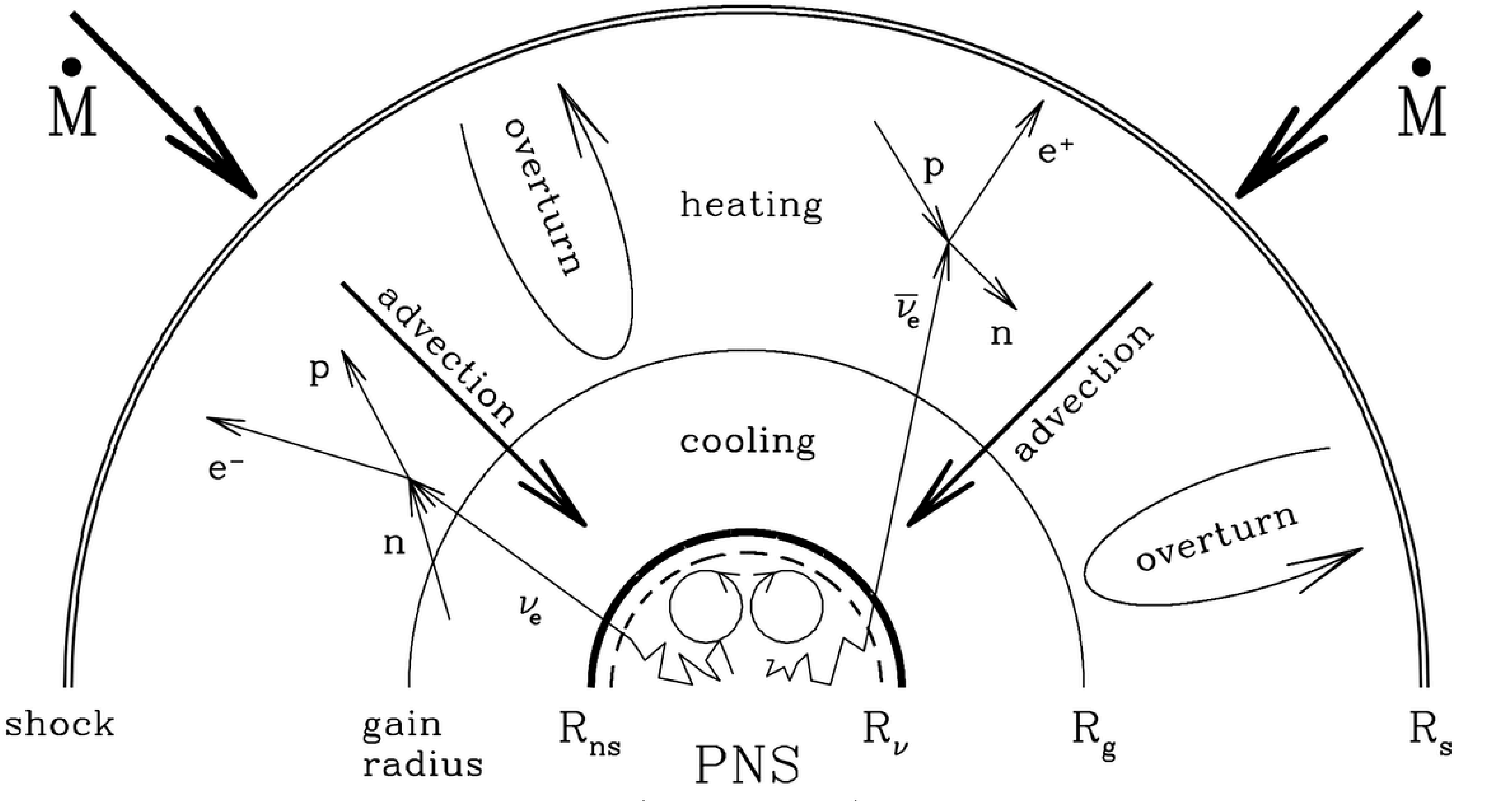
- Introduction
- Equation of motion
- Vacuum oscillations
- Fast conversions
- Slow conversions
- Summary

Introduction

- 99% of the output energy in Core-Collapse Supernova released in form of neutrino

- Neutrino dominant Core-Collapse Supernova dynamics
  - Cross-sections are flavor depentent
  - Shock revival

- Proton-neutron ratio
  - Nucleosynthesis in heavy elements



Janka, A&A 368, 527-560 (2001)

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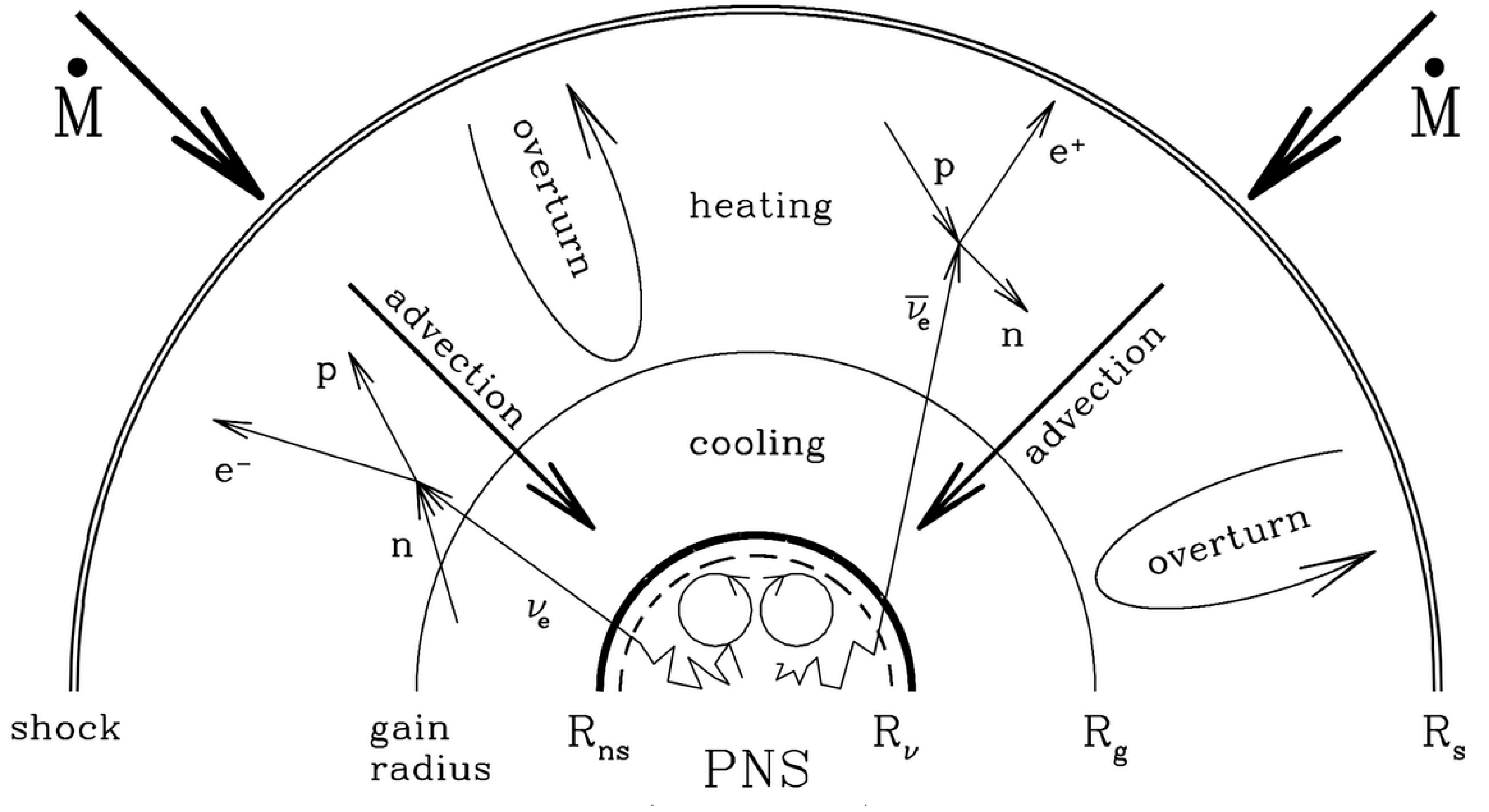

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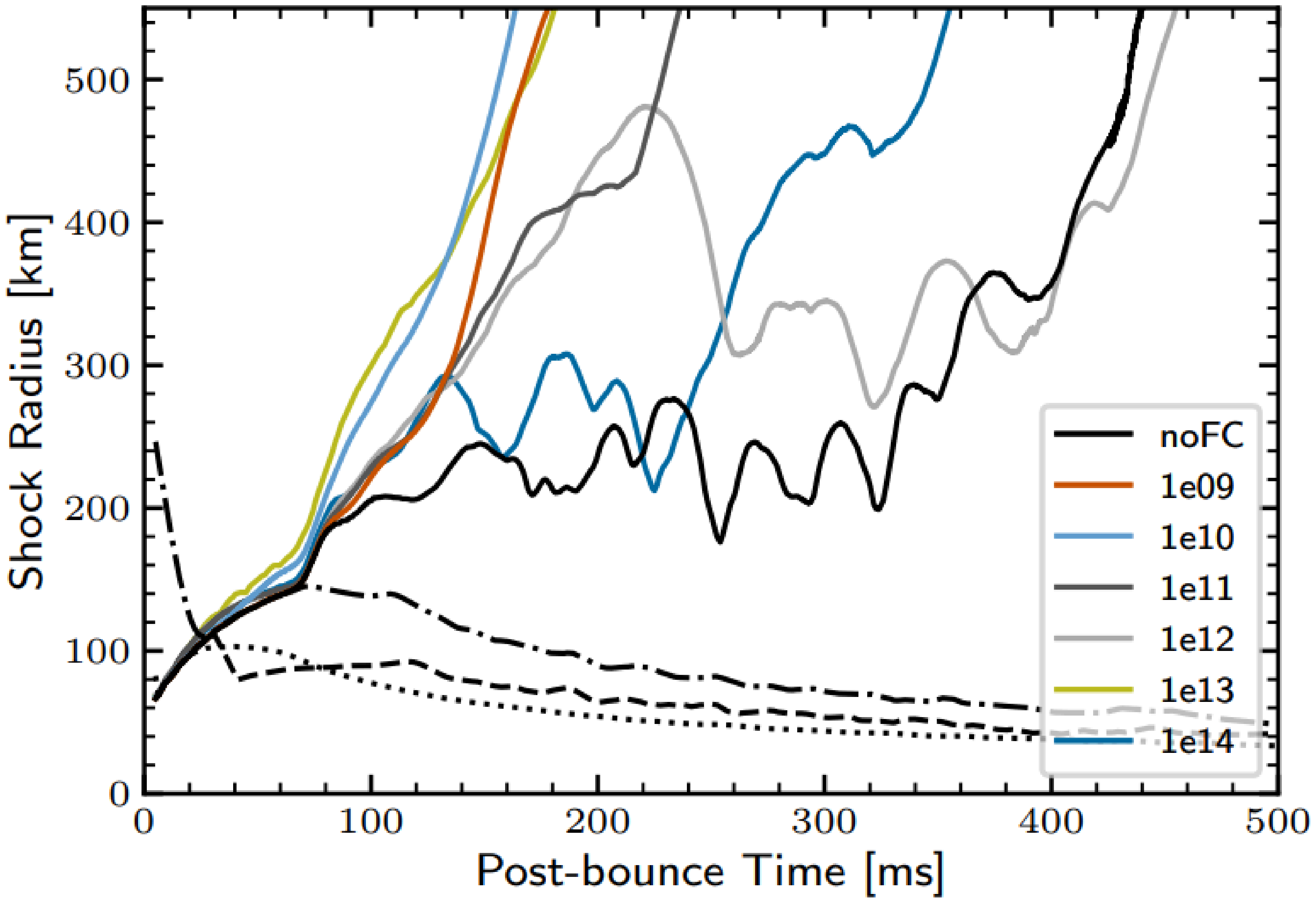

Ehring et al., Phys.Rev.Lett. 131 (2023) 6, 061401

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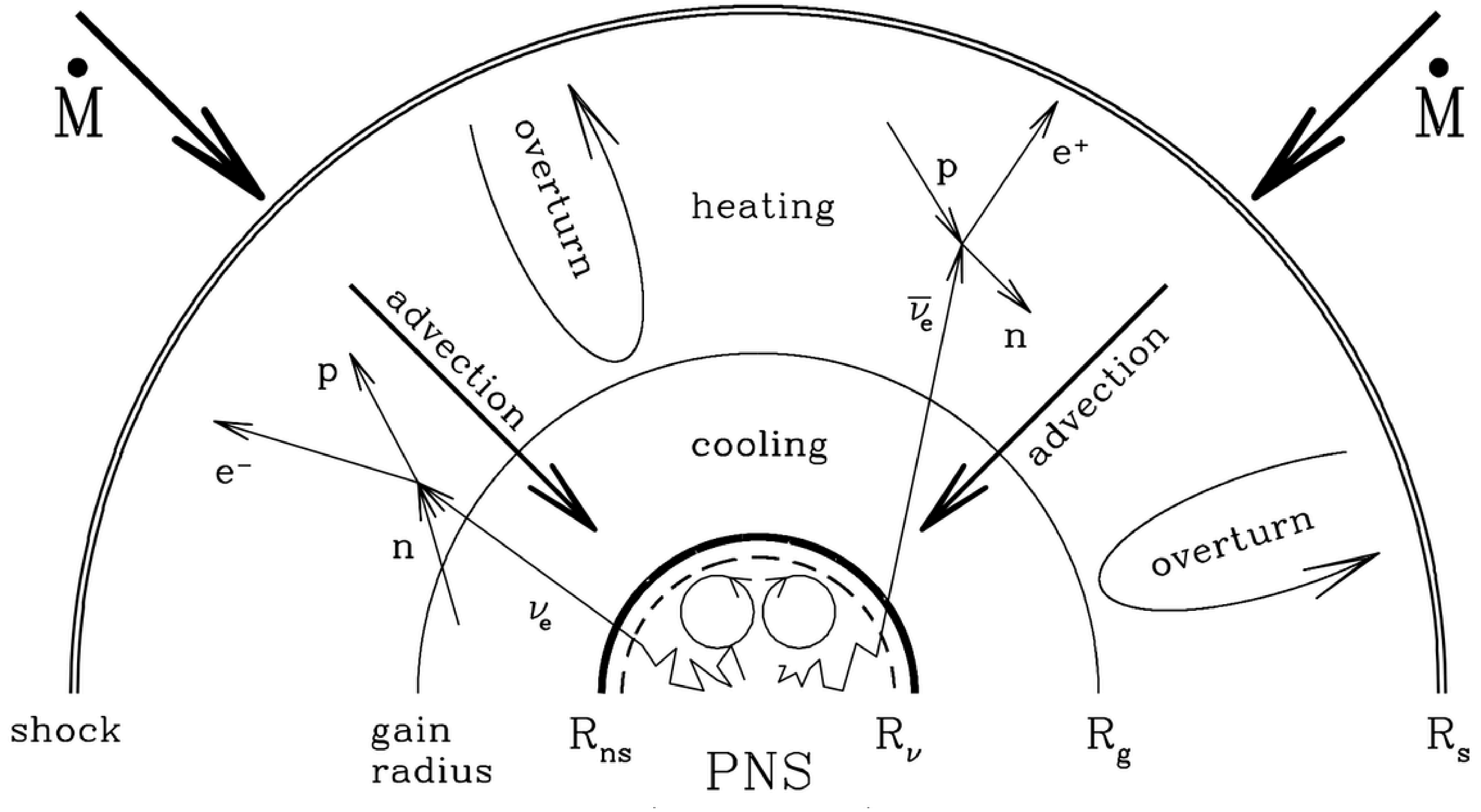

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Equation of motion

- Density matrix
  - Weak-interaction basis
  - Two-flavor for simplification
  - Flavor mixing in off-diagonal terms

$$\rho_{E,v}(x,t) = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}$$

- Translational symmetry in x, y, and axial symmetry in z are assumed

$$i(\partial_t + v_z\partial_z)\rho_{E,v_z} = [\mathrm{H}, \rho_{E,v_z}] + i\mathcal{C}$$

- Forward scattering
  - Momentum state unchanged
  - $\mathcal{O}(\mathrm{G_F})$

- Non-forward scattering
  - Momentum state changed
  - $\mathcal{O}(\mathrm{G_F^2})$

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Equation of motion

- Vacuum Hamiltonian
- Self-interaction Hamiltonian

$$\mathrm{H}_{\mathrm{vac}} \qquad \mathrm{H}_{\nu\nu}$$

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Vacuum oscillations

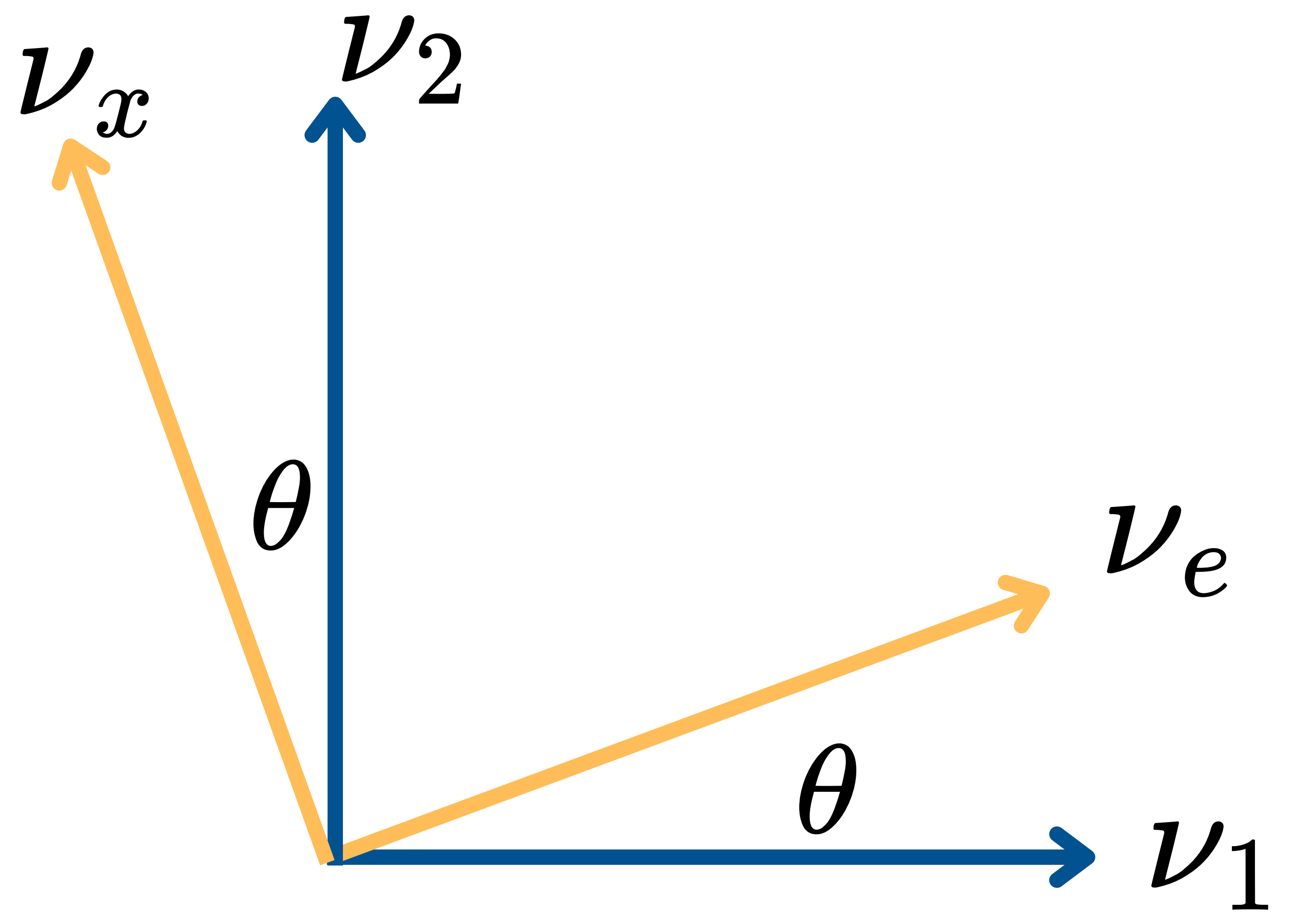
$$\mathrm{H}_{\mathrm{vac}} = \frac{\omega_E}{2}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\theta : \text{mixing angle},$$
$$\omega_E = \frac{\Delta m^2}{2E} : \text{vacuum frequency},$$
$$\Delta m^2 : \text{neutrino mass squared differences}$$

$$\omega_E > (<) 0 : \text{normal(invert) mass ordering}$$

$$P_{ee} \equiv \frac{\rho_{ee}(t)}{\rho_{ee}(t=0)} = 1 - \sin^2 2\theta \sin^2(\frac{\omega_E}{2} t)$$

- Vacuum Hamiltonian
  - misalignment between mass-eigenstate and flavor-eigenstates
- Kilometer scale



Fast conversions

- When neutrino number density is large
  - Vacuum Hamiltonian could be ignore

$$i(\partial_t + v_z\partial_z)\rho_{E,v_z} = [\mathrm{H}_{\nu\nu}(v_z), \rho_{E,v_z}]$$

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$$\mathrm{H}_{\nu\nu}(v_z) = \sqrt{2}\mathrm{G}_\mathrm{F} \int \mathrm{d}v' (\rho_{v'_z} - \bar{\rho}_{v'_z})(1 - v_z \cdot v'_z)$$

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- velocity dependent

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- Ambient (anti)neutrino
- Non-linearity

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$$\langle P_{ee}\rangle_z(t) = \frac{\int \mathrm{d}z\,\mathrm{d}v_z \rho_{ee}(z, v_z, t) g_\nu(v_z)}{\int \mathrm{d}z\,\mathrm{d}v_z \rho_{ee}(z, v_z, t=0) g_\nu(v_z)}$$

- $g_\nu(v_z)$: angular distribution function

$$\alpha \equiv \frac{n_{\bar{\nu}_e}}{n_{\nu_e}}$$

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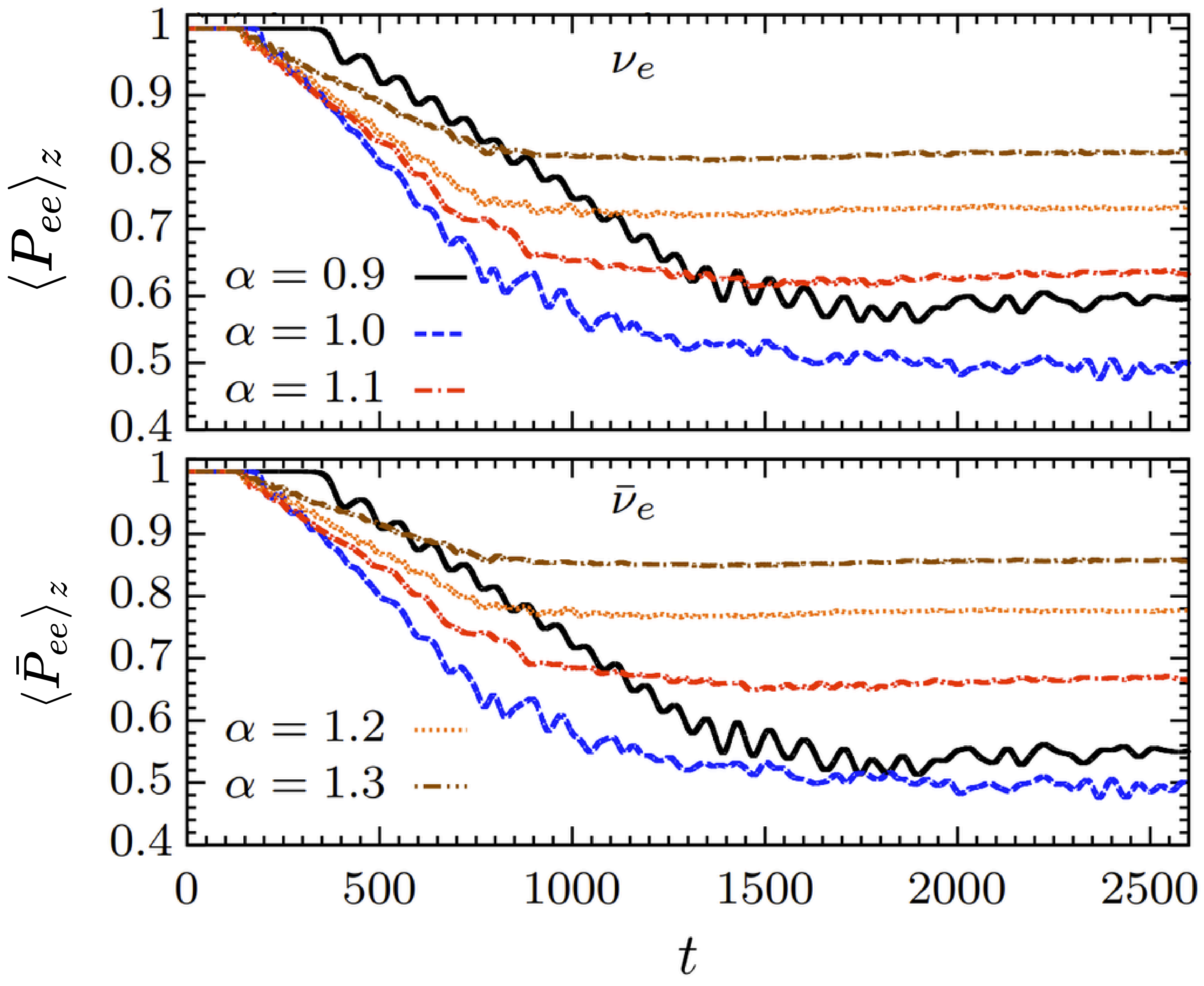
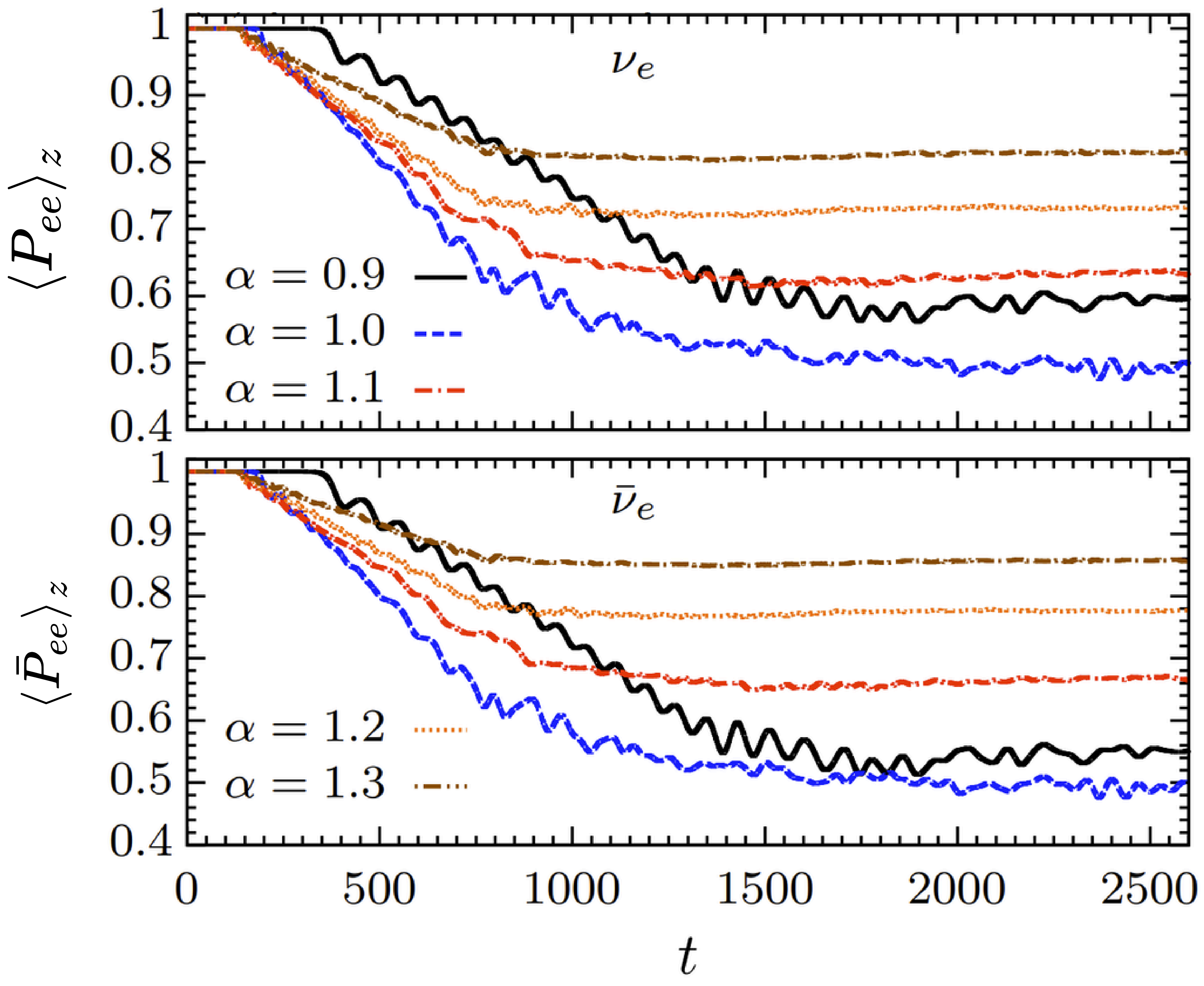


- Steady-state on coarse-grained level is reached at different α for neutrino and antineutrino

Wu et al., 0.1103/physrevd.104.103003

Does it always happen?



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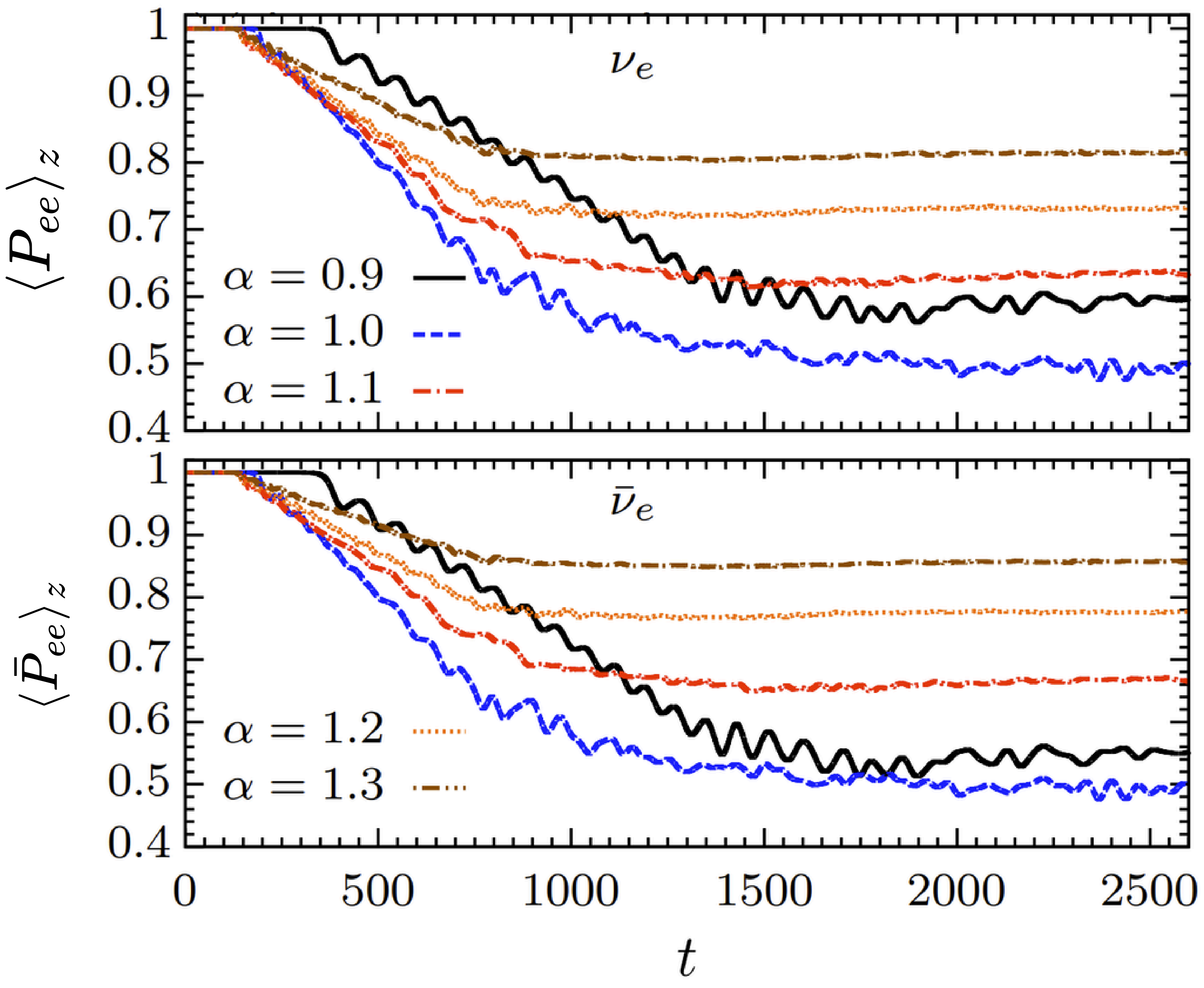

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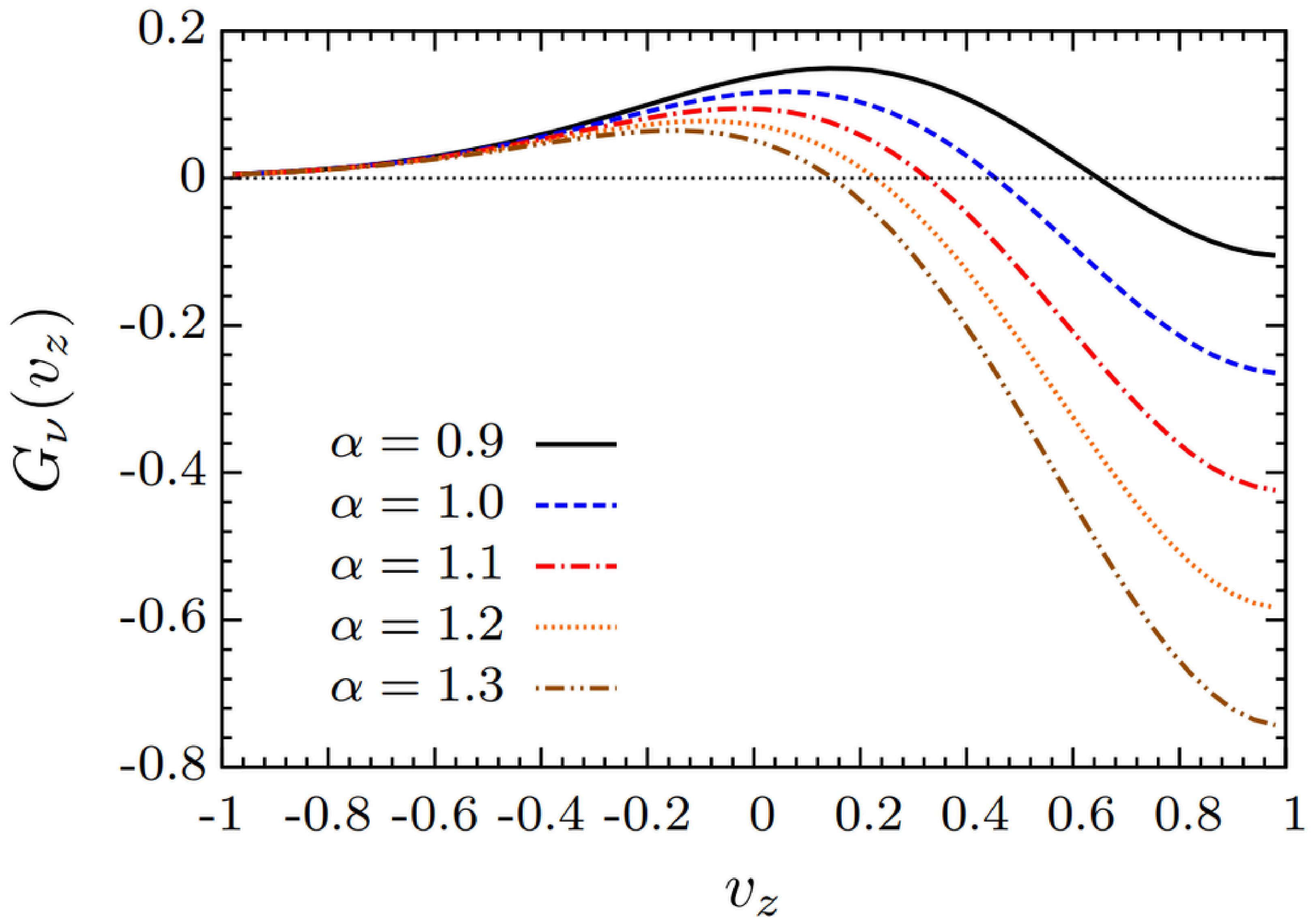
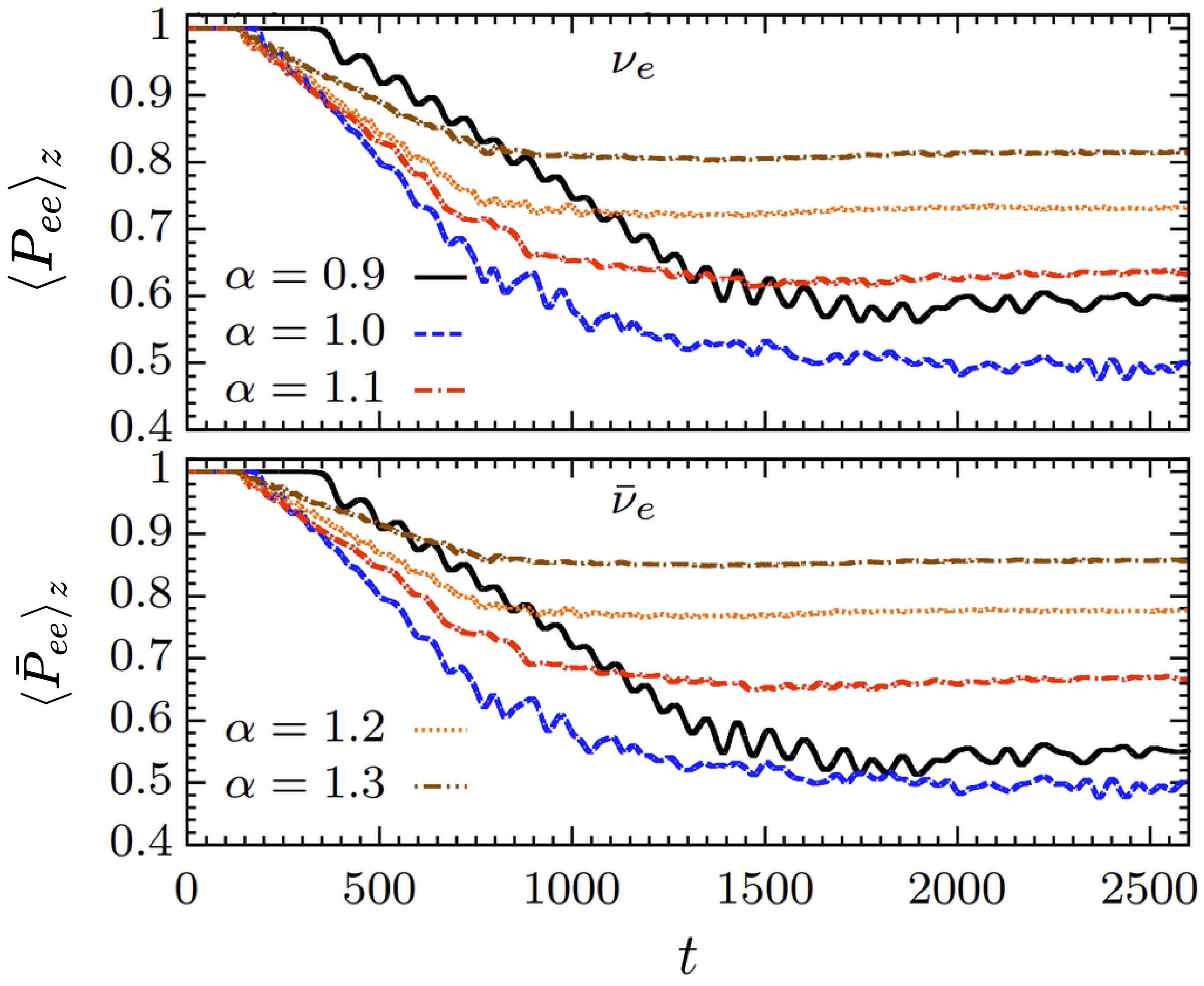
Does it always happen? NO.

- Electron lepton number crossing

  (ELN crossing)

  $$G_\nu(v_z) \equiv \rho_{ee}(v_z) - \bar{\rho}_{ee}(v_z)$$





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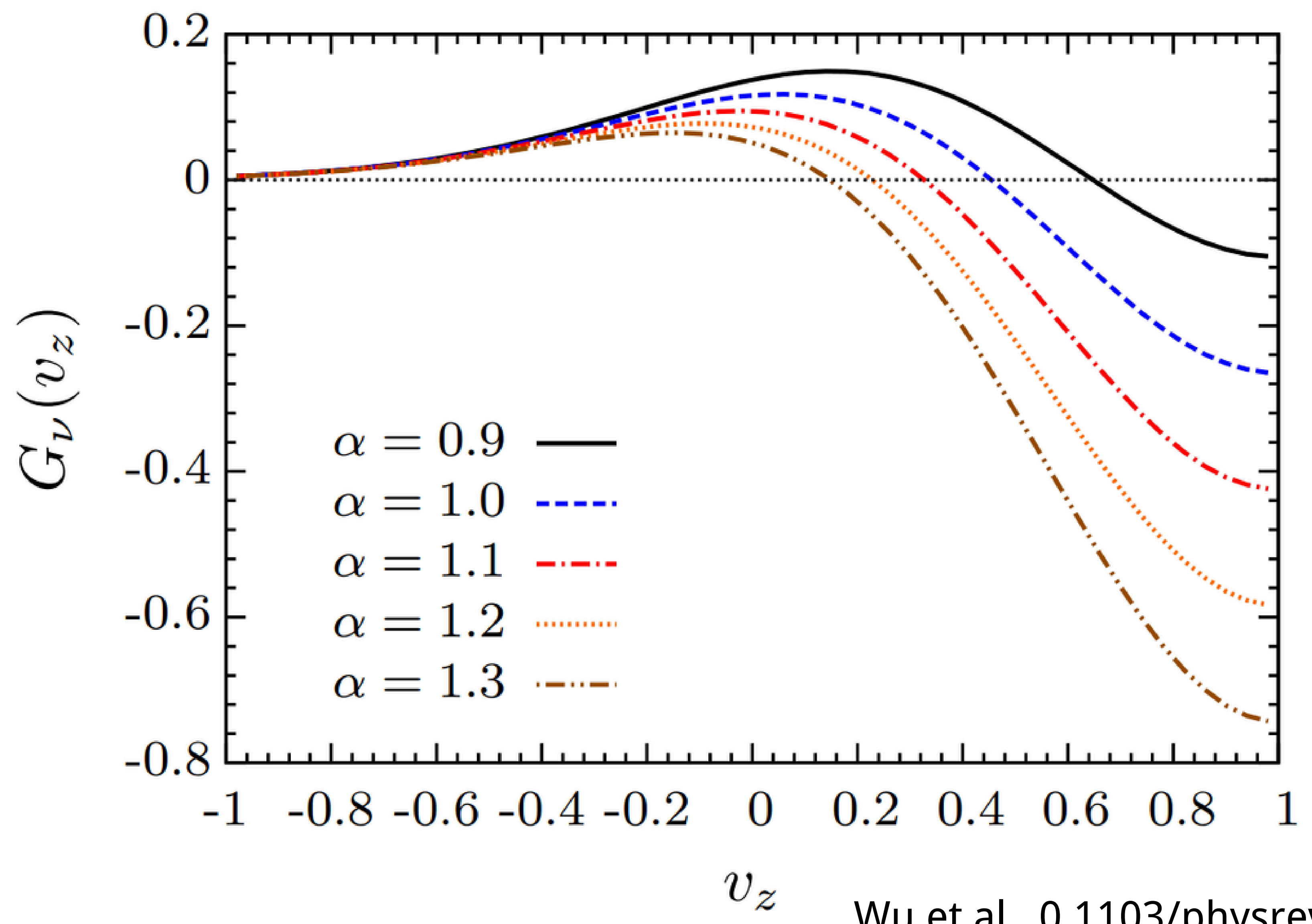
- Self-interaction Hamiltonian
  - velocity dependent
  - ambient (anti)neutrino
  - Non-linearity

- Condition to triger the conversion
  - (ELN crossing) $G_\nu(v_z) \equiv \rho_{ee}(v_z) - \bar{\rho}_{ee}(v_z)$
- Steady-state on coarse-grained level is reached



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Slow conversions $\mathrm{H} = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}$

- Generally the vacuum term could not be ignored

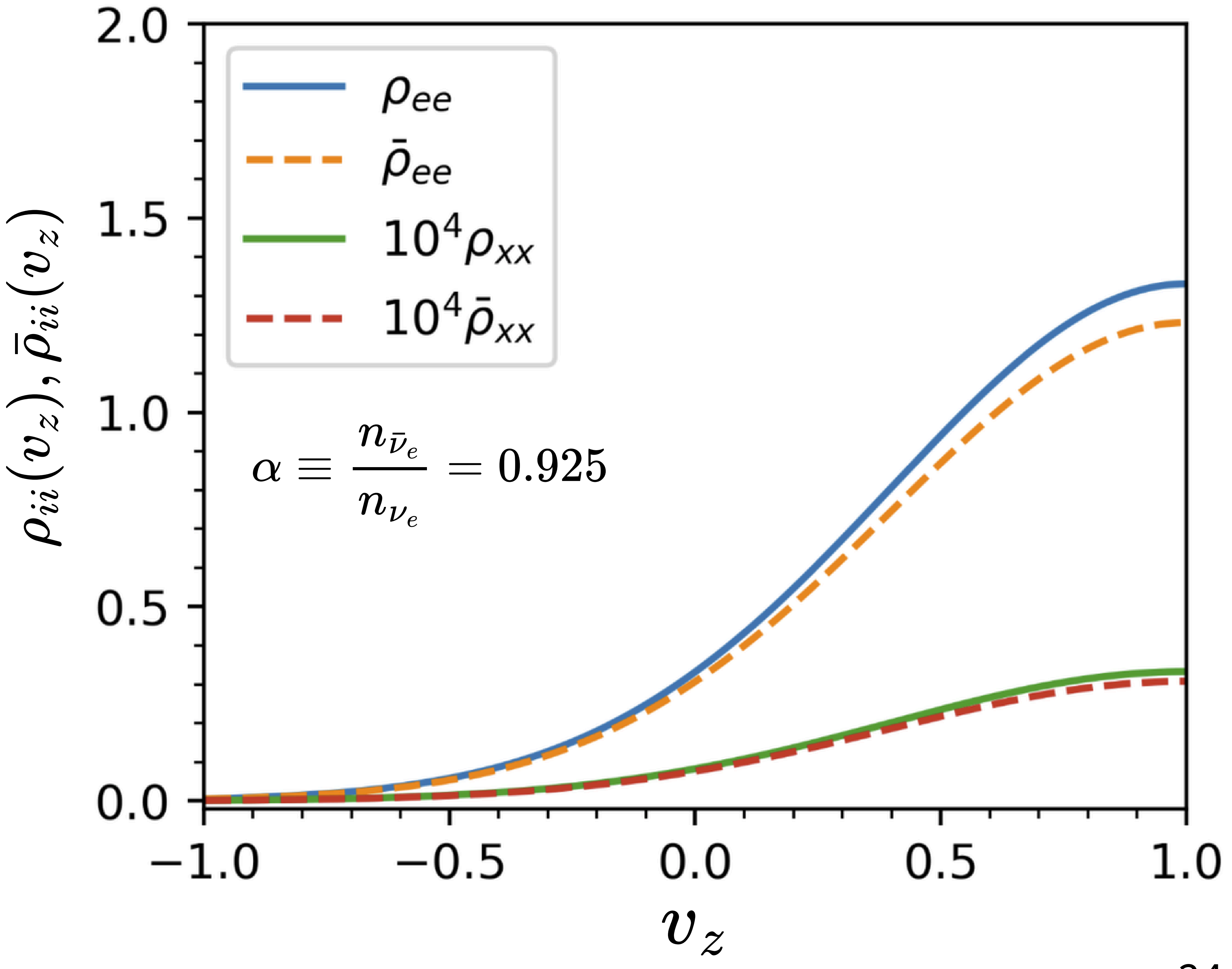
$$i(\partial_t + v_z\partial_z)\rho_{E,v_z} = [\mathrm{H}(v_z), \rho_{E,v_z}]$$

$$\mathrm{H}(v_z) = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}(v_z)$$

- ELN crossing NOT required

$$G_\nu(v_z) \equiv \rho_{ee}(v_z) - \bar{\rho}_{ee}(v_z)$$

- Half-gaussian angular spectrum
  - Forward-peaked
  - outside the $\nu -$ sphere



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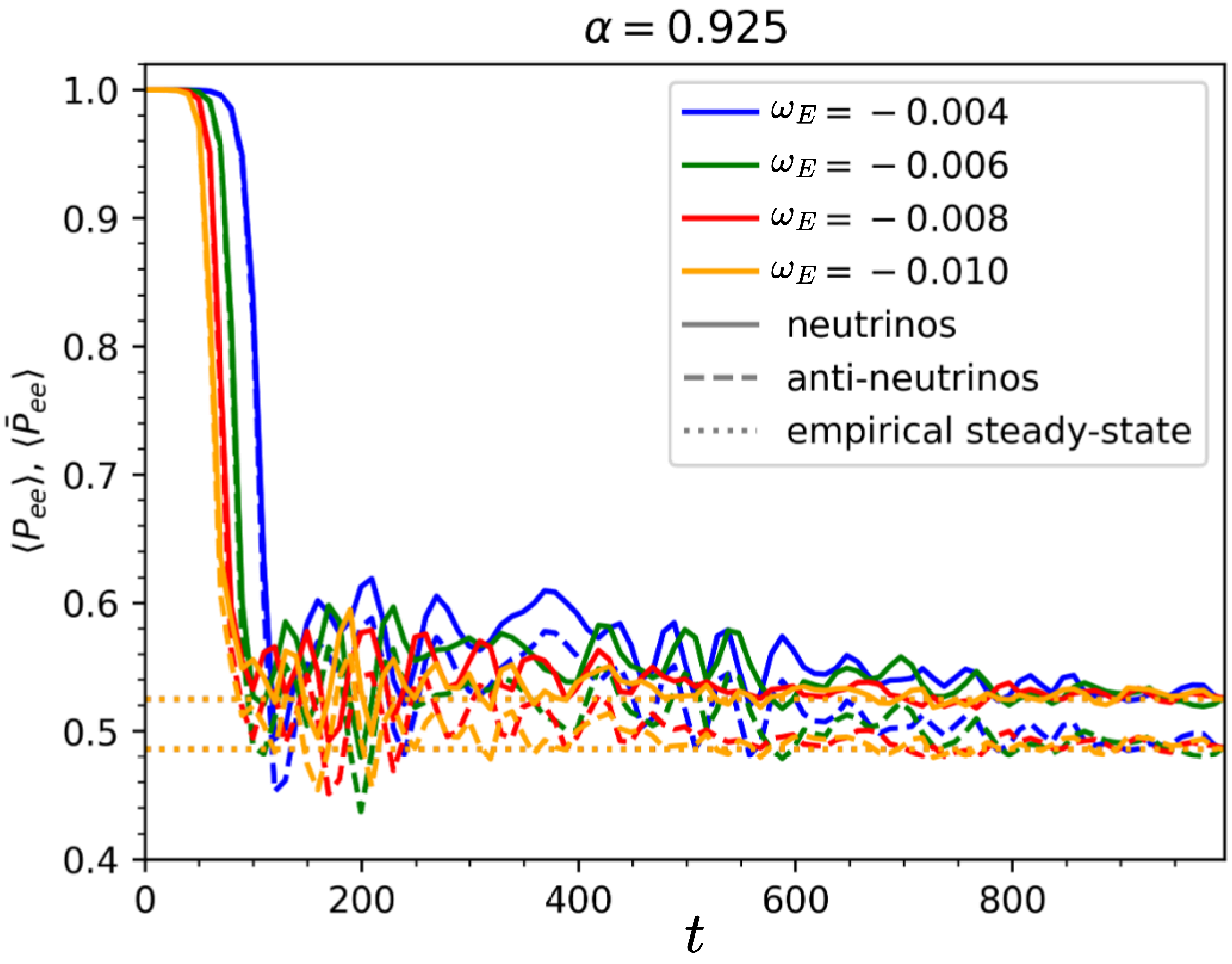
- Steady state
  - not sensitive to ω_E
  - depend on α



Slow conversions $\mathrm{H} = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}$

- Empirical steady-states is described by a formula

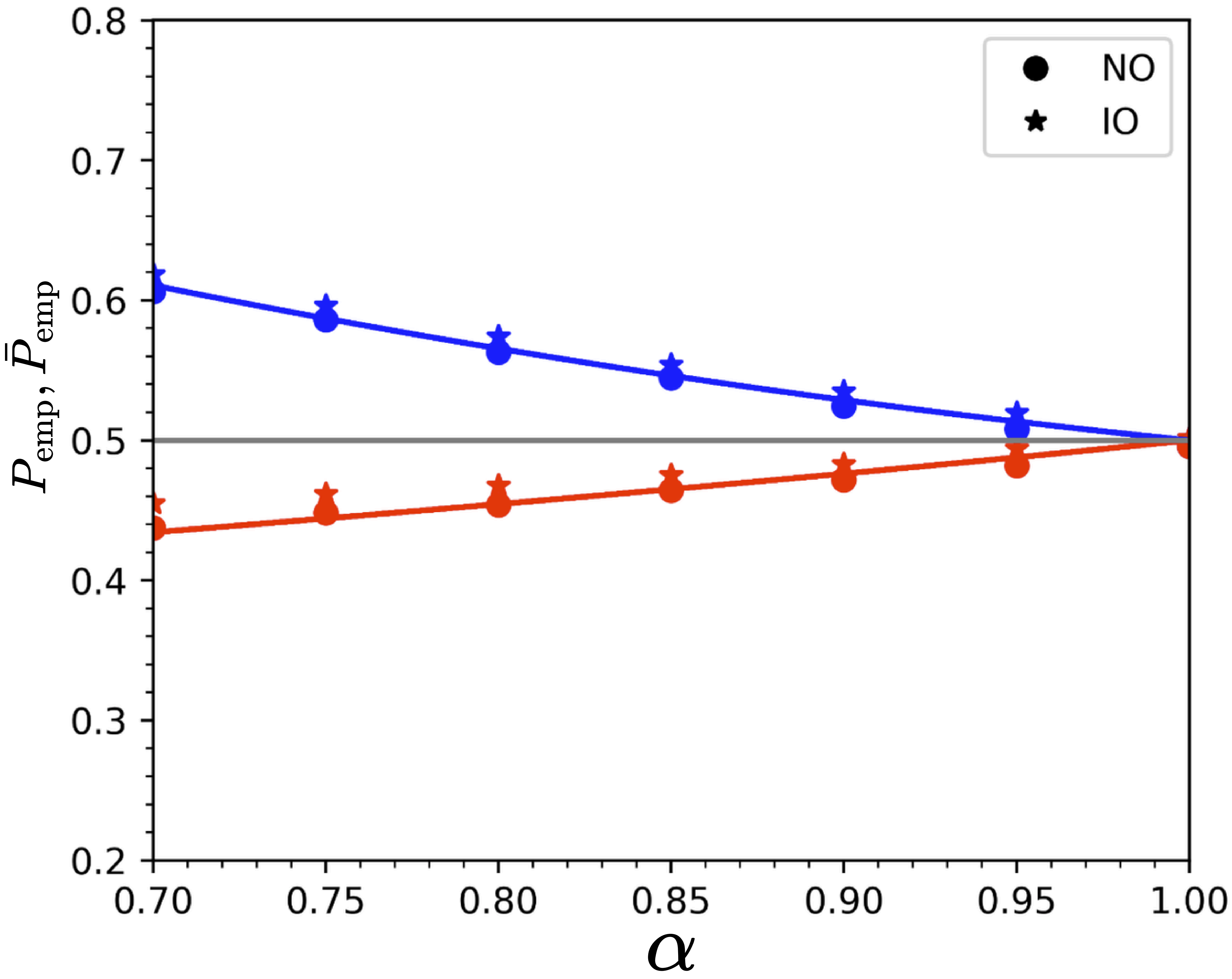
$$P_{\mathrm{emp}}(\alpha) = \frac{1+\epsilon}{2} + \frac{(1-\alpha)^2}{4}$$

$$\bar{P}_{\mathrm{emp}}(\alpha) = \frac{1-\epsilon}{2} + \frac{(1-\alpha)^2}{4}$$

with

$$\epsilon \equiv \frac{1-\alpha}{1+\alpha}$$

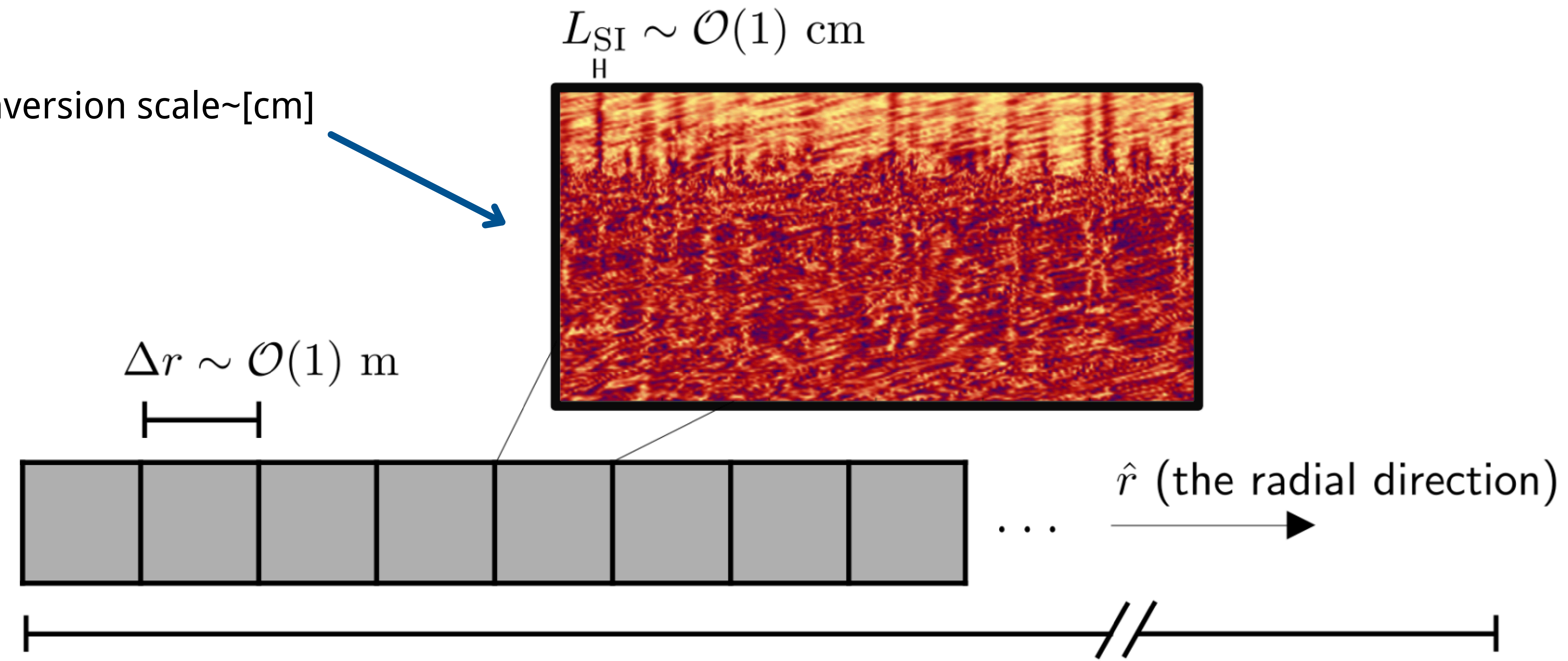
- Dependence of the mass ording contributes as much as a few percent
- Unaffected by initial conditions
- Applicable to future CCSN simulation



Scales of flavor conversions

Johns et al., arXiv:2503.05959

- Flavor conversion scale~[cm]



- Macroscopic scale~[km]
  - Density scale hight $H_\varrho = \varrho(\mathrm{d}\varrho/\mathrm{d}r)^{-1}$

Summary

- Neutrino oscillation is important to the
  - Core-Collapse Supernova dynamics
  - Nucleosynthesis in heavy elements
- Fast conversions
  - Self-interaction Hamiltonian only
  - Required electron-lepton-number crossing
- Slow conversions
  - Include both vacuum and self-interaction Hamiltonians
  - NOT required electron-lepton-number crossing
  - Empirical formula could be implemented in future CCSN simulations

References

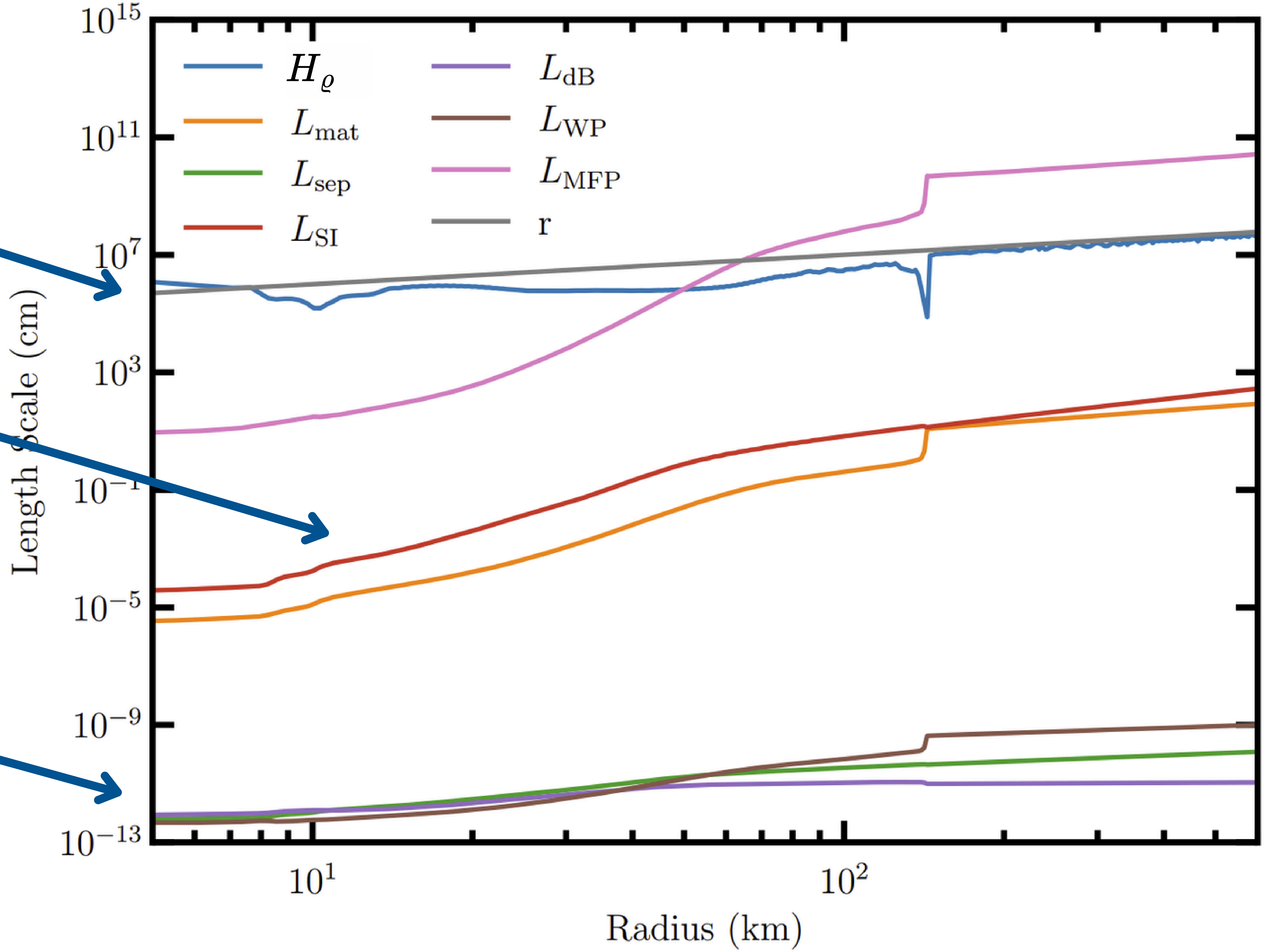
1. George et al., arXiv:2409.08833
2. Zaizen et al., arXiv:2211.09343
3. Wu et al., arXiv:2108.09886
4. Pantaleone, doi:10.1016/0370-2693(92)91887-f
5. Shalgar, https://indico.nbi.ku.dk/event/1532/contributions/11439/attachments/3550/5488/shalgar.pdf
6. George et al., arXiv:2203.12866
7. Dasgupta, arXiv:2110.00192

Scales of flavor conversions

- Macroscopic scale~[km]
  - Density scale hight $H_\varrho = \varrho(\mathrm{d}\varrho/\mathrm{d}r)^{-1}$
  - Mean free path L_{MFP}

- Mesoscopic scale~[cm]
  - Self-interaction term $L_{\mathrm{SI}} = (\sqrt{2}G_F n_{\nu_e})^{-1}$

- Microscopic scale~[pm]
  - Wave packet L_{WP}
  - Particle separation $L_{\mathrm{sep}} = n_{\nu_e}^{-1/3}$
  - de Brolie wavelength $L_{\mathrm{dB}} = 2\pi(\langle E_{\nu_e}\rangle)^{-1}$



Johns et al., arXiv:2503.05959

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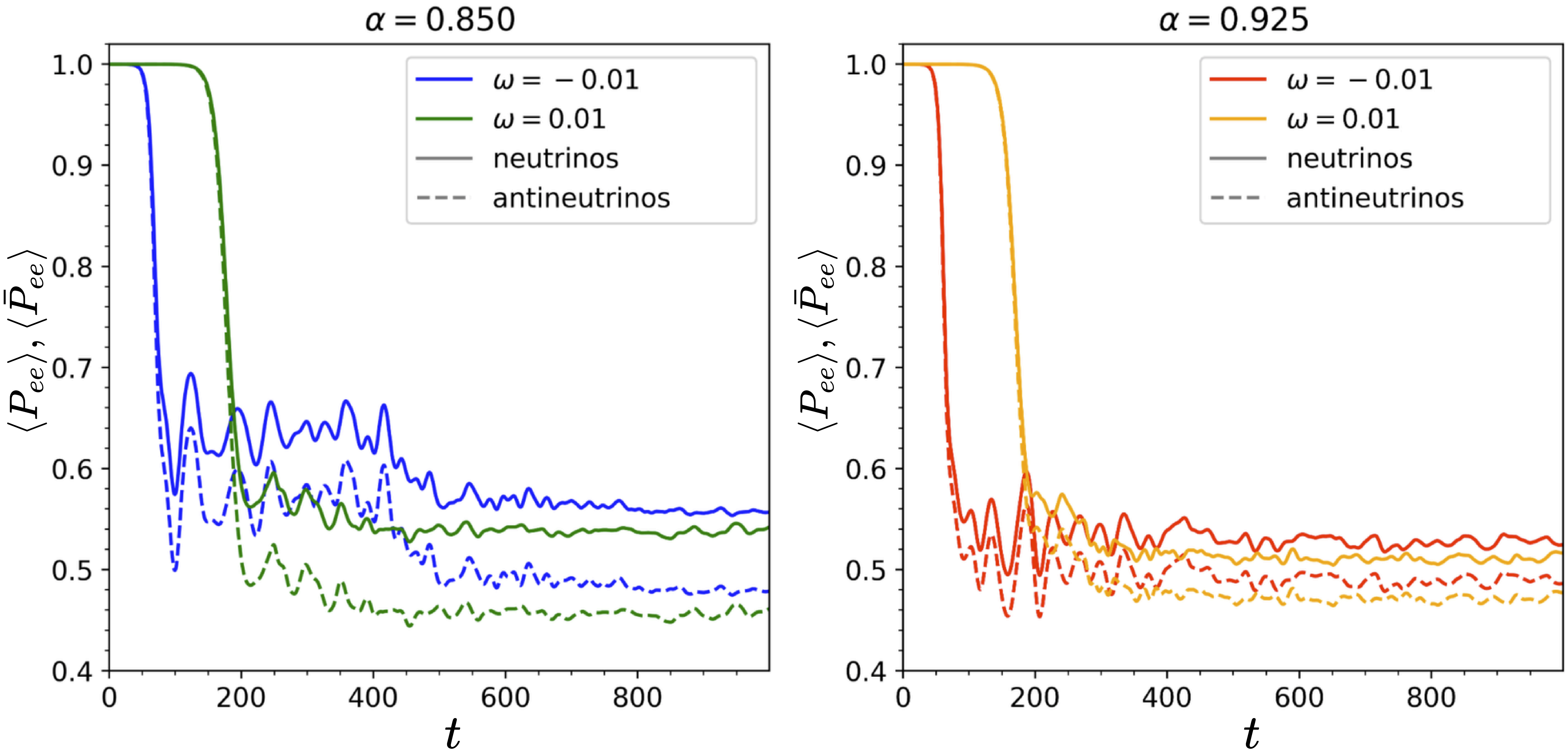
  - Density scale hight $H_\varrho = \varrho(\mathrm{d}\varrho/\mathrm{d}r)^{-1}$
  - Mean free path L_{MFP}

- Mesoscopic scale~[cm]
  - Matter term $L_{\mathrm{mat}} = (\sqrt{2}G_F n_e)^{-1}$
  - Self-interaction term $L_{\mathrm{SI}} = (\sqrt{2}G_F n_{\nu_e})^{-1}$

Slow conversions $\mathrm{H} = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}$



- Stationary states reached in different α

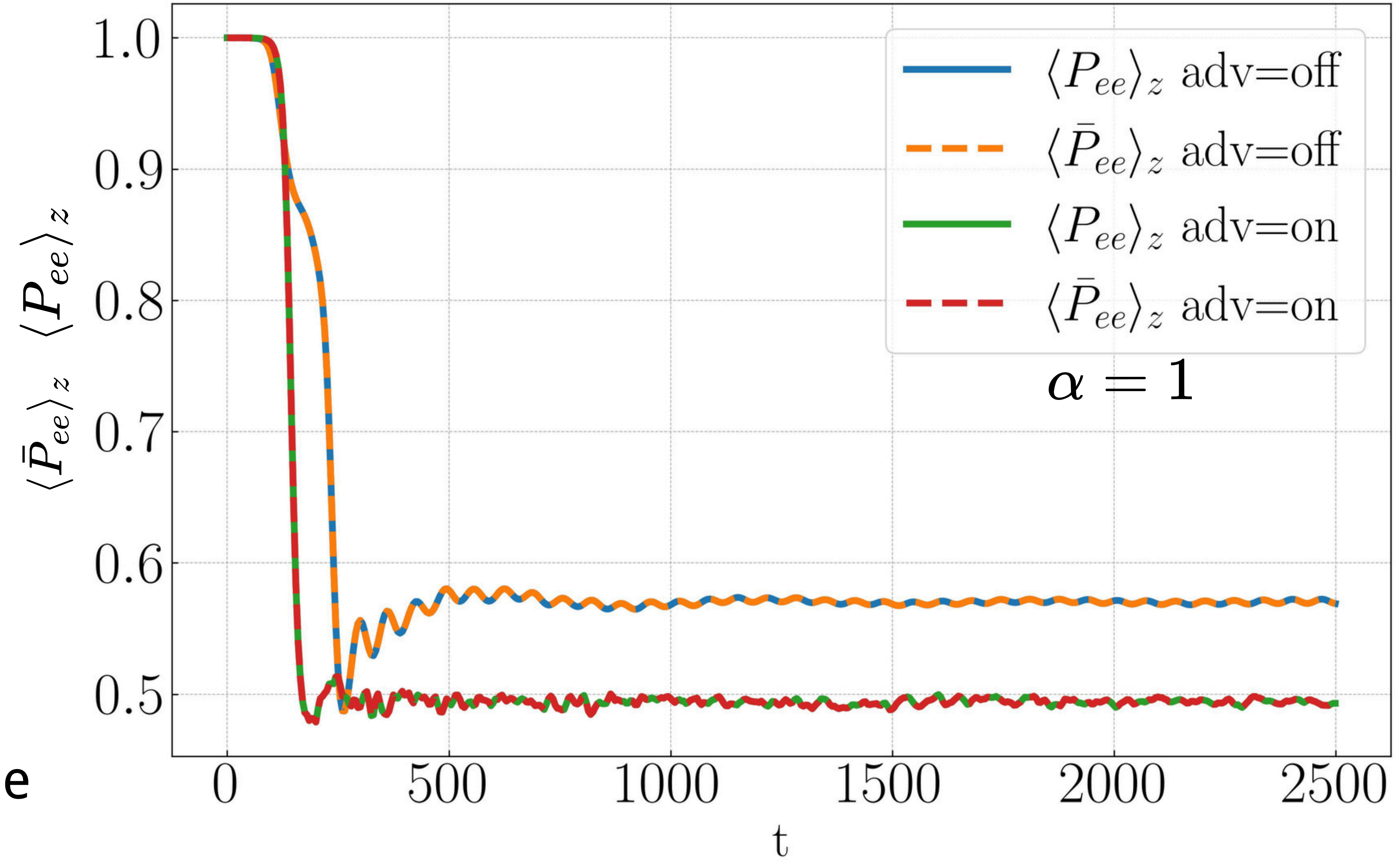
Slow conversions $\mathrm{H} = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}$

- Generally the vacuum term could not be ignored

$$i(\partial_t + v_z\partial_z)\rho_{E,v_z} = [\mathrm{H}(v_z), \rho_{E,v_z}]$$

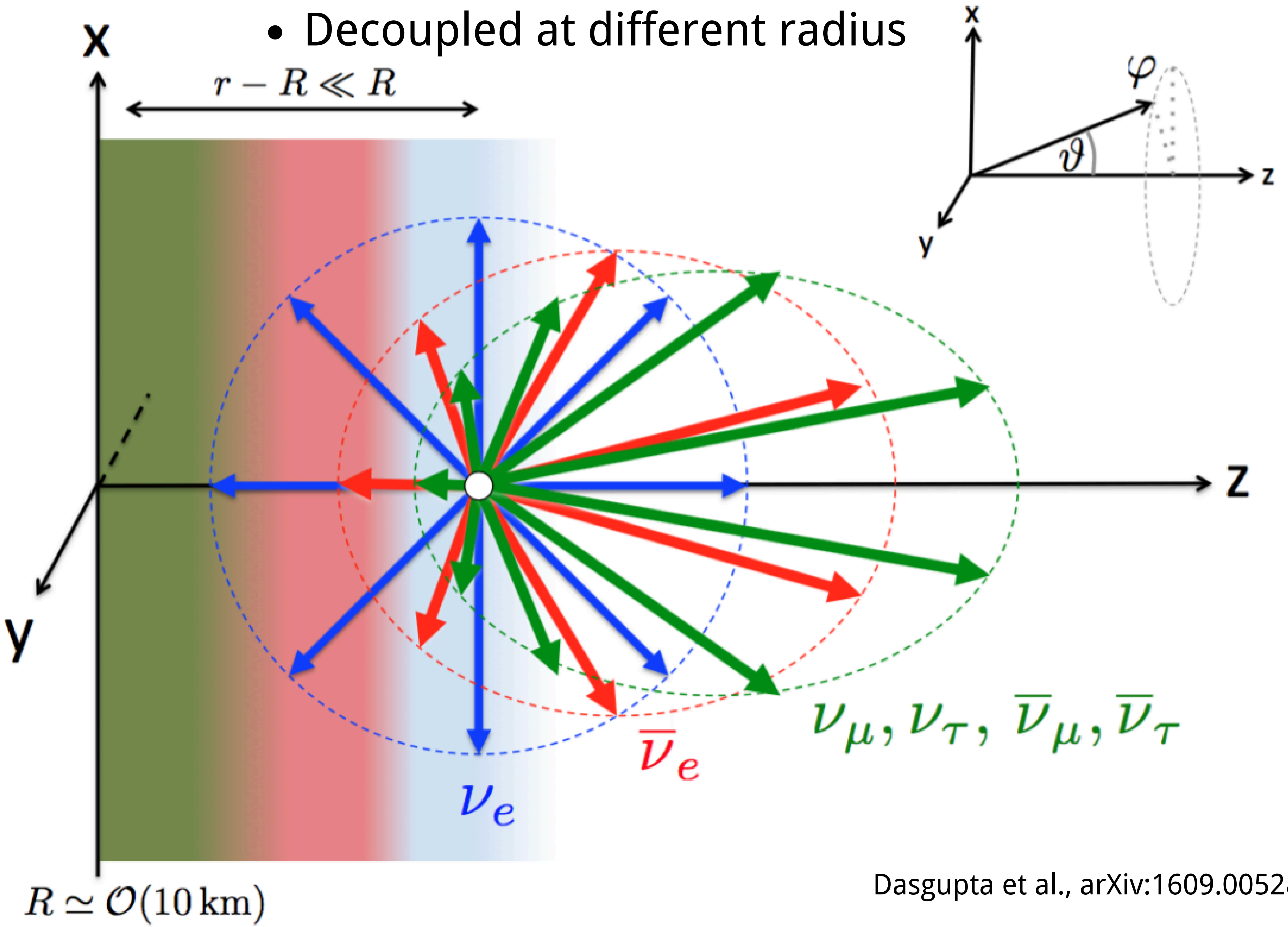
$$\mathrm{H}(v_z) = \mathrm{H}_{\mathrm{vac}} + \mathrm{H}_{\nu\nu}(v_z)$$

- Without advection term $v_z\partial_z$
  - Final state is deviate from equipartition
- Advection term $v_z\partial_z$, may change the final state to flavor equipartition on coarse-grained level

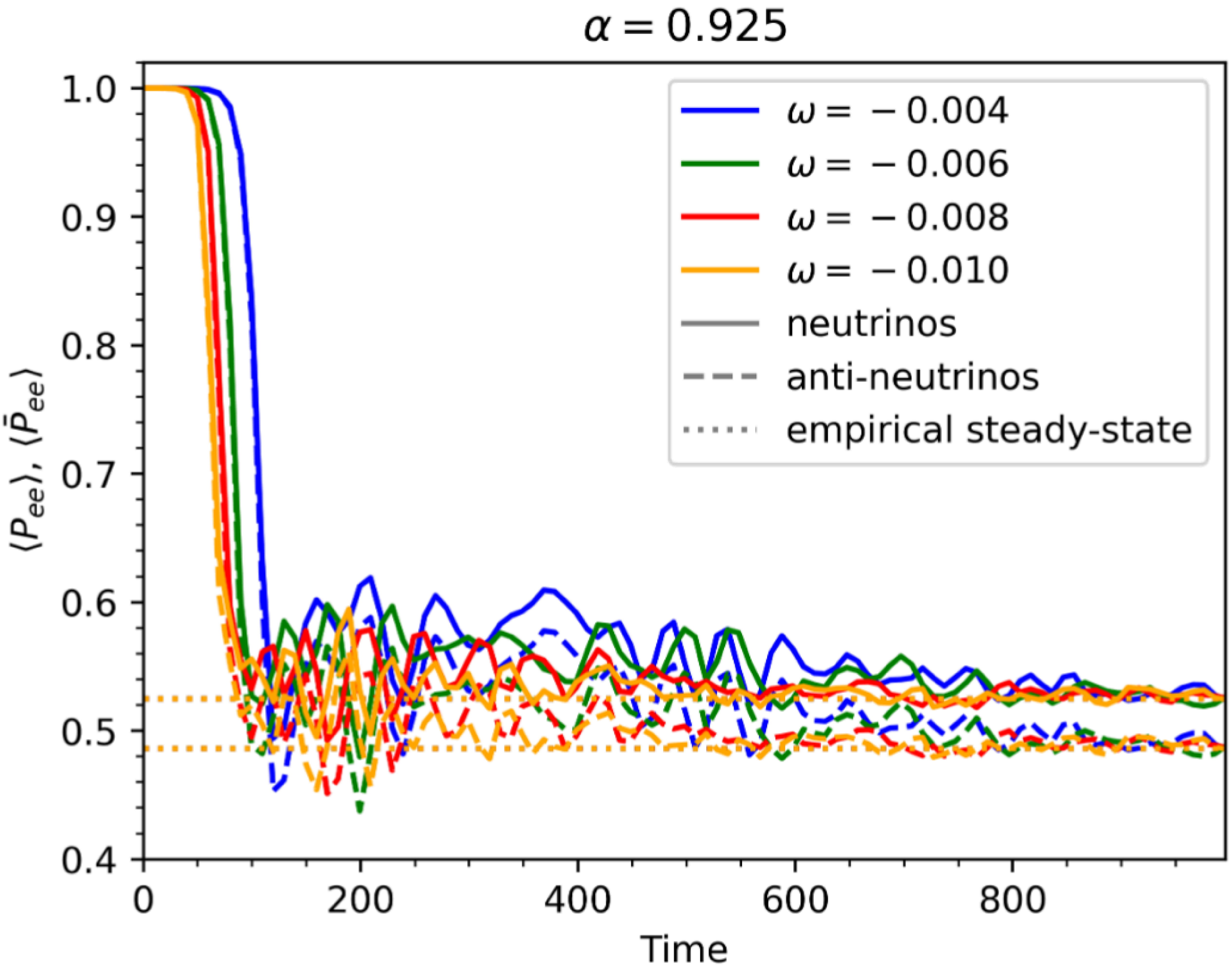


Angle distribution

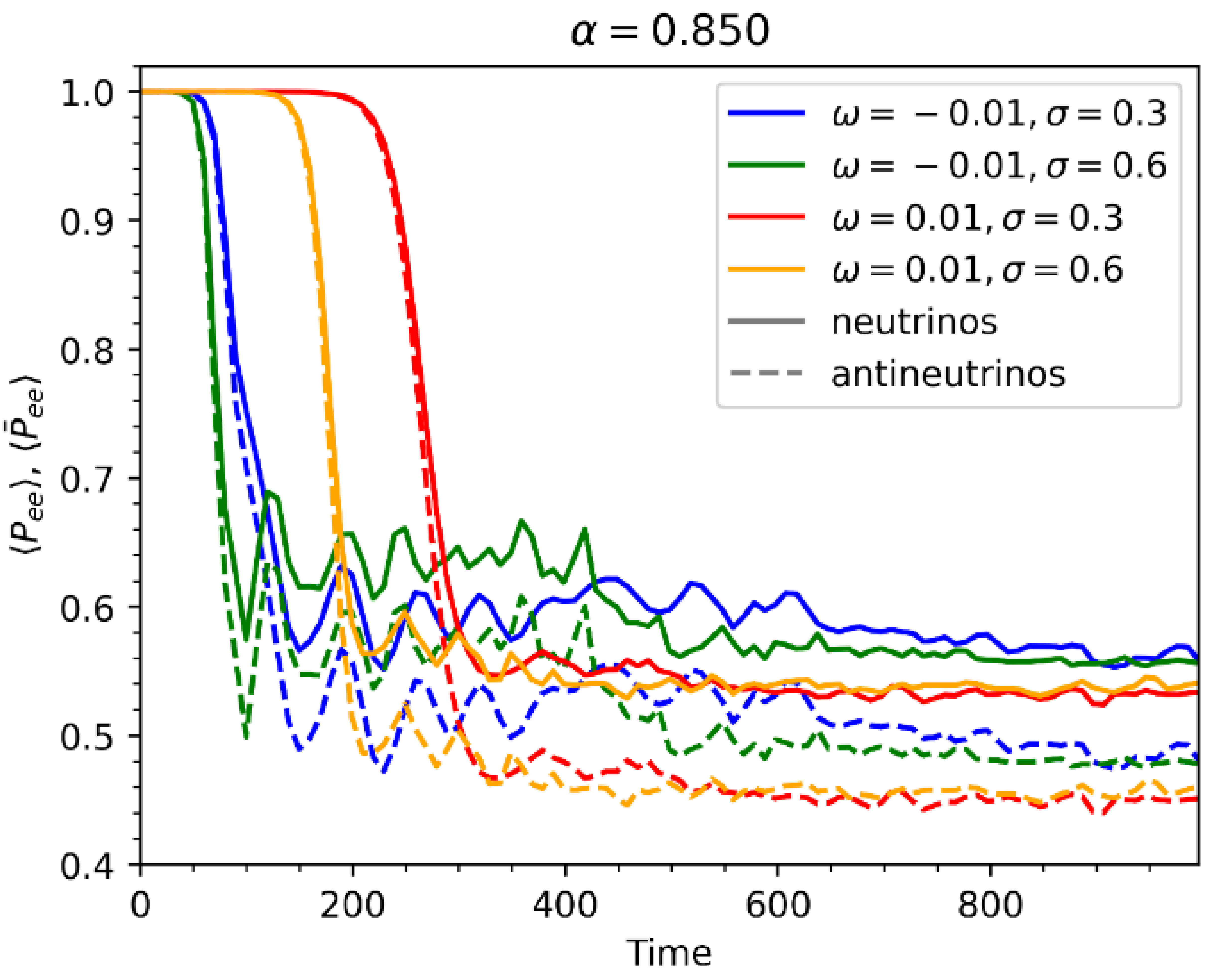
- Decoupled at different radius



Dasgupta et al., arXiv:1609.00528



$\alpha = 0.925$
$\omega = -0.004$
$\omega = -0.006$
$\omega = -0.008$
$\omega = -0.010$
neutrinos
anti-neutrinos
empirical steady-state
$\langle P_{ee} \rangle$, $\langle \bar{P}_{ee} \rangle$
Time
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0
200
400
600
800



$\alpha = 0.850$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
$\langle P_{ee} \rangle$, $\langle \bar{P}_{ee} \rangle$
0
200
400
600
800
Time
$\omega = -0.01, \sigma = 0.3$
$\omega = -0.01, \sigma = 0.6$
$\omega = 0.01, \sigma = 0.3$
$\omega = 0.01, \sigma = 0.6$
neutrinos
antineutrinos