"Feynman path Integral" 費曼路徑積分

Introduction : What is a "Path Integral" ?

Classical Mechanic : $S(x(t)) = \int L(x,\dot{x})\,dt$, $\exists\, x(t)$ is a "right path"

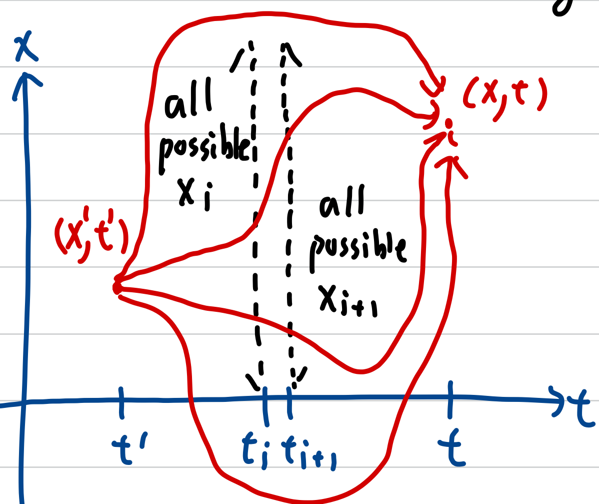
In QM : All the paths account for some probabilities

Def "Propagator" $K(x',t';x,t) \equiv \sum_i e^{\frac{i}{\hbar}\cdot S_i}$ (i → all the possible paths)

The probability of a particle propagate from

$(x',t') \xrightarrow{\text{to}} (x,t)$ $\quad P(x',t';x,t) = |K(x',t';x,t)|^2 \cdot dx$

Calculation of Path Integral (w/o detailed derivation)



$$K(x',t';x,t) \quad (n \to \infty)$$

$$= \frac{1}{A} \cdot \int_{\mathbb{R}^n} e^{\frac{iS(x(t))}{\hbar}} \cdot \frac{dx_1}{A}\frac{dx_2}{A}\cdots\frac{dx_n}{A}$$

A = normolized const.

Denoted as : $\int_{x'}^{x} e^{\frac{i}{\hbar}S(x(t))}\,\mathcal{D}x(t)$

Association with Schrödinger eq.

What is a wave function?

$$\Psi(x,t) = \sum_{\text{all possible } x} \Psi(x',t') \cdot K(x',t',x,t)$$

t': initial time

$$= \underbrace{\int_{-\infty}^{\infty} dx' \, \Psi(x',t')}_{\text{sum up all possible initial states}} \cdot \underbrace{K(x',t',x,t)}_{\text{propagation to } (x,t)}$$

cosider a free particle in 1-D: (1st order)

$$L = \frac{1}{2} m \dot{x}^2 , \quad S = \int_{t'}^{t} \frac{1}{2} m \dot{x}^2 dt \doteq \frac{m}{2} \cdot \frac{(x-x')^2}{t-t'}$$

$$K(x',t',x,t) = \sum_i e^{\frac{i}{\hbar} S_i} = \sum_i e^{\left(\frac{im}{2\hbar} \cdot \frac{(x-x')^2}{t-t'}\right)_i}$$

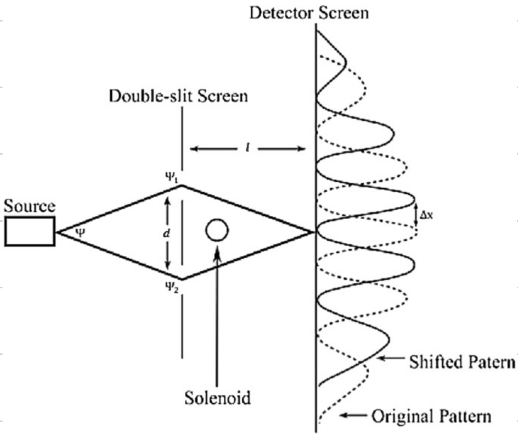
is a guassian dist. (sol. of diffusion eq.) with complex coeff.

$$\Rightarrow i\hbar \frac{\partial K}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 K}{\partial x^2} \quad \text{or} \quad i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \quad \text{(sch. eq.)}$$

(put the $\frac{\partial}{\partial t}, \frac{\partial^2}{\partial x^2}$ into the integral)

Impurtant application:

"A-B effect" in double slit experiment



momentum $m\dot{\vec{x}} \rightarrow m\dot{\vec{x}} + q\vec{A}$

the action:

$$S = \int \frac{1}{2} m \dot{\vec{x}}^2 + q\dot{\vec{x}} \cdot \vec{A} \, dt$$

Since there are only two paths!!

$$\Psi_{total} = \Psi_L + \Psi_R = Ae^{i\phi_L} + Ae^{i\phi_R}$$

$$\phi = \frac{1}{\hbar}\underbrace{\int \frac{1}{2} m\dot{\vec{x}}^2 \, dt}_{\text{same in two paths}} + \frac{q}{\hbar}\int \vec{A}\cdot\dot{\vec{x}} \, dt = \int \vec{A}\cdot d\vec{x}$$

$$\Rightarrow \Delta\phi = \frac{q}{\hbar}\left(\int_R \vec{A}\cdot d\vec{x} - \int_L \vec{A}\cdot d\vec{x}\right) = \frac{q}{\hbar}\oint \vec{A}\cdot d\vec{x}$$

$$= \int \vec{\nabla}\times\vec{A}\cdot d\vec{a} = \int \vec{B}\cdot d\vec{a} = \Phi \text{ (mag. flux)}$$

phase difference $\Delta\phi = \frac{q\Phi}{\hbar} = \pi, 3\pi, 5\pi$ the pattern will shift!

Further application

1. Quantum tunneling

In classical Mechanic, the particle cannot propagate the region of $V(x) > E$, but it's possible in QM.

2. Association with Statistical Mechanic

Taking the "wick rotation" ($\tau = it$)

then we can use the concept of "partition func." to evaluate the system.

3. Association with Quantum field theory.

The formalism of path integral is easier to transist to quatum field theory (compared with wave eq.)

Thanks!