DIRAC SPINOR

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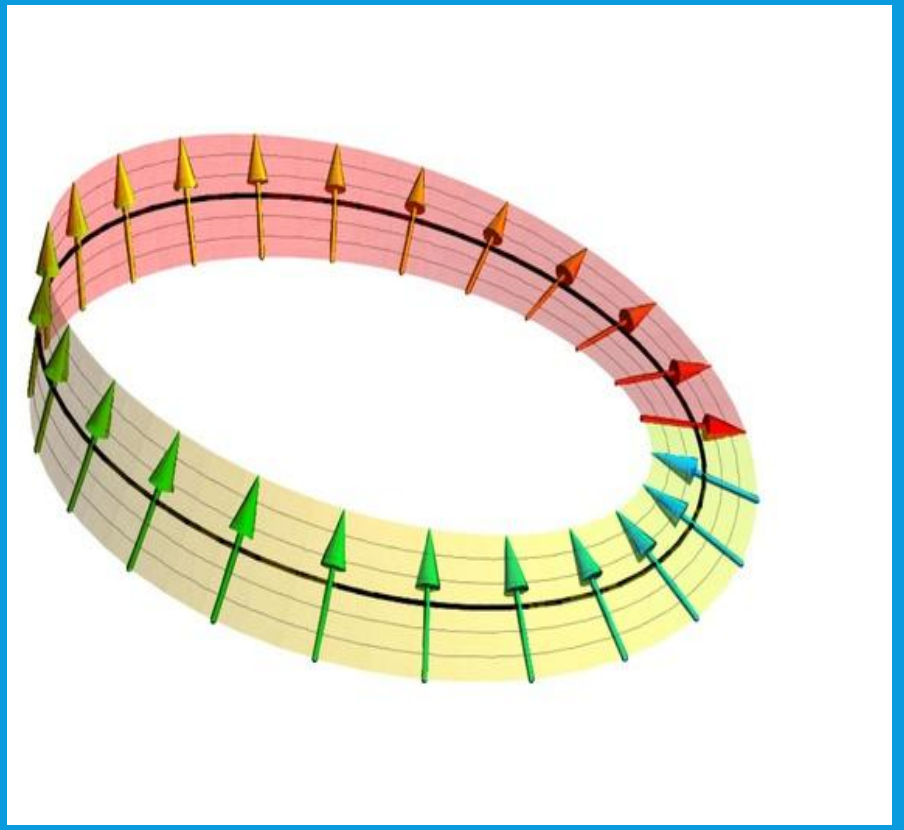
WHAT IS A SPINOR ?

- A mathematical tool usually used in quantum mechanics to describe a particle with spin
- Main property: It change its sign after a full rotation (see the figure)
- Sometimes it calls: the square root of a vector (pauli vector and spinor)



SPINOR VS VECTOR

	spinor	vector
Main fields	Quantum mechanics	Classical mechanics
Rotate 360°	Change the sign of state	Returns to its original state
Transform in 3D space	Under SU(2) group	Under SO(3) group
Components in 3D space	3	2 (Weyl) or 4 (Dirac)

SO(N) AND SU(N)

The corresponding matrices have following properties

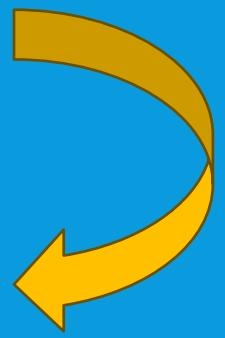
- SO(n): Special (determinant = +1), Orthogonal, n dimension
- SU(n): Special (determinant = +1), Unitary, n dimension

Difference between Orthogonal and Unitary: Real and complex

COMMON SPINOR

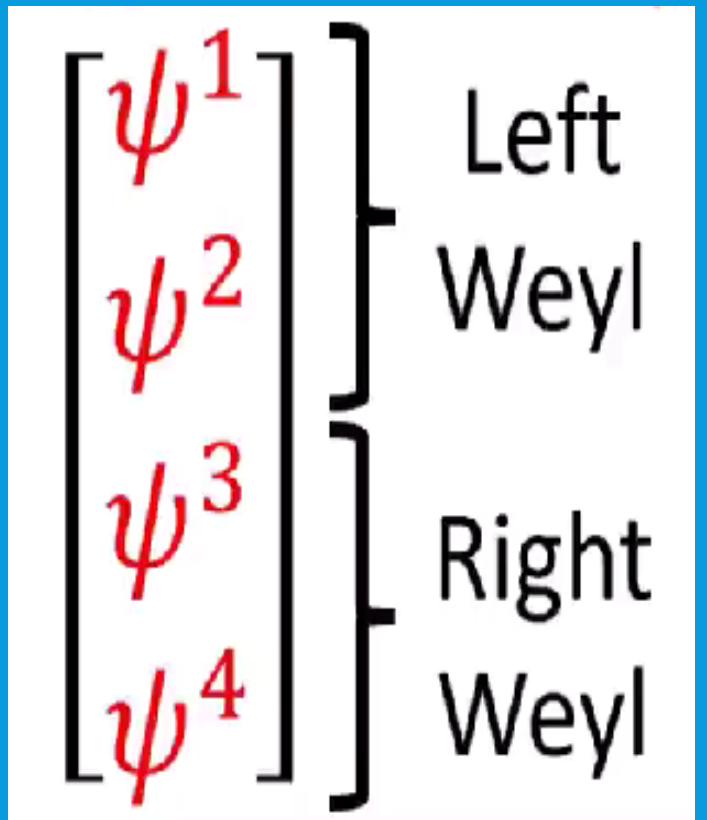
- Pauli spinor (3D space, 2 components)
- Weyl spinor (4D spacetime, 2 components)
- Dirac spinor (4D spacetime, 4 components)

Main topic



DIRAC SPINOR

- One of the most typical spinor
- There are 4 components, consisted of left and right Weyl spinor (each with 2 components)
- Describing relativistic spin-½ particles, for particle and antiparticle



DIRAC FUNCTION

- Dirac spinor is originated from Dirac function
  (Dirac spinor is the solution of Dirac function)

$$(i\hbar\gamma^{\mu}\partial_{\mu} - mc)\psi = 0$$

Ψ : Dirac spinor

γ^{μ} : Dirac gamma matrices

μ = 0,1,2,3 : spacetime indices

m : mass of the particle

Originated from the Schrödinger equation

SPIN PROJECTION OPERATORS

- Operators used in quantum mechanics to project out a specific spin state of a particle from a general quantum state
- Generally used in spin-1/2 particles system
- Example: spin up and down projection operator for spin-1/2:

$$P_+ = |\uparrow\rangle\langle\uparrow| = \frac{1}{2}(I + \sigma_z)$$

$$P_- = |\downarrow\rangle\langle\downarrow| = \frac{1}{2}(I - \sigma_z)$$

CHIRAL (HELICITY) PROJECTION OPERATORS

- Consider one spin projection operator acts on Dirac spinor
- This operator project out the left-handed and right-handed components of the Dirac spinor.
- Math form: left-handed and right-handed projections operators

$$P_L = \frac{1-\gamma^5}{2}, P_R = \frac{1+\gamma^5}{2}$$

ENERGY PROJECTION OPERATORS

- Consider one spin projection operator acts on Dirac spinor
- This operator project out the positive and negative energy solutions of the Dirac equation.
- Math form: positive (particle) or negative (antiparticle) energy projections operators

$$\Lambda_{\pm}(p) = \frac{1}{2m}\left(\gamma^{\mu} p_{\mu} \pm m\right)$$

REFERENCE

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- https://physics.stackexchange.com/questions/440478/meaning-of-the-subscripts-l-r-for-the-two-component-weyl-spinors-phi-l-r
- https://sites.ualberta.ca/~gingrich/courses/phys512/node60.html
- https://www.youtube.com/watch?v=j5soqexrwqY&list=PLJHszsWbB6hoOo_wMbob6T44KM_ABZtBs (graph source)

謝謝大家