DIFFERENTIAL FORMS

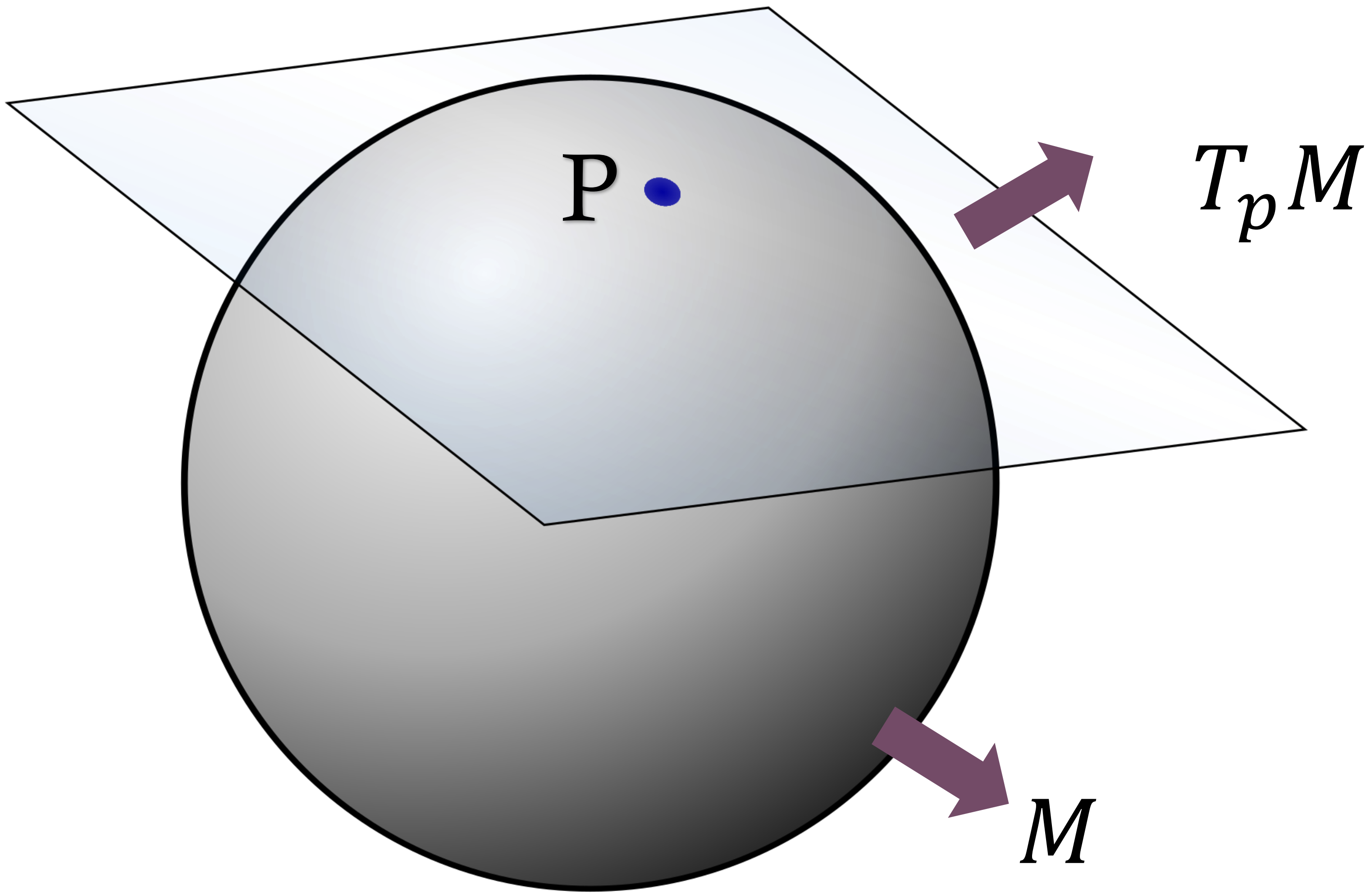
A Geometric Language from Calculus to Electromagnetism

WHAT ARE FORMS?

DEFINITION OF FORMS

$\omega \in V^*$

$\omega \in T_p^* M$



https://en.wikipedia.org/wiki/Tangent_space

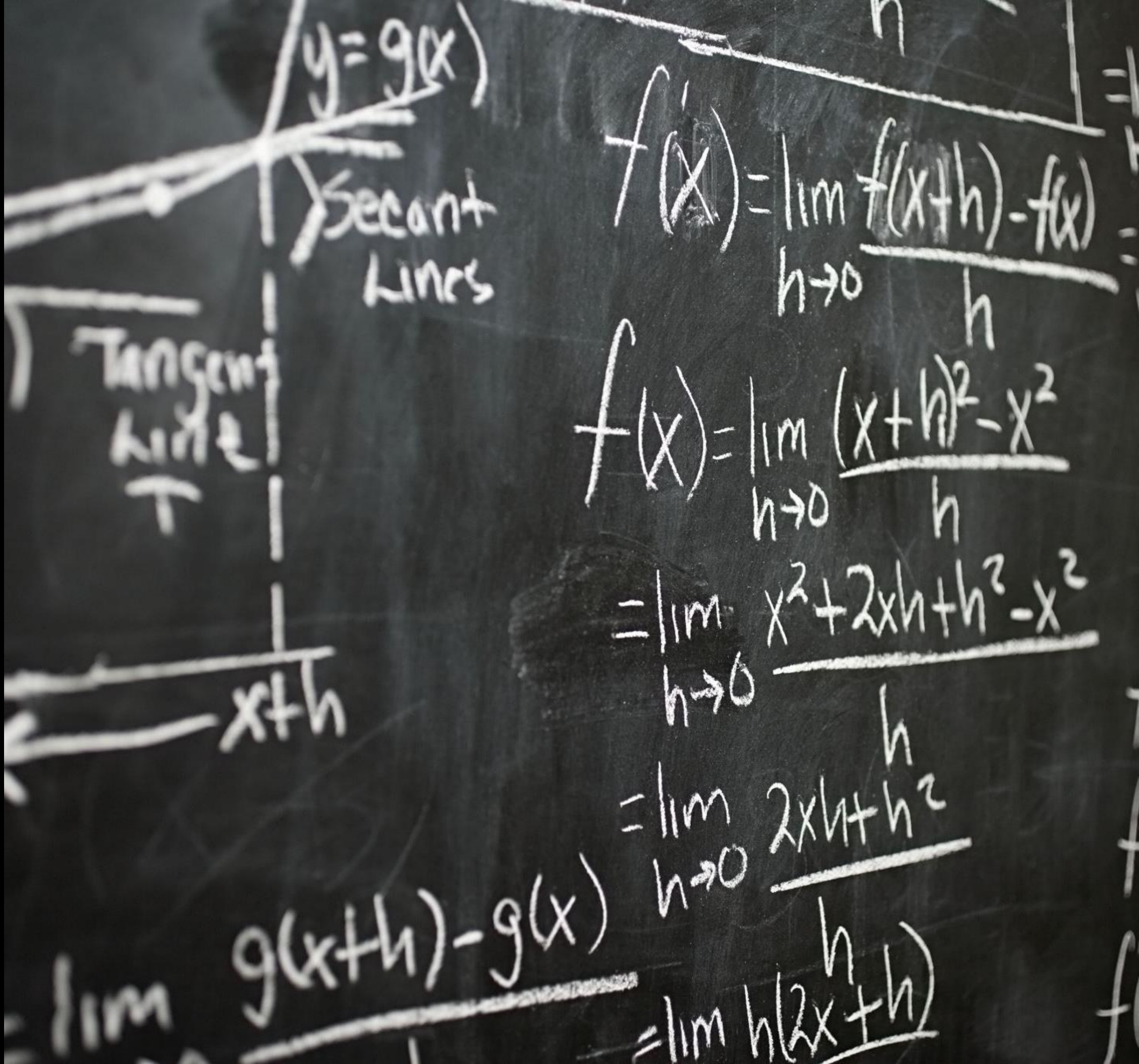
WHY USING FORMS?

This is definitely more important!

WHAT'S THE DIFFERENCE?

Vector	Form
Describe the directions	Describe how quantities change along those directions
Ex. Arrows	Ex. Contour Lines

PHYSICS IS A SCIENCE THAT RELATES TO CHANGES.



APPLICATIONS

How do we use forms in physics?

FIRST LAW OF THERMODYNAMICS

$$dU = \delta Q - \delta W$$

State-dependent

Path-dependent

FIRST LAW OF THERMODYNAMICS

$$\delta Q$$

$$\delta W$$

They are something that eat a path (described by vectors) and return energy (scalar).

They are forms!

ELECTROMAGNETIC TENSOR

$$F = \frac{1}{2} F_{\mu\nu} dx^{\mu} \wedge dx^{\nu},$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$F_{0i} = -E_i$$
$$F_{ij} = \epsilon_{ijk} B_k$$

ELECTROMAGNETIC TENSOR

Let's do the integral.

$$\int_{\Sigma} F = \int_{\Sigma} E_i dt \wedge dx^i + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy$$

$$= \int_{t_1}^{t_2} dt \oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} + \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0$$

A scalar

The result 0 comes from the Stokes' theorem, which states that $\int_{\Sigma} F = \int_{\partial\Sigma} dF$, and the fact that $dF = 0$.

ELECTROMAGNETIC TENSOR

$$\int_{t_1}^{t_2} dt \oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} + \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0$$

Σ is a close surface
(Gauss's Law for magnetism)

$$\oiint_{\Sigma} \mathbf{B} \cdot d\mathbf{A} = 0$$

Σ is a open surface
(Faraday's Law)

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} = \frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{A}$$

Q&A

Q&A

1. What is d?

2. What is the wedge product?

3. Why does $dF = 0$?

4. Where are the other two missing equations? (Maxwell's equations)

5. What can we do next?