CW complex

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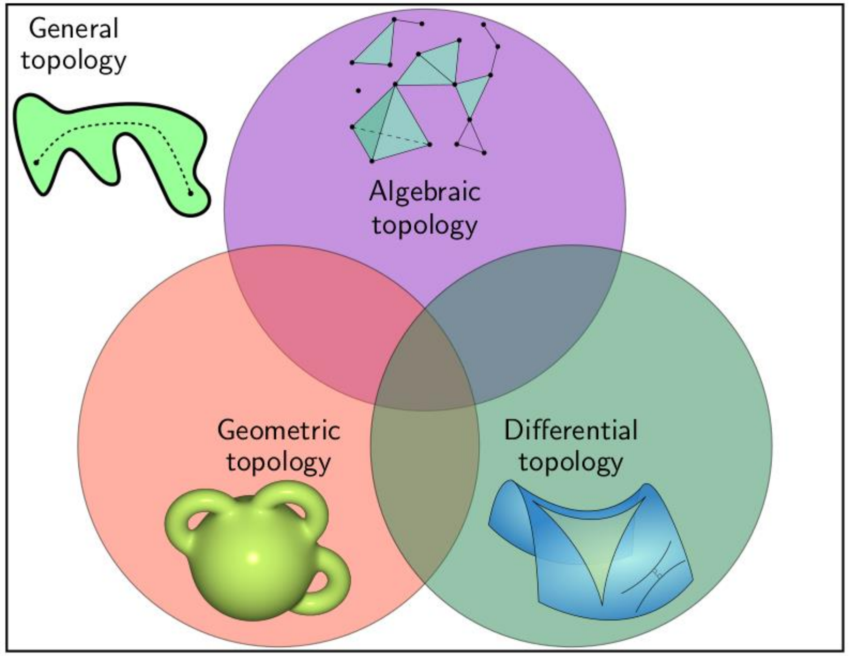
Content

What is a CW Complex?



- **Algebraic Topology** Studies shapes and connectivity of spaces using algebraic tools (e.g., homotopy, homology).
- "C" = closure-finite, "W" = weak topology

A **CW complex** is a Hausdorff space X, built from disjoint open cells $e_\alpha^{(k)}$, satisfying:

1. Each $e_\alpha^{(k)}$ is the image of the interior of a disk D^k via a continuous map, with the boundary mapped into the (k−1)-skeleton.
2. X has the **weak topology**: a set is closed in X if its intersection with each cell closure is closed.

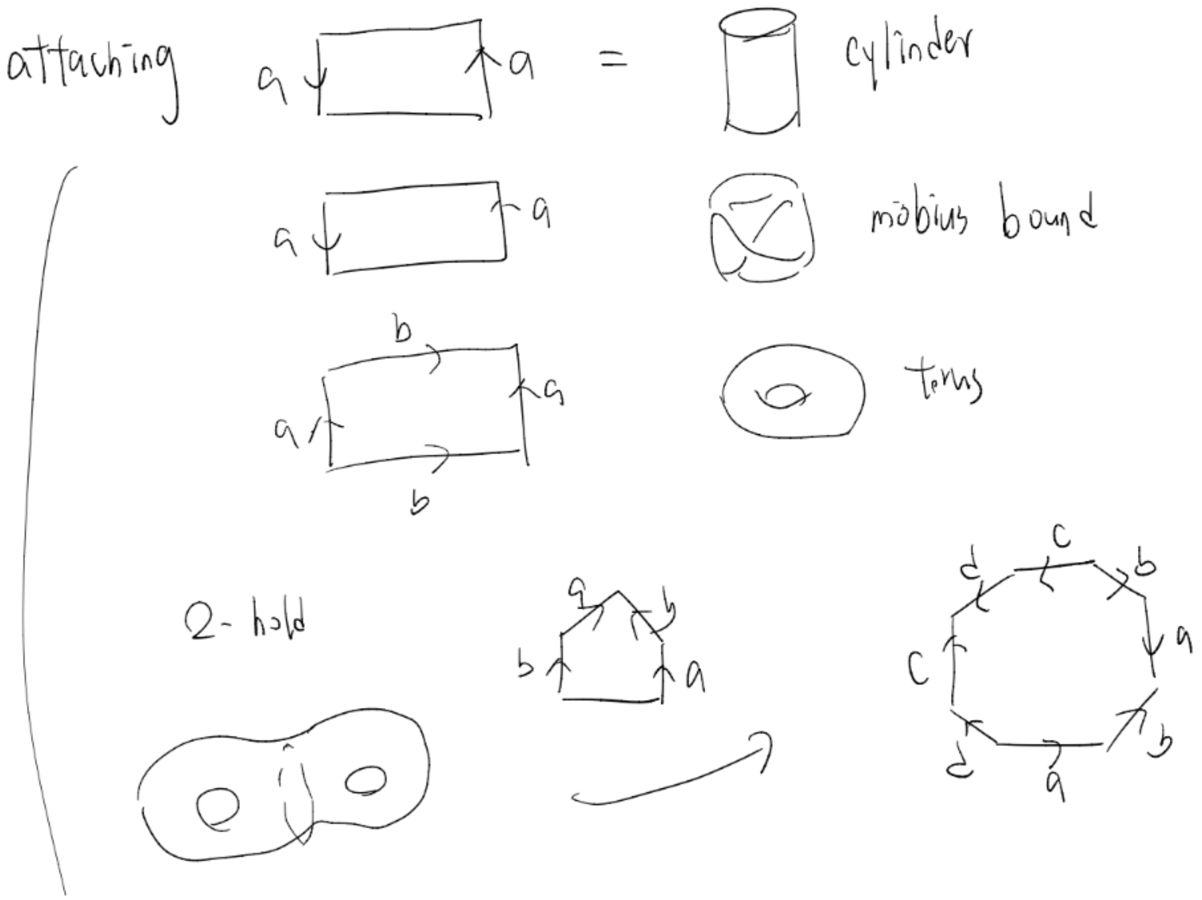
Building CW Complexes with Cell Attachment

0-cell •

1-cell —, attach via two S_0

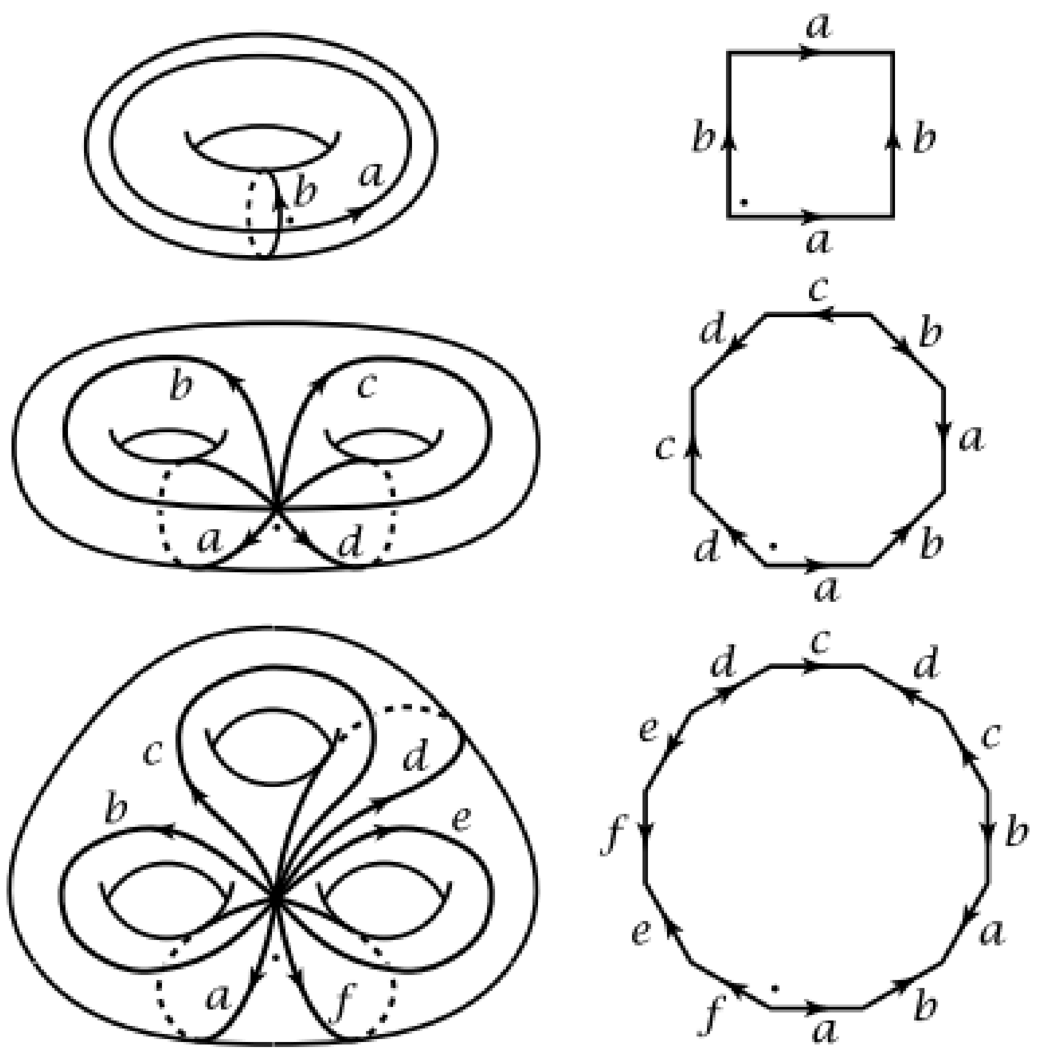
2-cell ○, attach via S_1

n-cell D_n, attached via boundary S_{n-1}



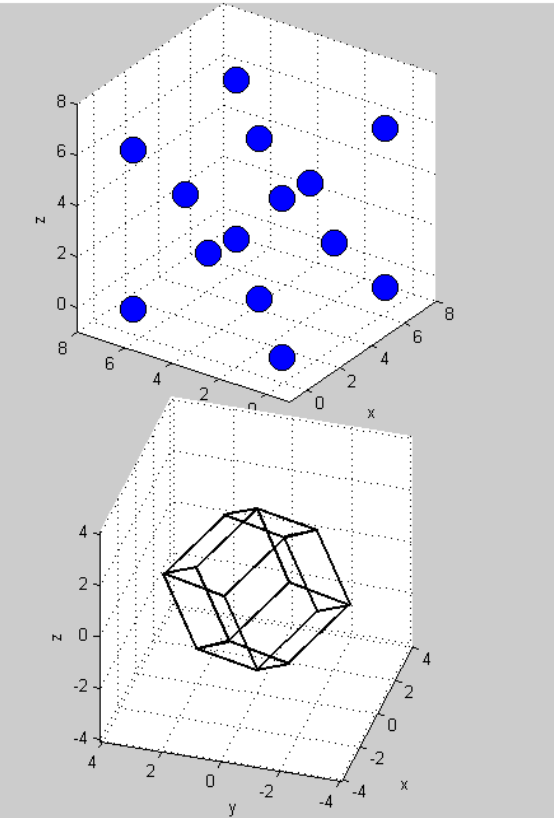
Building CW Complexes with Cell Attachment

- **CW Complex: A Building Block for Spaces** A topological space built by attaching "cells" (points, lines, disks, etc.) step-by-step.



CW Complexes in Brillouin Zones

- **First Brillouin Zone**:
  A unit cell in reciprocal space that describes allowed electron wavevectors.
- **Topological Identification**:
  - In 1D: interval $[-\pi/a, \pi/a]$ → endpoints identified ⇒ topologically a **circle** S^1
  - In 2D (square lattice): edges identified ⇒ a **2D torus** $T^2 = S^1 \times S^1$
  - In 3D: forms a **3-torus** T^3

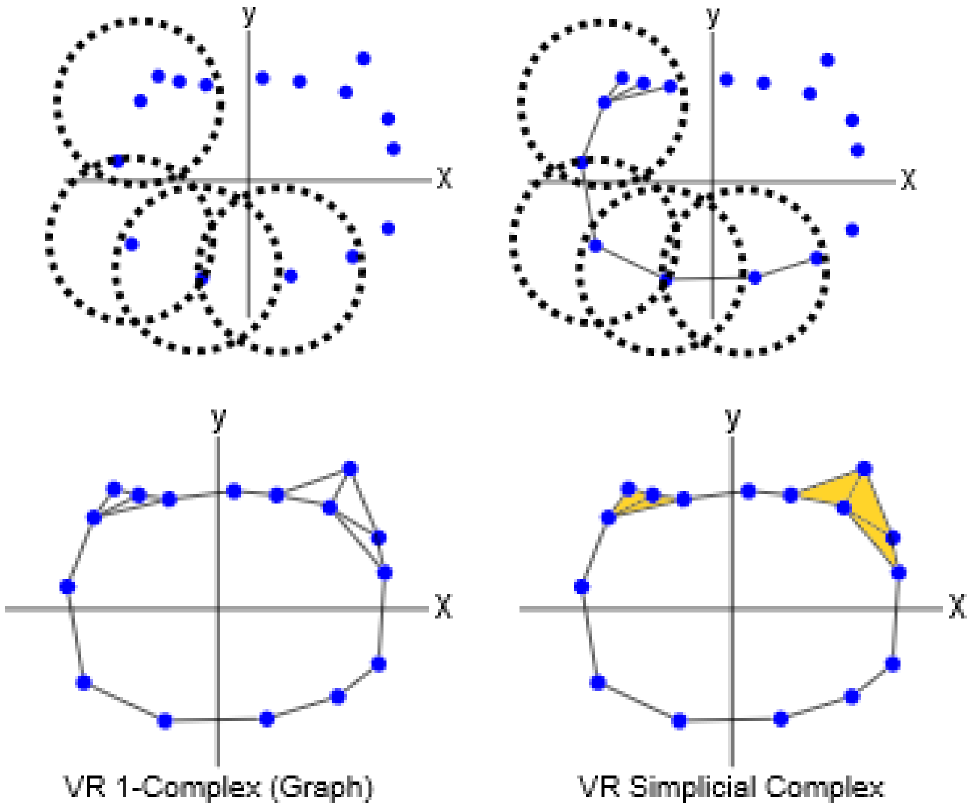


topological data analysis

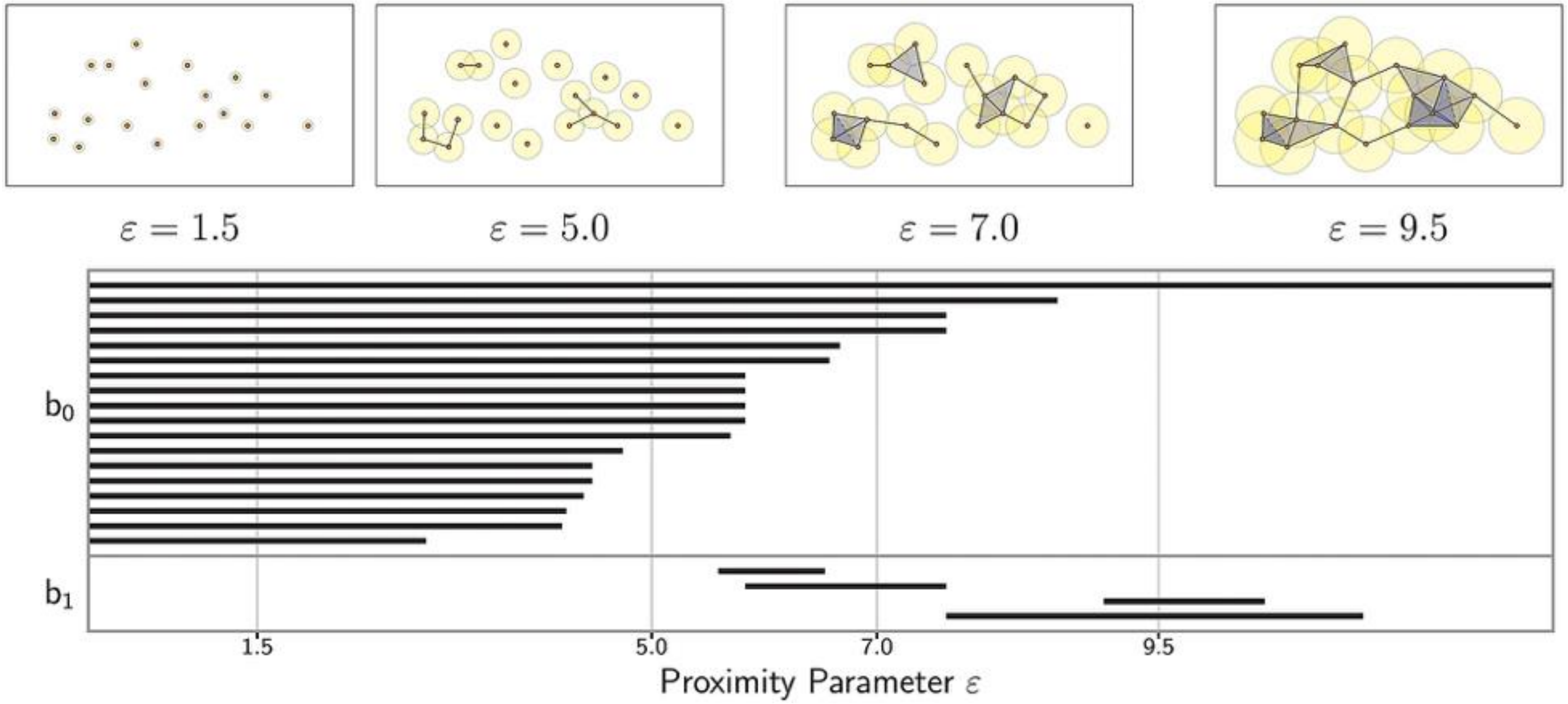
- **Core idea:** Uses algebraic topology to study the shape and structure of data
- **Focus:** Captures global features (e.g., connected components, loops, voids) rather than local statistics

Key Steps in TDA

- Convert data into a **simplicial complex** (e.g., Vietoris–Rips complex)
- Build a **filtration** — a nested sequence of complexes
- Compute **persistent homology** to detect stable topological features
- Visualize results using **persistence diagrams** or **barcodes**



topological data analysis




Thank you for listening