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Impact of Vertical Boundaries on Convective Overstability in Protoplanetary Discs

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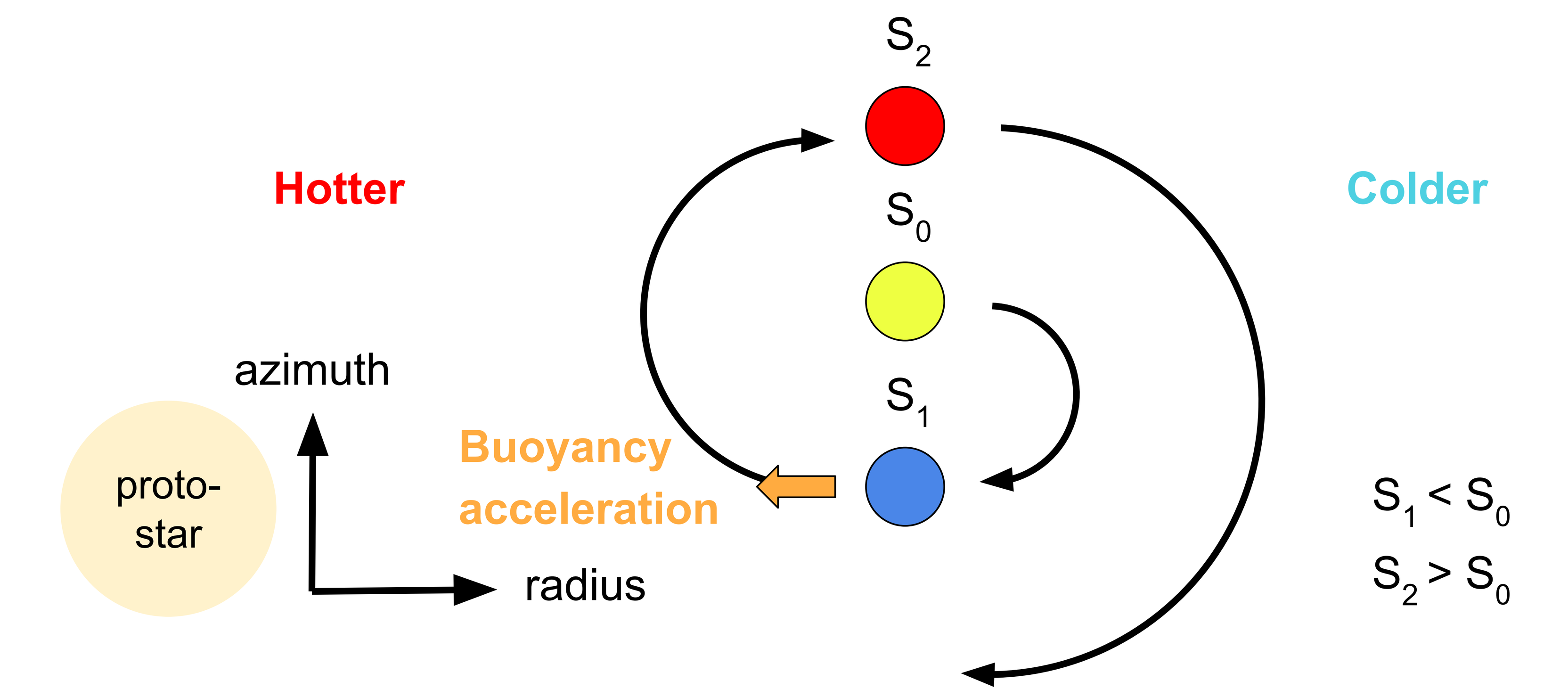
Protoplanetary Discs & Planet Formation

Protoplanetary discs (PPDs) are gas-rich environments surrounding young stars where planets form through the accumulation of dust grains. Within these discs, hydrodynamic instabilities play a crucial role by creating large-scale fluid structures that concentrate dust and facilitate planetesimal formation.

Convective Overstability (COS)

Mechanism: COS requires two conditions: (1) a negative radial entropy (S) gradient, and (2) thermal relaxation on orbital timescales. As illustrated, an oscillating fluid blob overcools moving outward (**blue**, $S_1 < S_0$) and overheats moving inward (**red**, $S_2 > S_0$). This thermal mismatch creates a resonant buoyancy acceleration that exponentially amplifies its epicyclic motion.



Structure & Planet Formation: Upon nonlinear saturation, COS generates large-scale coherent structures:

- **Zonal flows:** Radial bands of sub- and super-Keplerian motion that create pressure bumps.
- **Elevator flows:** Coherent vertical upwellings and downwellings.

Standard **periodic simulations** assume **"infinite" discs** where these dust traps artificially persist. However, their physical robustness is highly sensitive to realistic vertical boundaries.

Vertical Boundary Conditions (VBCs)

- **Periodic:** Continuous boundary matching across the vertical domain.
- **Zero heat flux:** Thermal gradients vanish at the boundary ($\partial z\, \theta' = 0$).
- **Zero perturbation:** All perturbation variables are strictly forced to zero at the boundary.

Framework & Methodology

- **Framework: Unstratified Boussinesq shearing box** (Latter & Papaloizou, 2017), capturing small-scale and subsonic phenomena of COS.
- **Linear Theory:**
  - **Local:** Linearized as a matrix eigenvalue problem.
  - **Vertically Global:** Solve linearized equations as 1D ODEs to obtain growth rates under different VBCs.
- **Nonlinear Simulation:** We use the DEDALUS code (Burns et al., 2020) to evolve the full equations in a 2D (axisymmetric) domain.

Linear Theory Growth Rates

- **Large scales (small vertical wavenumber k_z):** Periodic and Zero Heat Flux VBCs yield almost identical growth. Conversely, the Zero Perturbation VBC significantly suppresses growth.
- **Small scales (large k_z):** Modes become highly localized, reducing sensitivity to boundary conditions and causing all growth rates to converge.

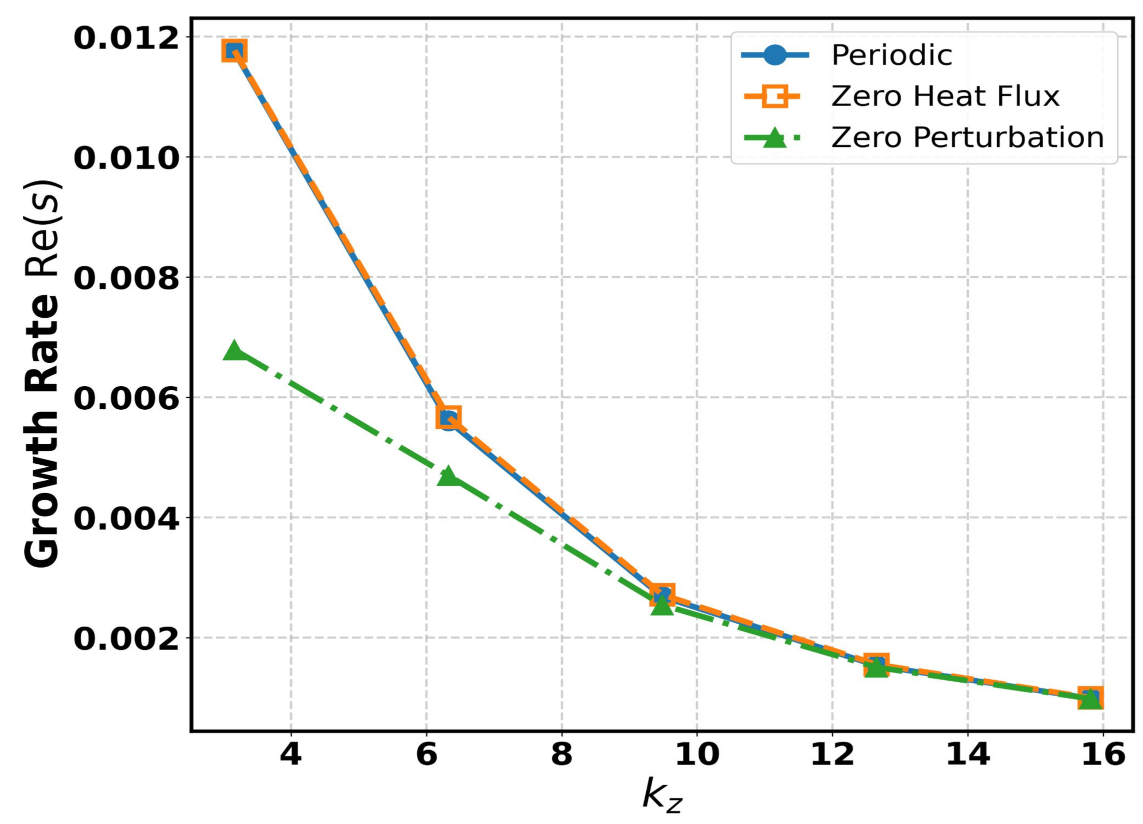
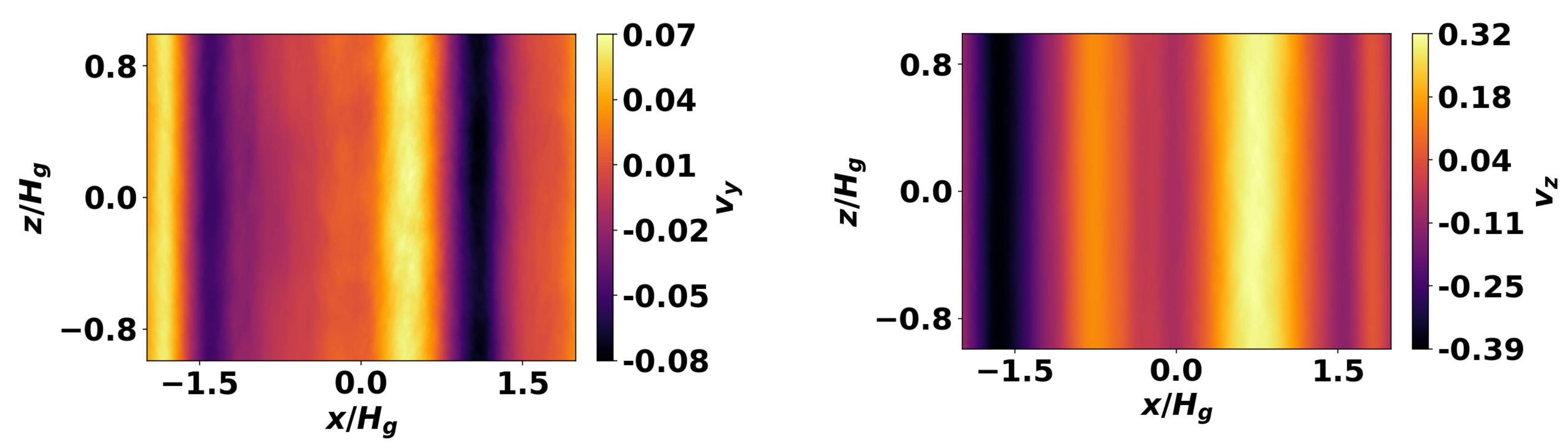


Figure: Linear growth rate vs. vertical wavenumber (k_z) under different VBCs.

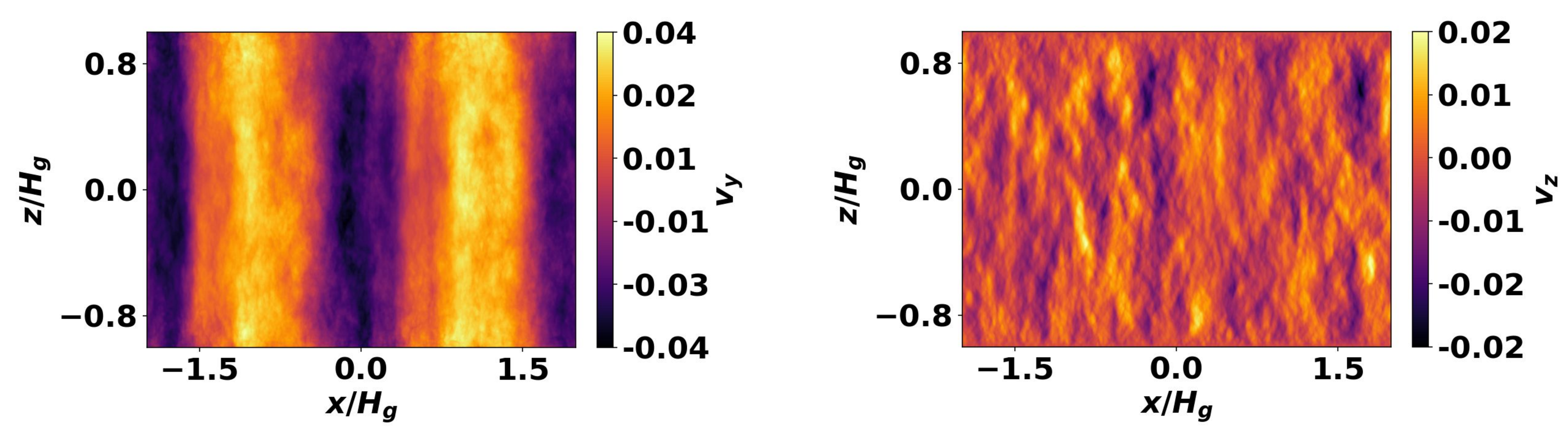
Nonlinear Simulation Morphologies

Table: Average 2D contour plots of **azimuthal (left) and vertical (right) velocity field** after reaching the saturation of COS.

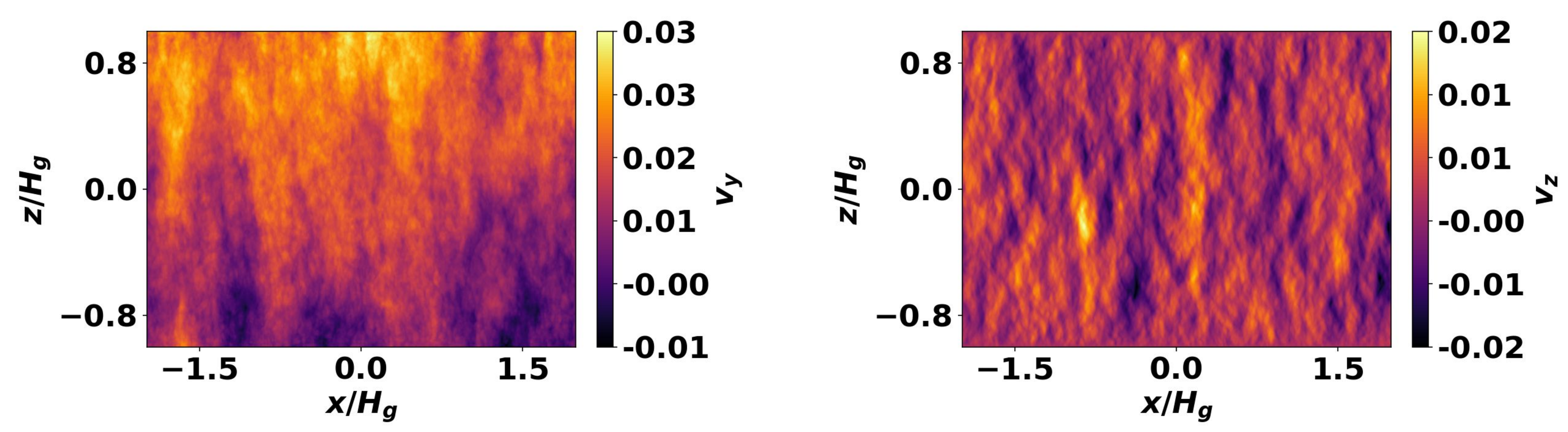
Periodic: zonal flows and elevator flows



Zero Heat Flux: zonal flows only



Zero Perturbation: no structures



Conclusion

- Linear growth rates do not fully predict non-linear outcomes. For instance, while Periodic and Zero Heat Flux VBCs share similar linear growth rates, their saturated flow morphologies diverge significantly.
- **Realistic, non-periodic vertical boundaries disrupt the coherence of zonal and elevator flows**, causing them to become chaotic or completely suppressed.
- The lack of robust pressure bumps under realistic VBCs challenges the assumption that COS can efficiently trap dust to facilitate planetesimal formation.