

The background is a complex, abstract geometric pattern composed of numerous triangles of various sizes and colors. The colors include shades of yellow, orange, red, pink, purple, blue, and green, creating a vibrant, low-poly aesthetic. The triangles are arranged in a way that creates a sense of depth and movement, with some areas appearing more prominent than others.

DIFFERENTIAL FORMS

A Geometric Language from
Calculus to Electromagnetism

WHAT ARE FORMS?



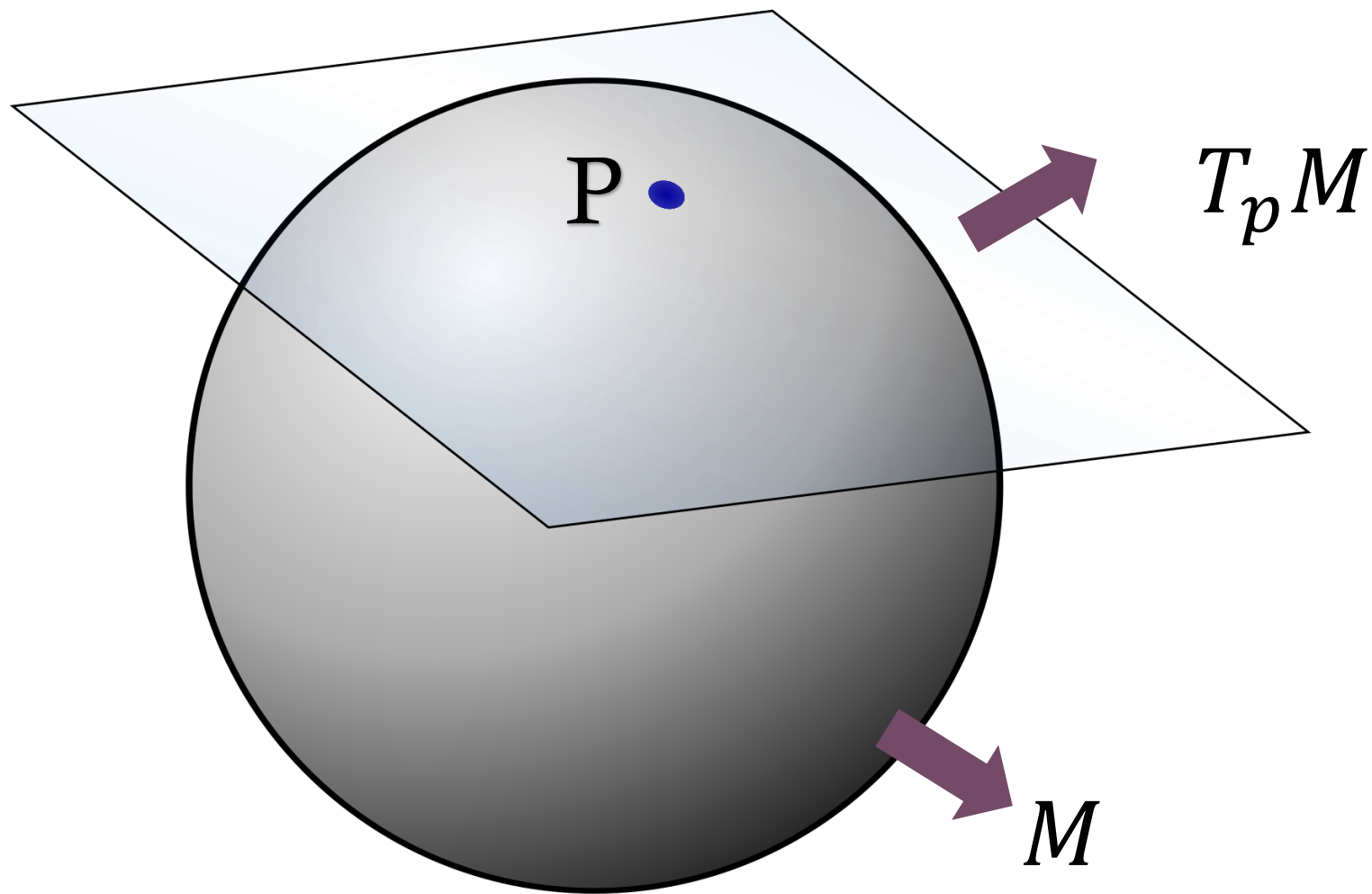
DEFINITION OF FORMS



$$\omega \in V^*$$



$$\omega \in T_p^*M$$

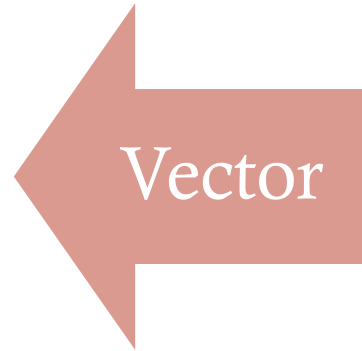


WHY USING FORMS?

This is definitely more important!

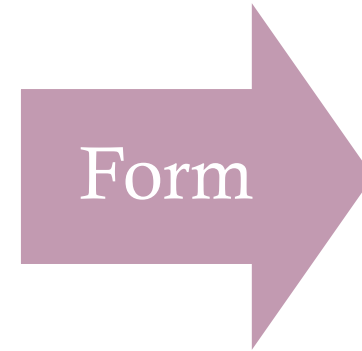


WHAT'S THE DIFFERENCE?



Describe the directions

Ex. Arrows



Describe how quantities
change along those directions

Ex. Contour Lines

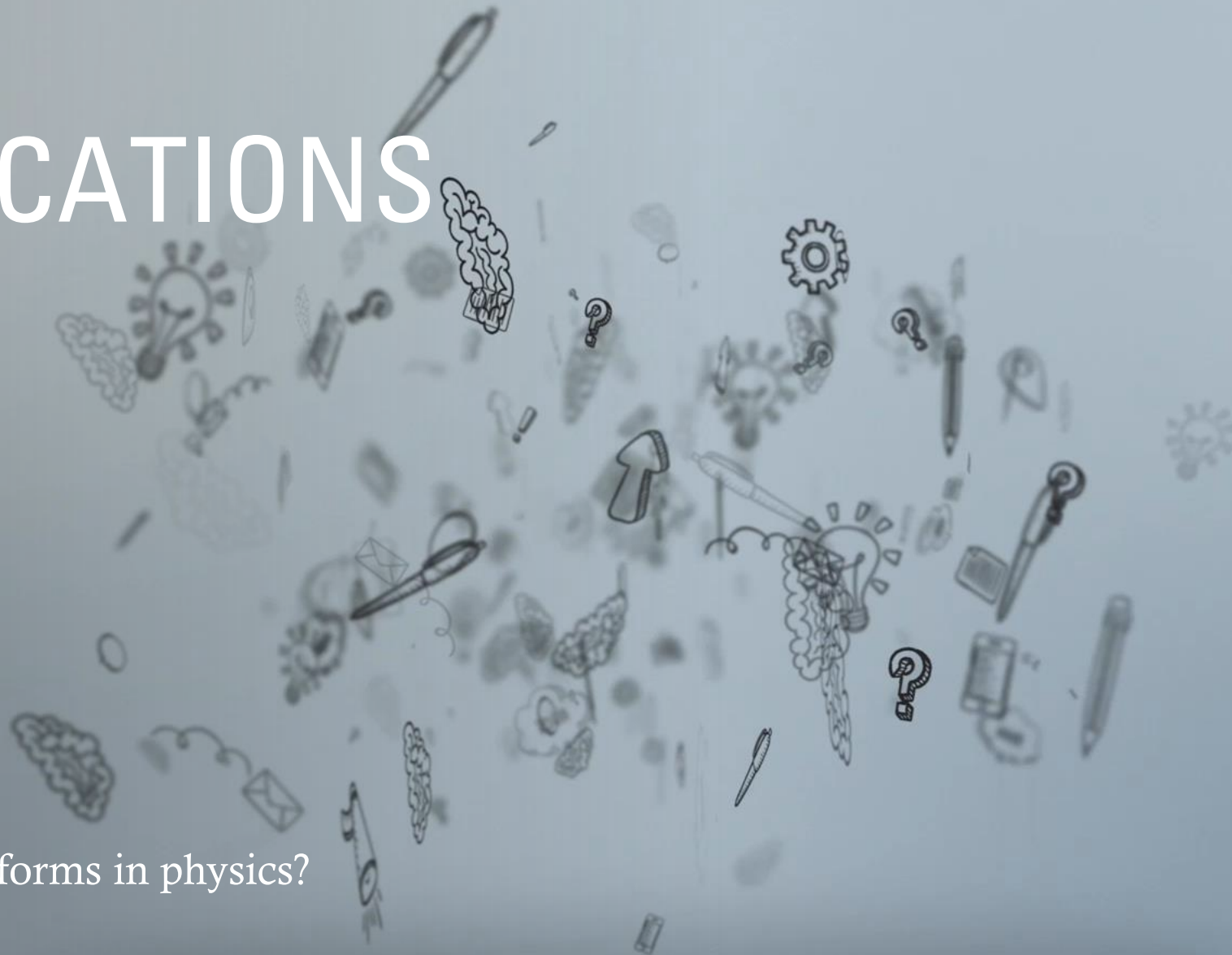
PHYSICS IS A
SCIENCE THAT
RELATES TO
CHANGES.

The image shows a chalkboard with handwritten mathematical derivations. At the top left, a graph shows a curve $y = g(x)$ with a secant line and a tangent line. The secant line is labeled "Secant Lines" and the tangent line is labeled "Tangent Line". The x-axis is labeled $x+h$. The derivative is defined as $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. The derivation for $f(x) = x^2$ is shown as follows:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} h(2x + h) \end{aligned}$$

APPLICATIONS

How do we use forms in physics?



FIRST LAW OF THERMODYNAMICS

$$dU = \delta Q - \delta W$$



State-dependent



Path-dependent

FIRST LAW OF THERMODYNAMICS

δQ

They are something that eat a path (described by vectors) and return energy (scalar).

δW

They are forms!

ELECTROMAGNETIC TENSOR


$$F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{0i} = -E_i$$

$$F_{ij} = \epsilon_{ijk} B_k$$

ELECTROMAGNETIC TENSOR

Let's do the integral.

$$\begin{aligned}\int_{\Sigma} F &= \int_{\Sigma} E_i dt \wedge dx^i + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy \\ &= \int_{t_1}^{t_2} dt \oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} + \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0\end{aligned}$$


A scalar

The result 0 comes from the Stokes' theorem, which states that $\int_{\Sigma} F = \int_{\partial\Sigma} dF$, and the fact that $dF = 0$.

ELECTROMAGNETIC TENSOR

$$\int_{t_1}^{t_2} dt \oint_{\partial \Sigma} \mathbf{E} \cdot d\mathbf{l} + \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0$$

Σ is a close surface
(Gauss's Law for magnetism)

$$\oiint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0$$

Σ is a open surface
(Faraday's Law)

$$\oint_{\partial \Sigma} \mathbf{E} \cdot d\mathbf{l} = \frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A}$$



Q&A



Q&A

1. What is d ?
 2. What is the wedge product?
 3. Why does $dF = 0$?
 4. Where are the other two missing equations?
(Maxwell's equations)
 5. What can we do next?
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