First-Principles Lattice Study of Dense QCD-like Theories

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)

Recent Review:
EI "Lattice results for the equation of state in dense QCD-like theories"
arXiv: 2508.03090
K. Iida, EI, K. Murakami, D. Suenaga, arXiv:2606.13974 (Yesterday!!)

The 16th Particle Physics Phenomenology Workshop, National Tsing Hua University (NTHU), Taiwan, 2026/06/16

Another Workshop Series: "PPP" in Japan

PPP2026 @ Kyoto

基研研究会『素粒子物理学の進展2026』（Progress in Particles Physics 2026, PPP2026）

Aug 24 – 28, 2026
京都大学基礎物理学研究所
Asia/Tokyo timezone

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- Timetable
- Contribution List
- Registration
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- Code of conduct
- Past Workshops

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ppp202x@googlegroups...

概要

本研究会は、現象論的な側面を念頭に置き、 素粒子物理学の発展を意図した研究交流を目的としています。 近年、素粒子物理学の細分化が進んでおりますが、本研究会では素粒子物理学の広範な話題をカバーし、分野を超える総合的な討論を行えるものを目指しています。 また同時に、個別分野の専門家だけでなく、専門分野を特定していない若手の方々の積極的な参加をサポートしたいと考えています。

この研究会でとりあげる主なテーマ
- 「LHCおよび各種将来加速器実験における物理」
- 「フレーバーの関与する物理」
- 「非摂動的な方法と強結合場の理論のダイナミクス」
- 「宇宙・天体物理と素粒子物理」
- 「統一理論、超対称理論、世代構造に関する模型構成」
- 「高次元理論や弦理論からのアプローチ」

- Annual workshop series forcus on phenomenology of particle physics since 2003 (Taiwan PPP (since1992) has a longer history!)
- Held every summer in Kyoto
- Around 200 participants
- Mainly domestic, but English talks are welcome
- This year: Aug. 24–28
- Registration is now open!
  (Ask Ishida-san, the chair person)

First-Principles Lattice Study of Dense QCD-like Theories

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

iTHEMS Interdisciplinary Theoretical & Mathematical Sciences

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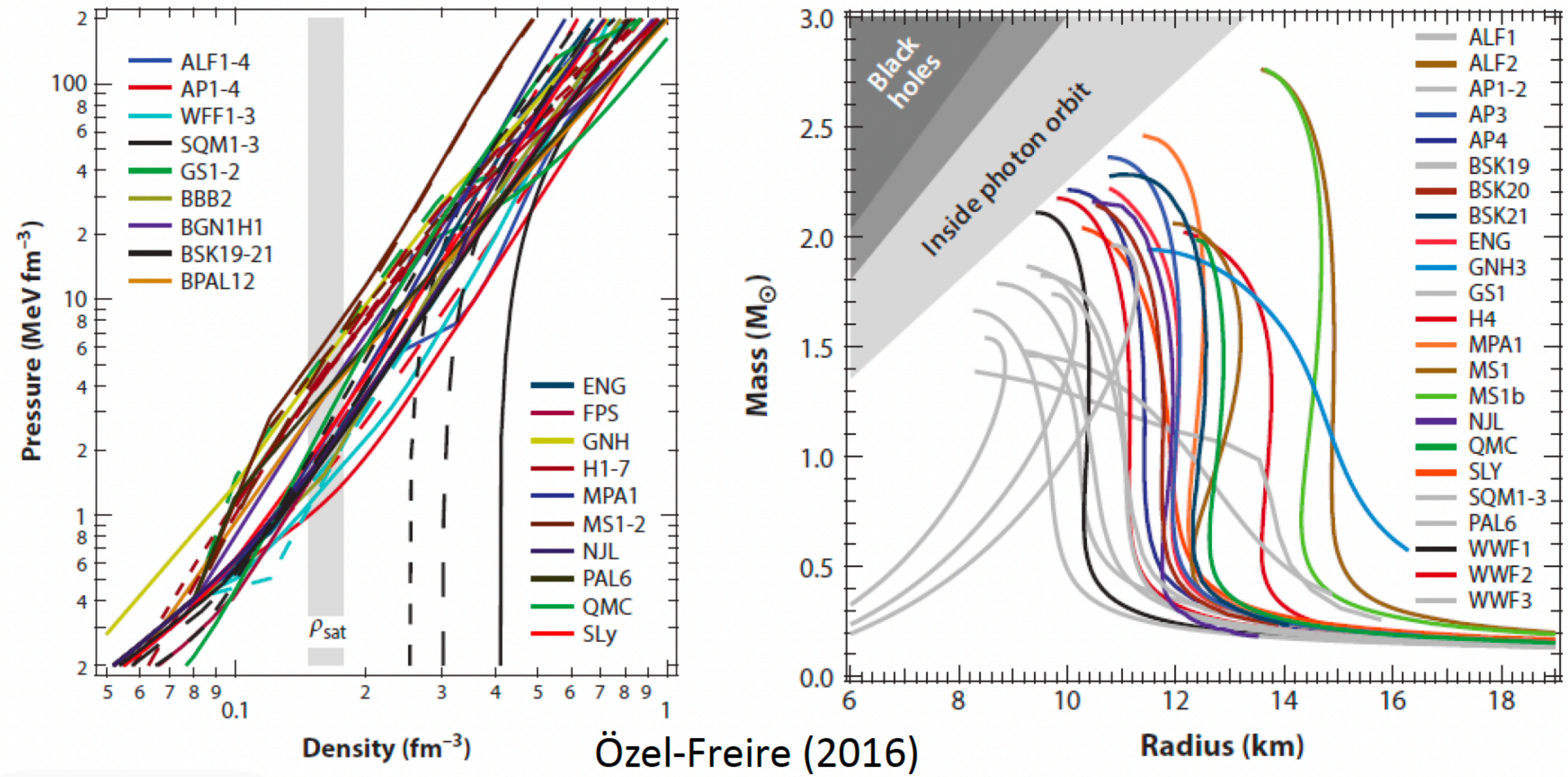
科研費 KAKENHI

CREST

The 16th Particle Physics Phenomenology Workshop, National Tsing Hua University (NTHU), Taiwan, 2026/06/16

A One-to-One Correspondence

dense QCD Eq. of State ⟵ **Tolman–Oppenheimer–Volkoff equation** ⟶ **M-R relation of neutron star**



Özel-Freire (2016)

The discovery of 2-solar-mass neutron stars strongly constrained dense-matter EoSs. Many **soft** EoSs predicted by conventional dense-QCD effective models cannot support such massive stars.

Dense QCD phenomena

- Cold dense QCD: $\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$

Dense QCD-like phenomena

- Cold dense QCD(-like): $\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$

Analytical study
(perturbation, symmetries,
effective model)

Neutron star experiment

low-T and high-density
QCD-like phenomena

high dense and low-T
Collider exp. is difficult

~~infamous sign problem~~

lattice calculation
(numerical experiment)

QCD-like theory=2color QCD
- same Lagrangian form
- asymptotic freedom
- at $\mu = 0$
  confinement
  chiral symmetry breaking
  instanton configuration$\cdots$

Our 2color QCD projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation (T=80MeV)
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, PoS, Lattice 2021
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation (T=80MeV)
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023) and
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, PoS, Lattice 2022
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, JHEP 02 (2024) 152
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, JHEP 10 (2024) 022
  Phase diagram and EoS by Lattice simulation (T=40MeV)

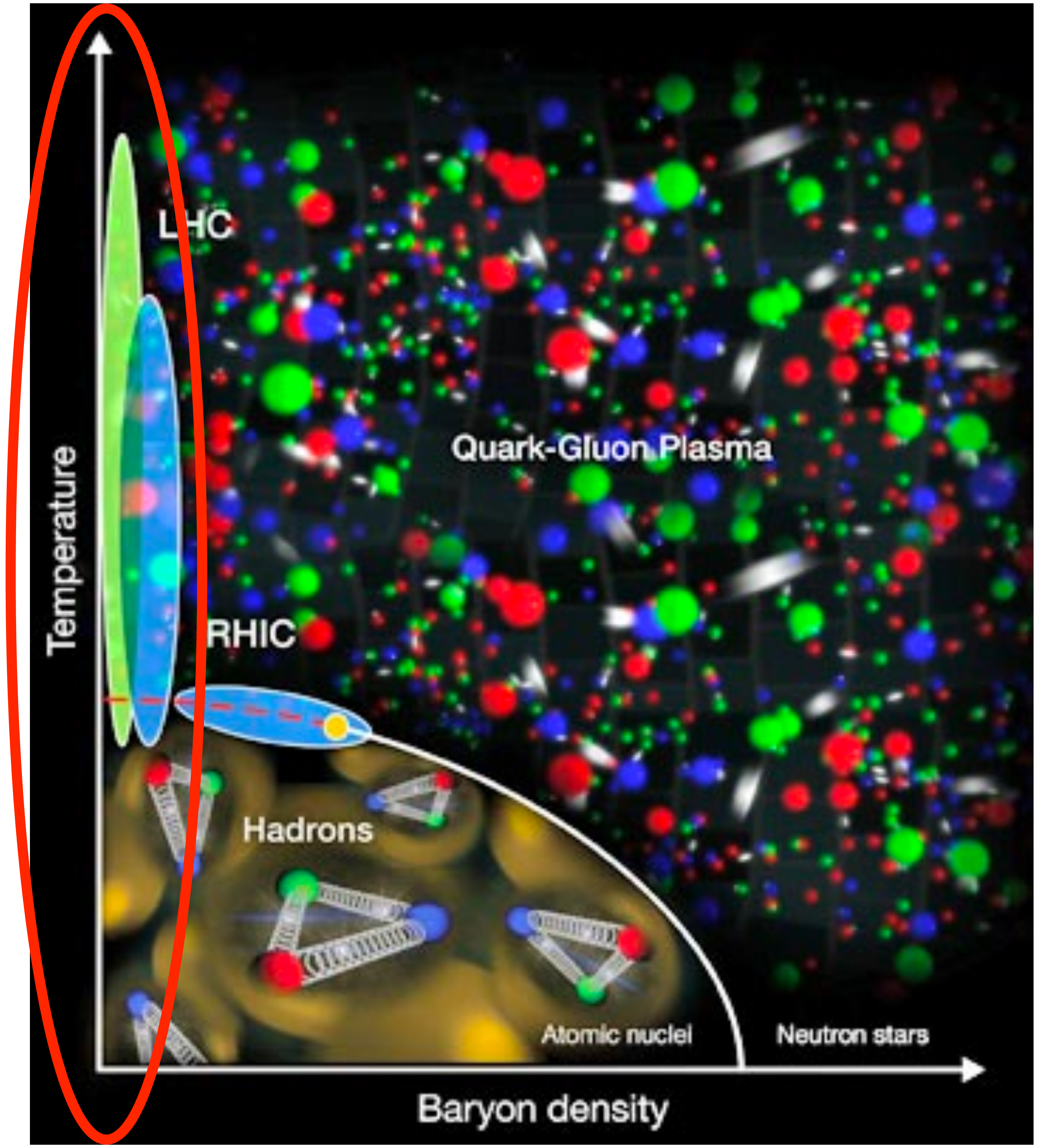
Outline

- Introduction
- Phase diagram
  (Hadronic , BEC, BCS phases)
- Equation of State and sound velocity
- Mass spectra
- Summary

Introduction

EoS and sound velocity at zero μ

expected QCD phase diagram



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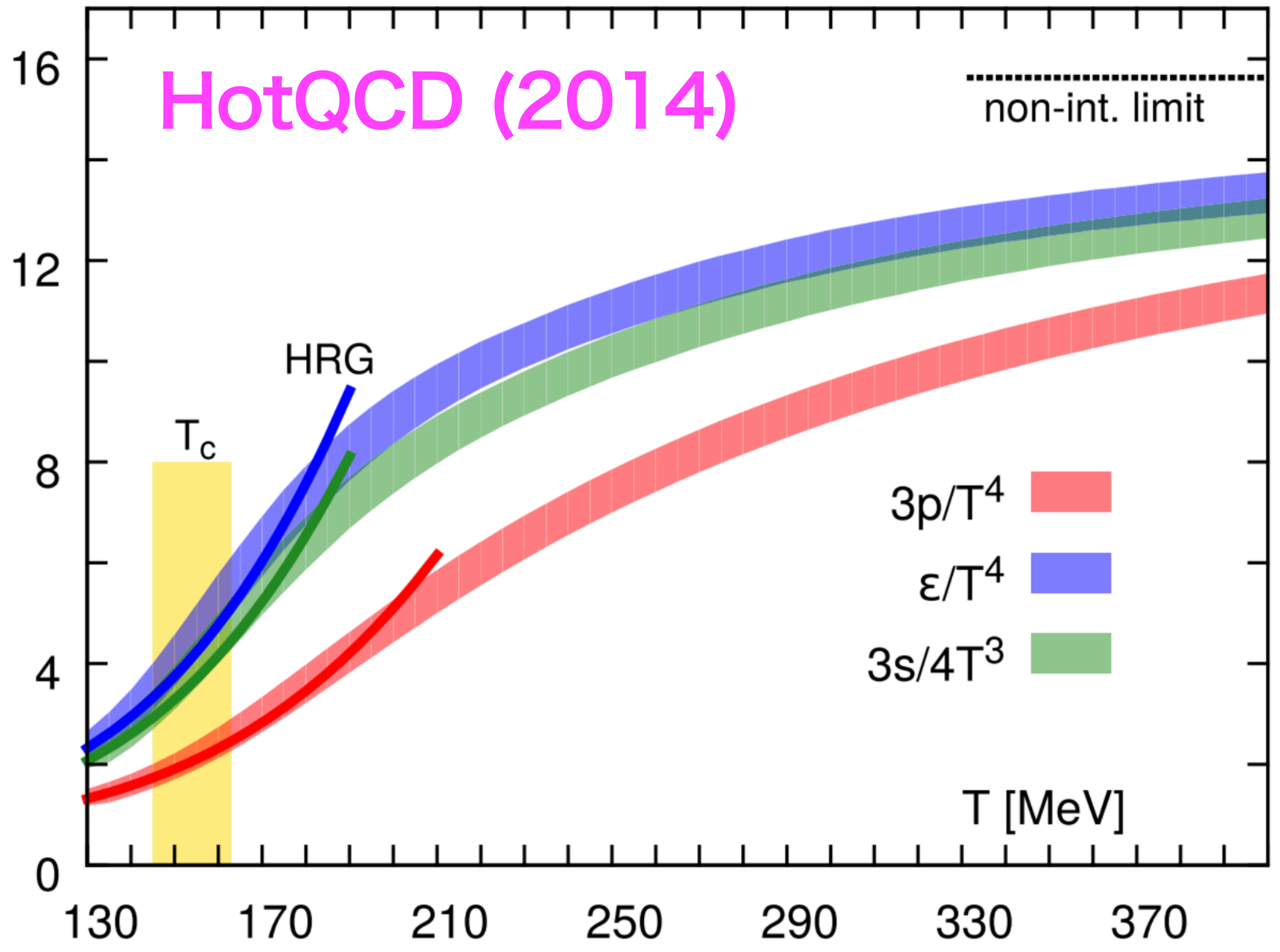
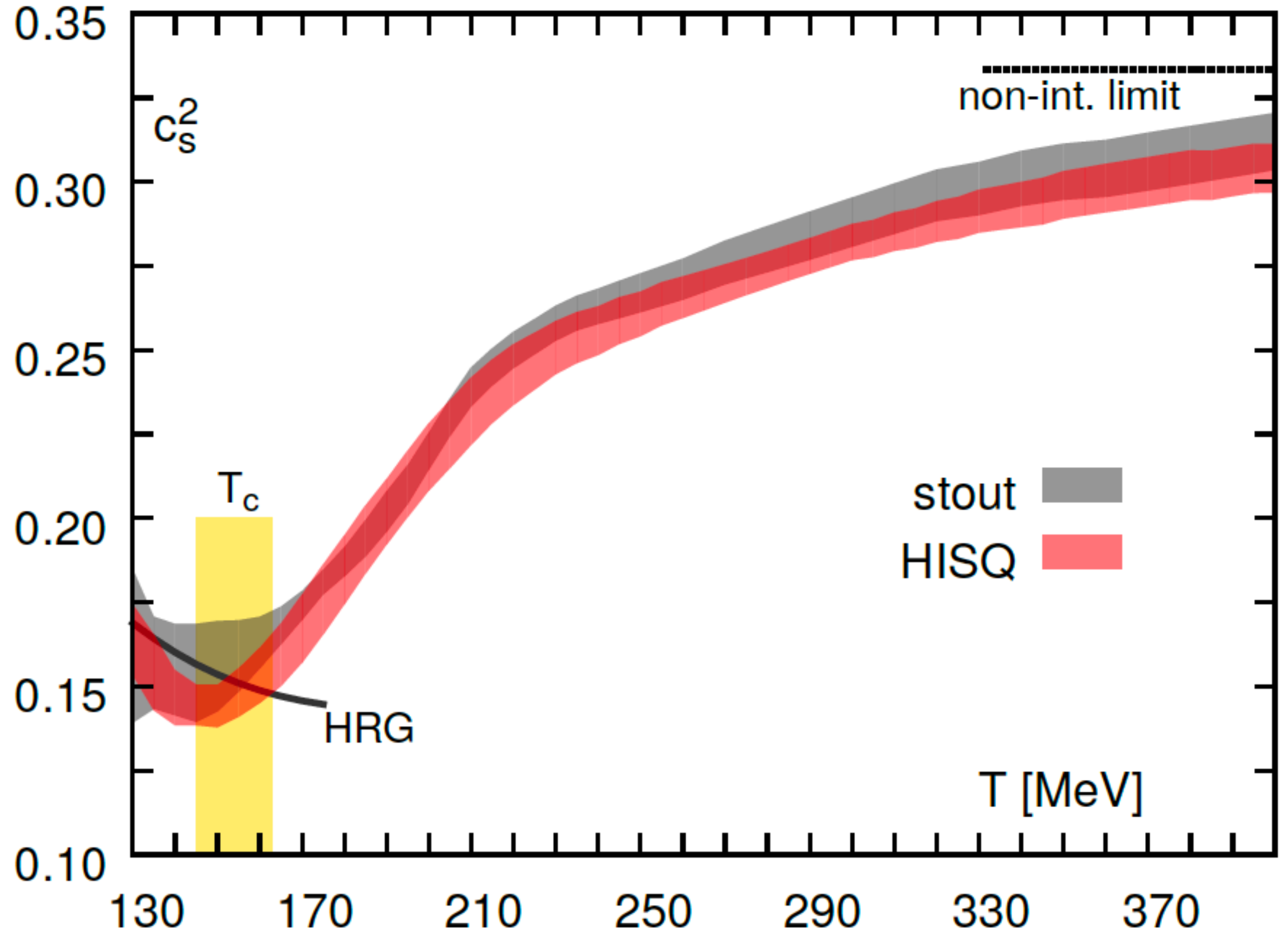
Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)



Sound velocity $c_s^2/c^2 = \partial p/\partial \epsilon$



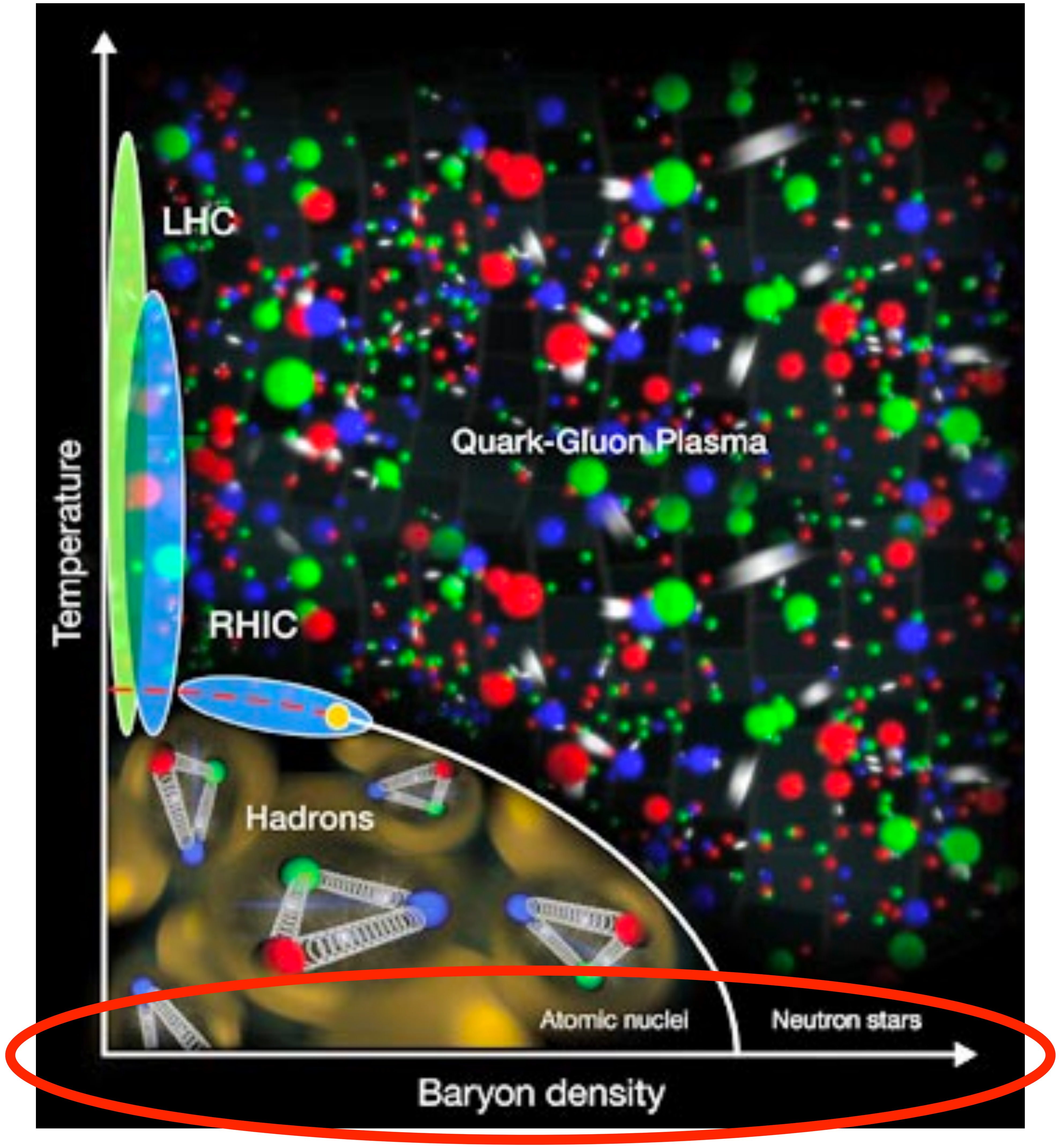
cf.) $c_s^2/c^2 = 1/3$: relativistic free theory where $e = 3p$

($c_s^2/c^2 \leq 1/3$: conformal bound)

It gives the input for $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

Sound velocity in finite density regime?

expected QCD phase diagram



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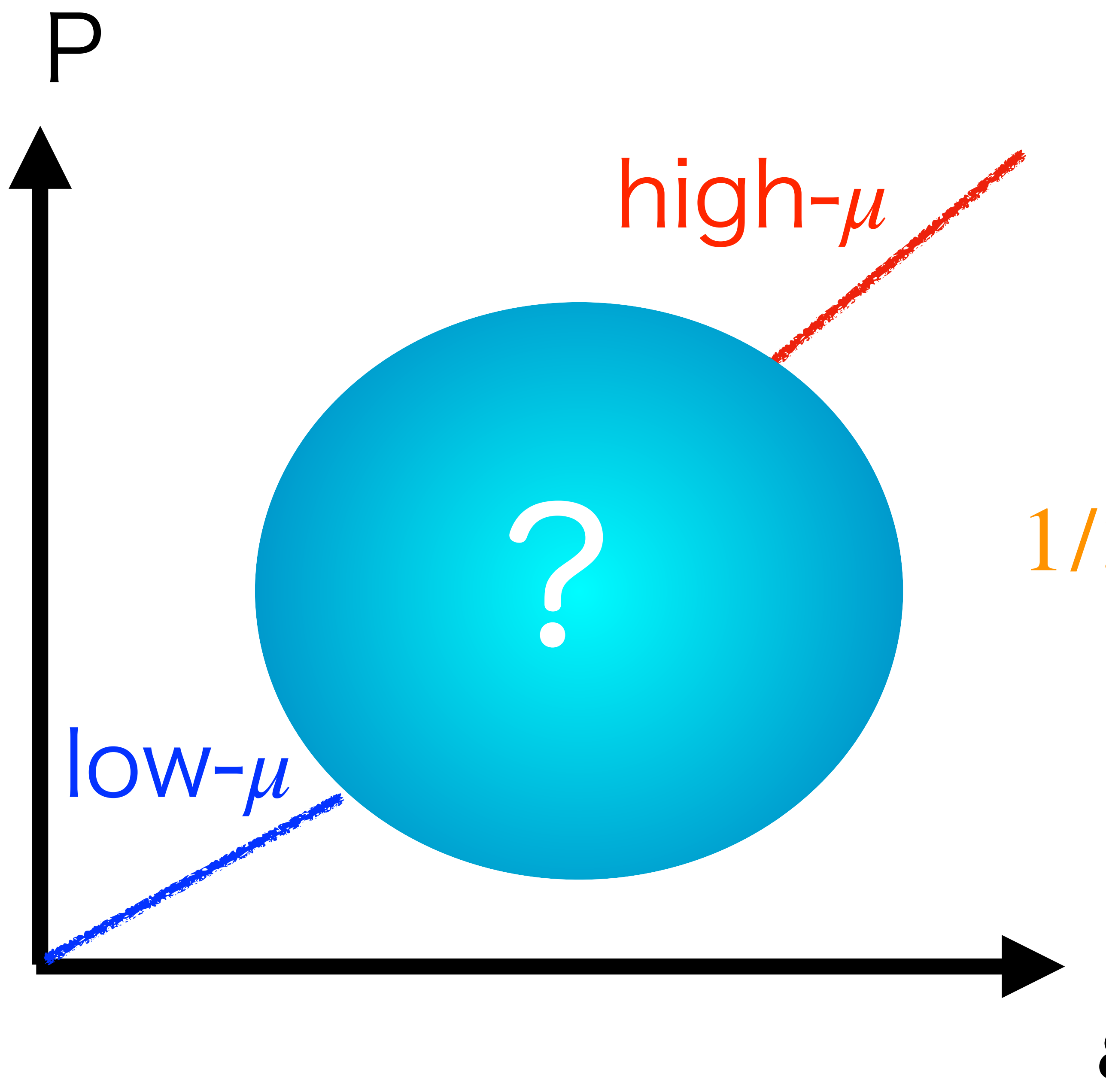
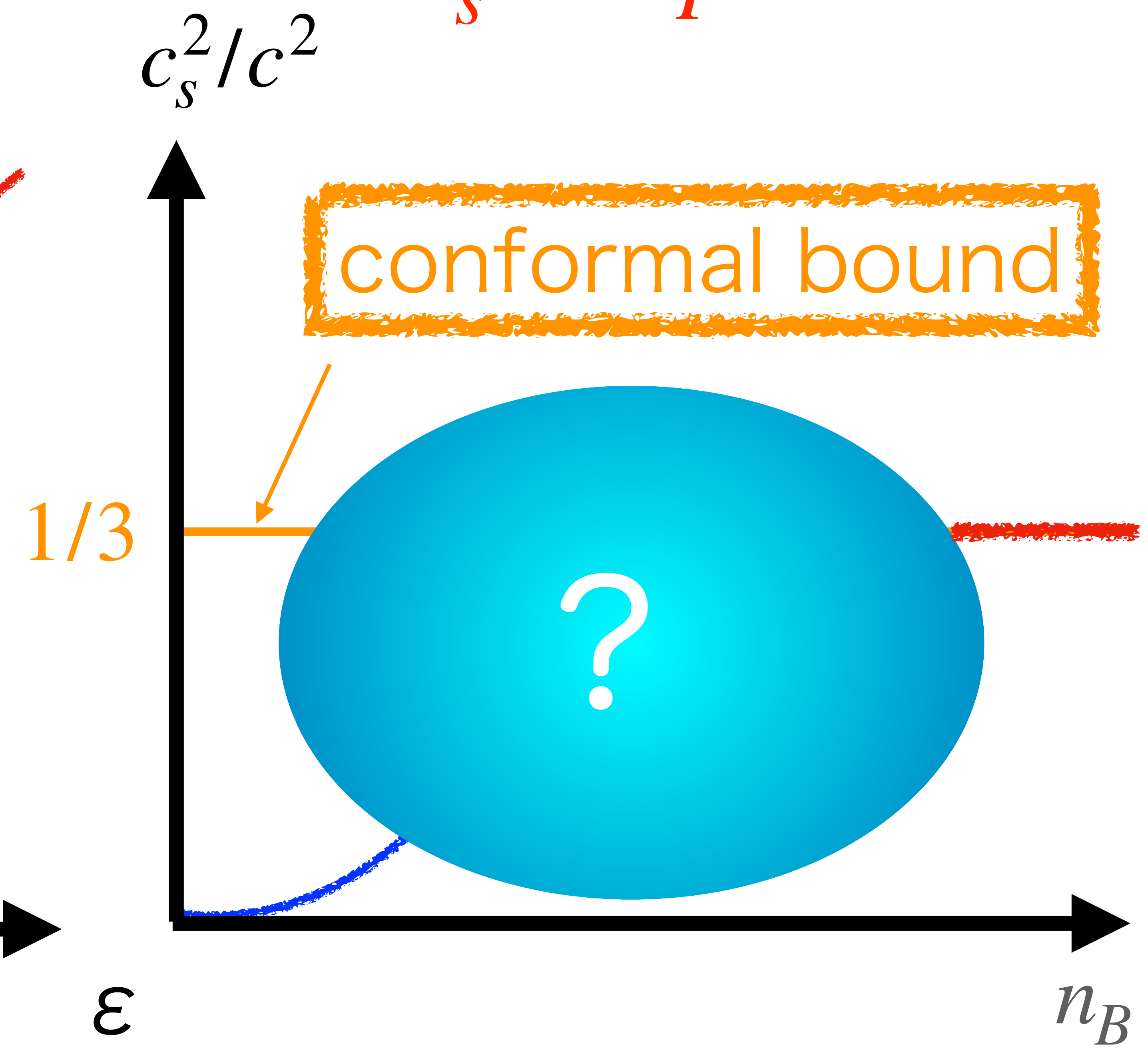
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$

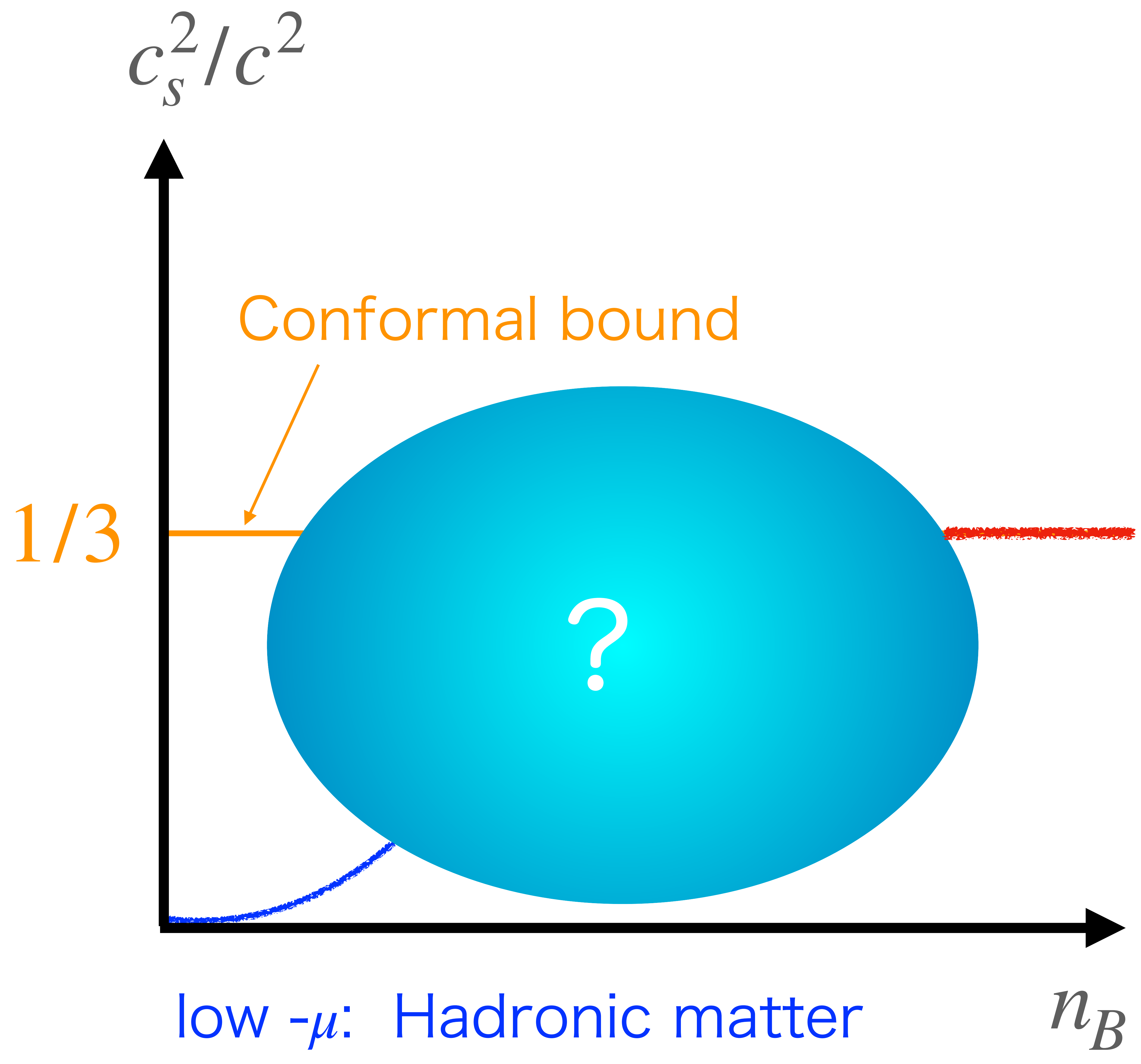


low -μ ($n_B \lesssim 2n_0$): Hadronic matter

high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

Prediction by phenomenology and effective models



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

  Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD
the sign problem is absent!!

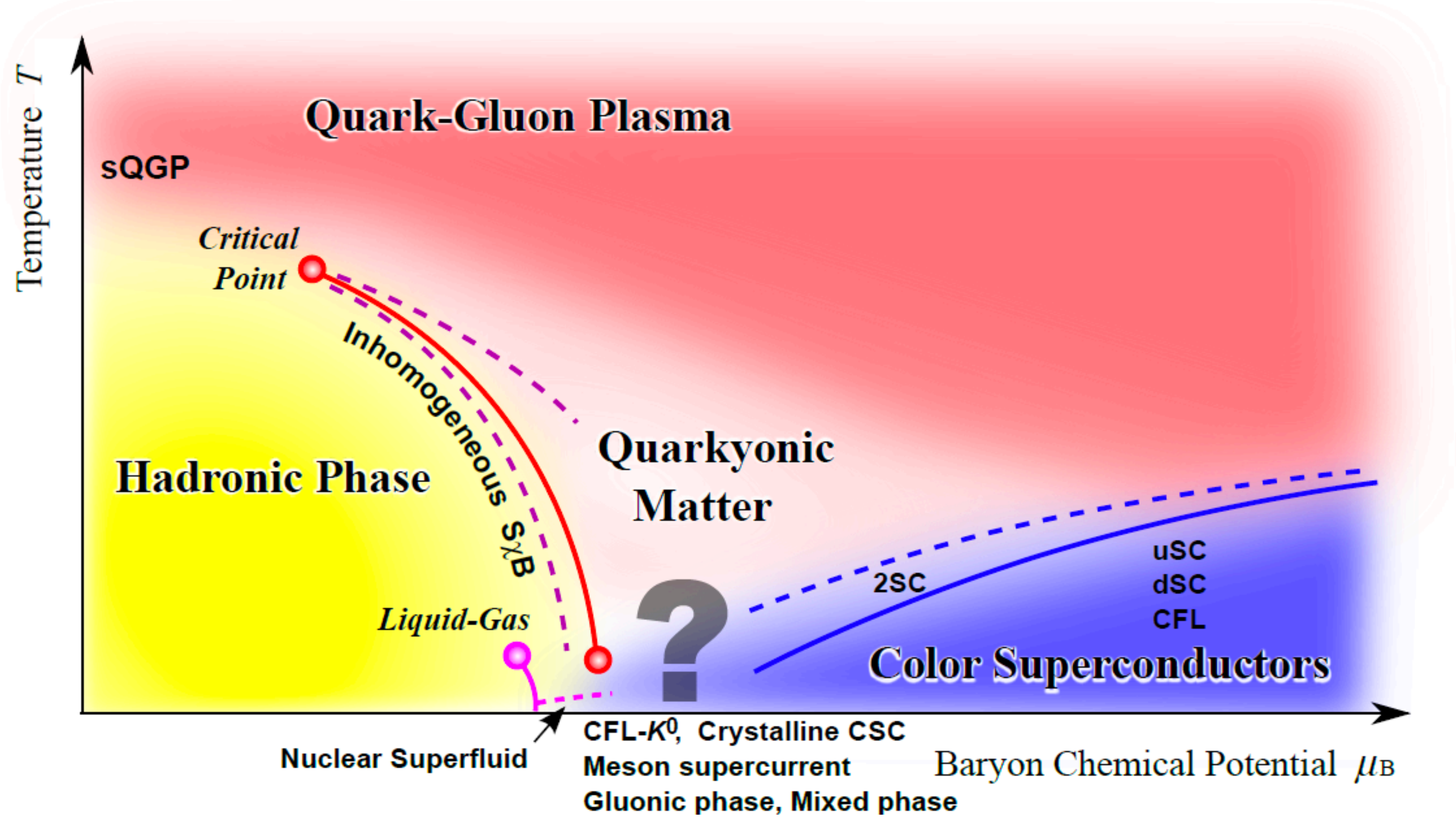
2color QCD phase diagram

(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat]
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(5) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

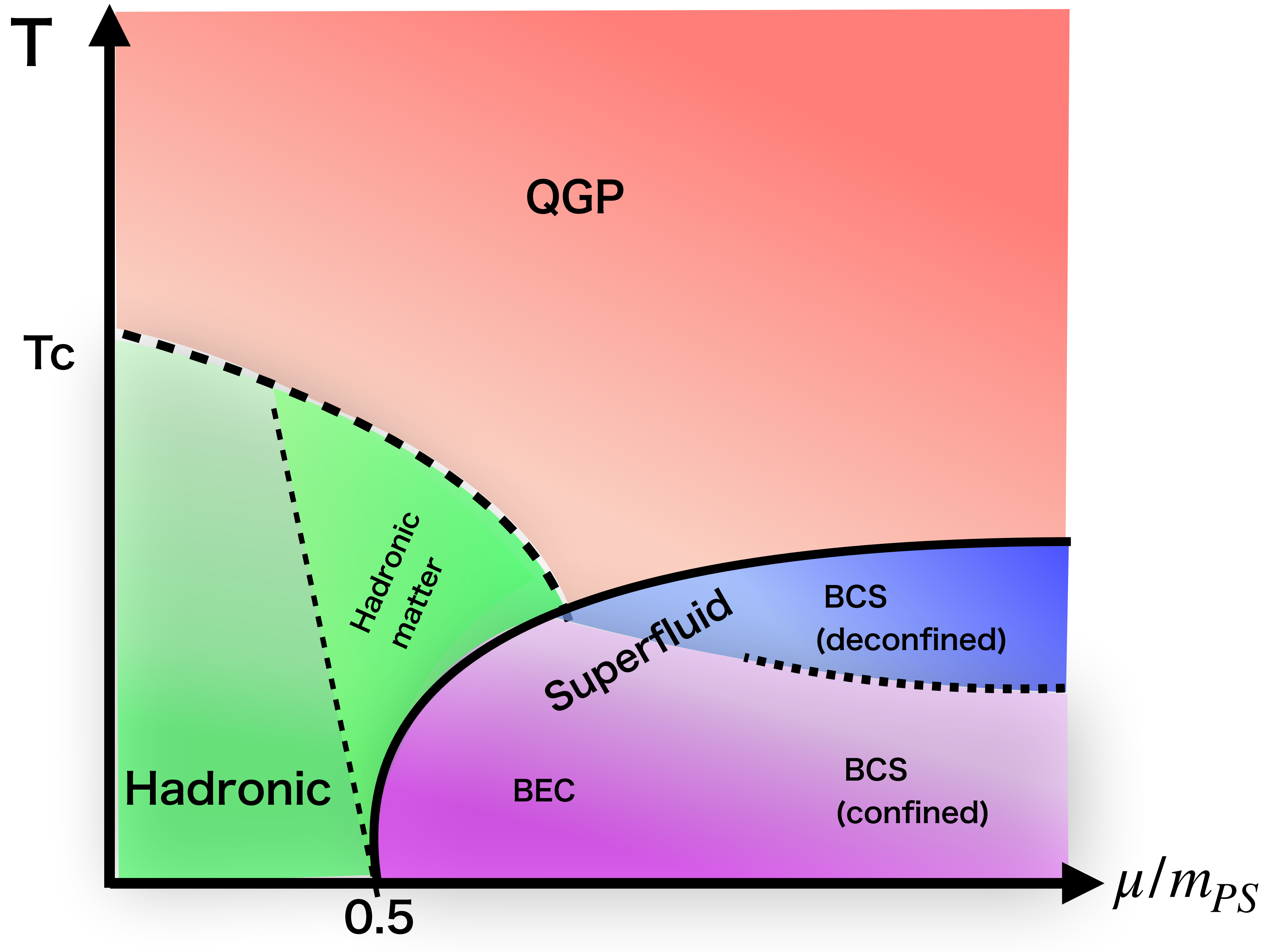
Phase diagram in T-μ plane

3 color QCD
expected phase diagram

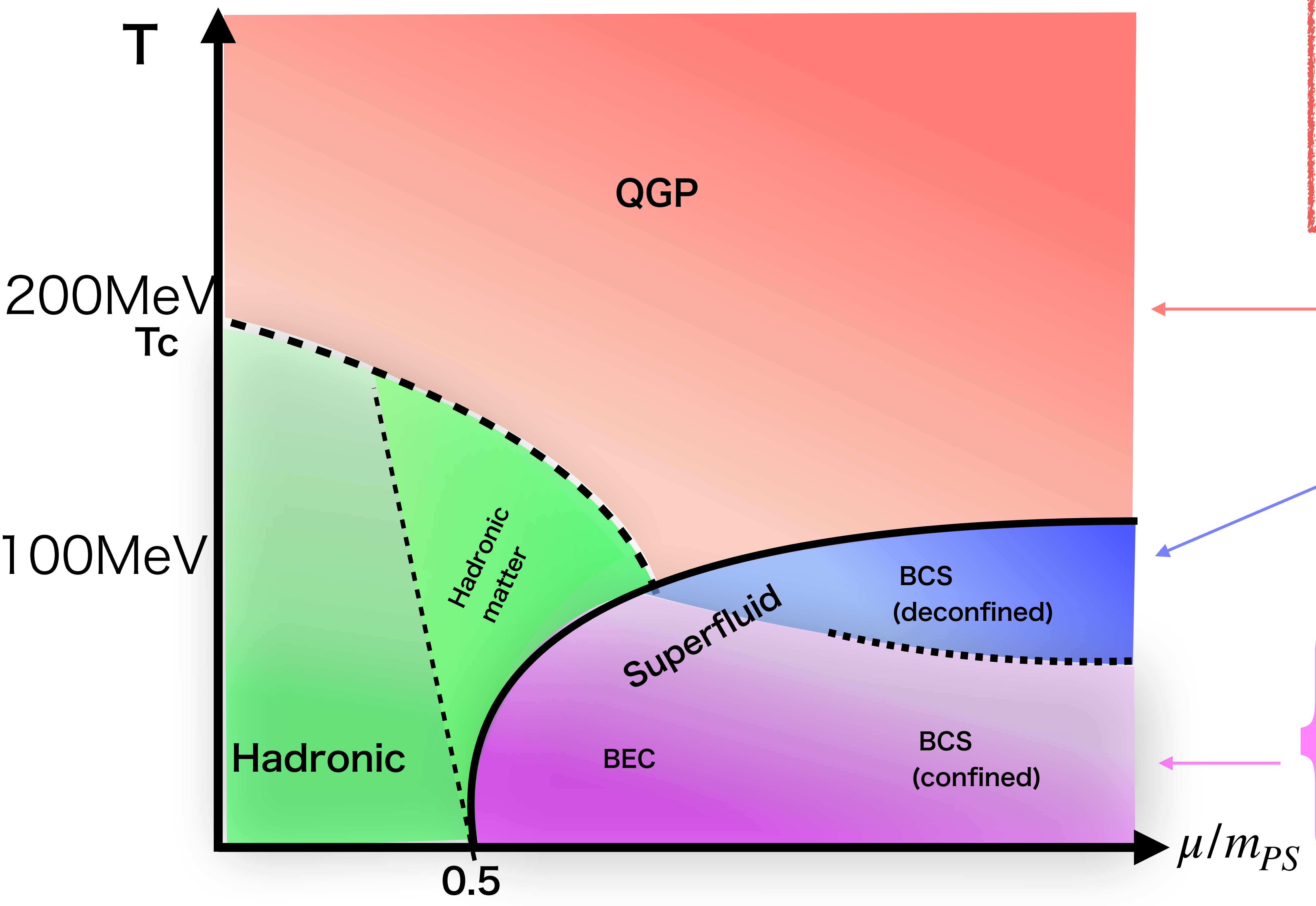
Fukushima-Hatsuda (2010)



2 color QCD
numerically determined phase diagram



Current status on 2color QCD phase diagram



At least 4 independent groups are studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

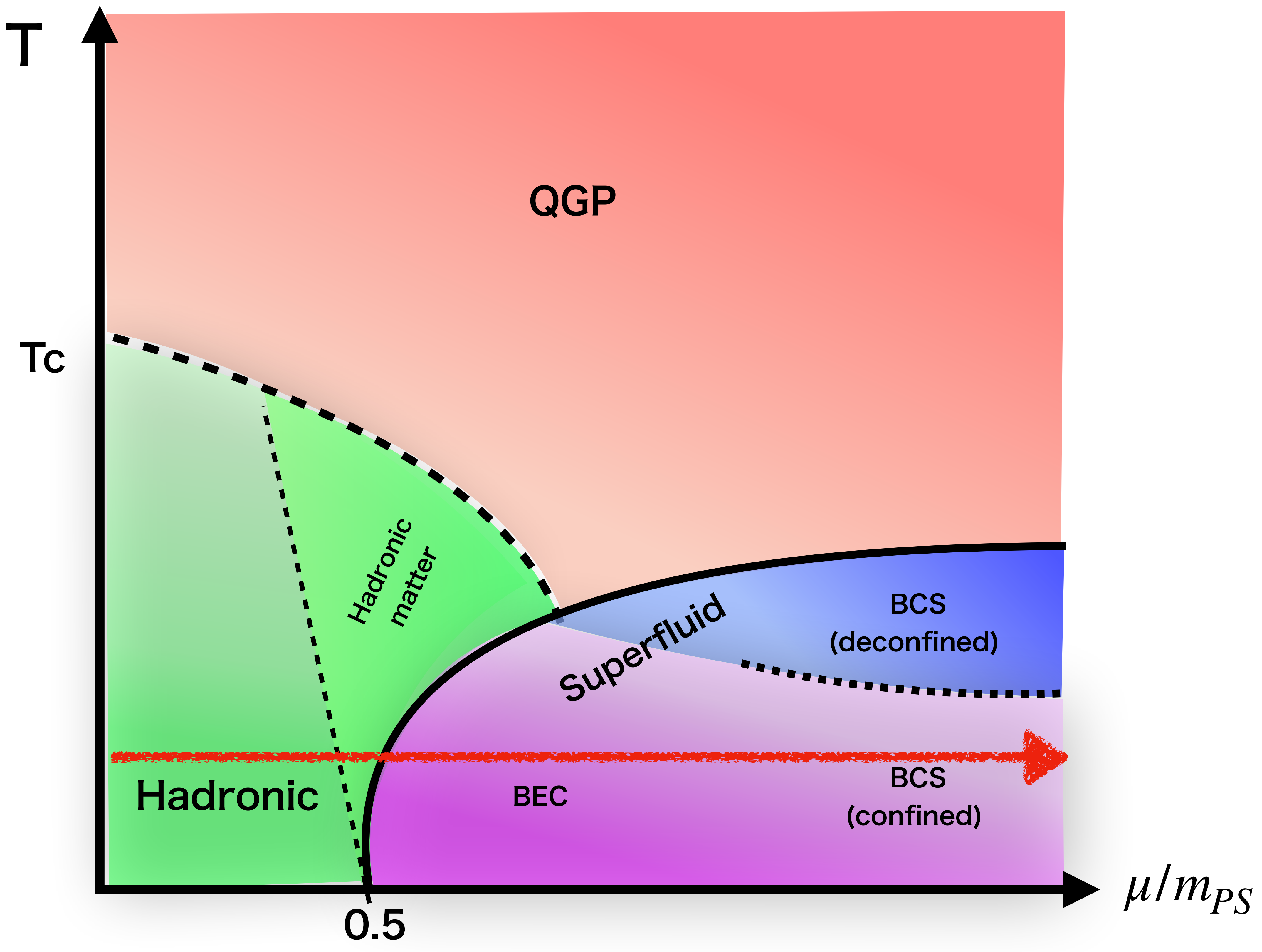
T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu.2019)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)
T=40 MeV (**confined, 2024**)

- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)≤ T_{SF} (superfluid/QGP) : constraint from ’t Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Our model: 2color 2flavor dense-QCD

2color QCD Phase diagram



40MeV (32^4 lattices)

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

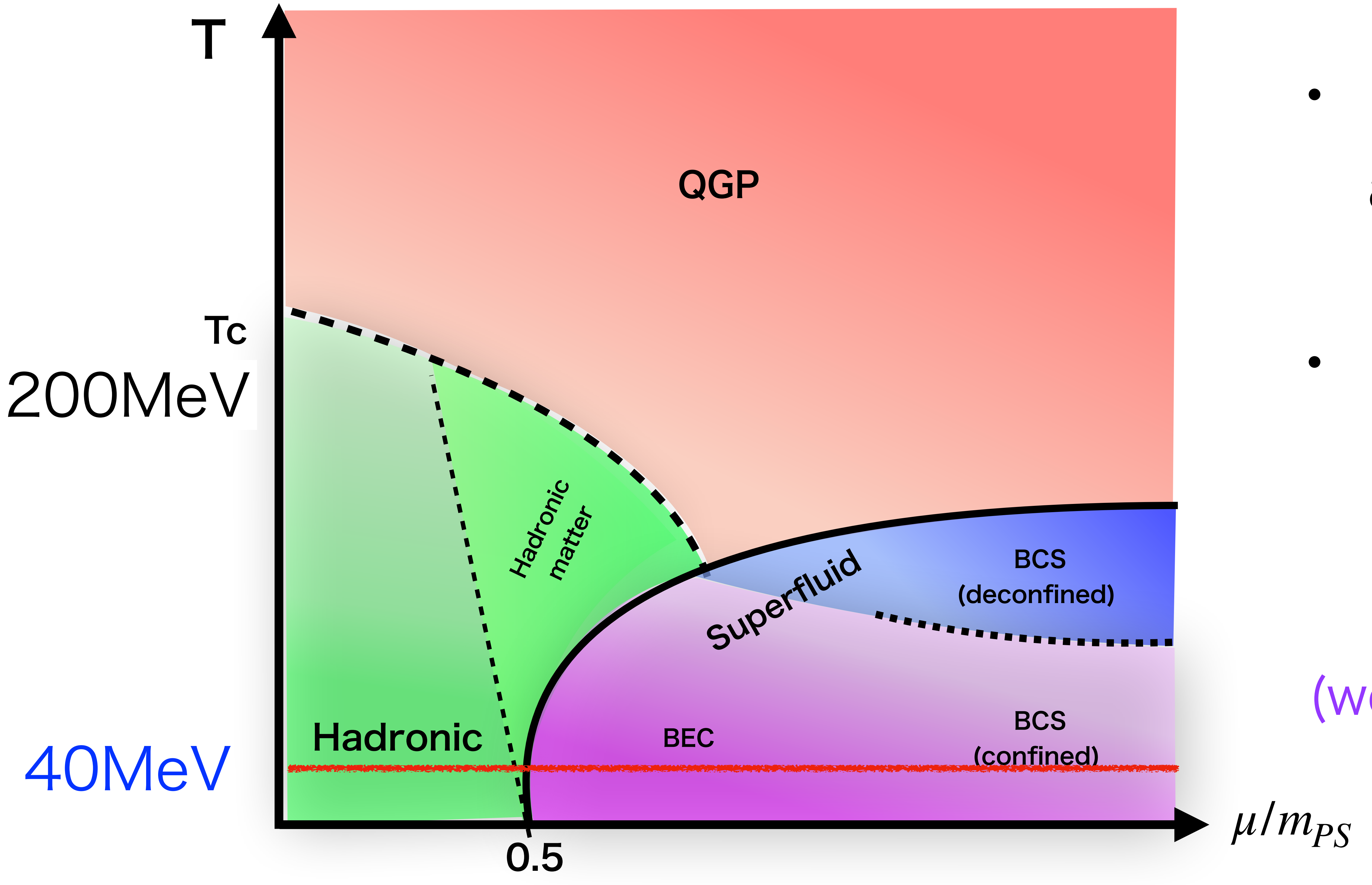
$$-J[\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T - \psi_2^T(C\gamma_5)\tau_2\psi_1]$$

diquark source term ($J \ll 1$)

- Physical obs. is calculated in $J \neq 0$ for each μ, and then take the $J \to 0$ limit

- In Superfluid phase, diquark cond.$\langle qq \rangle \neq 0$ (color-singlet in 2color QCD)

Three phases in low temperature regions

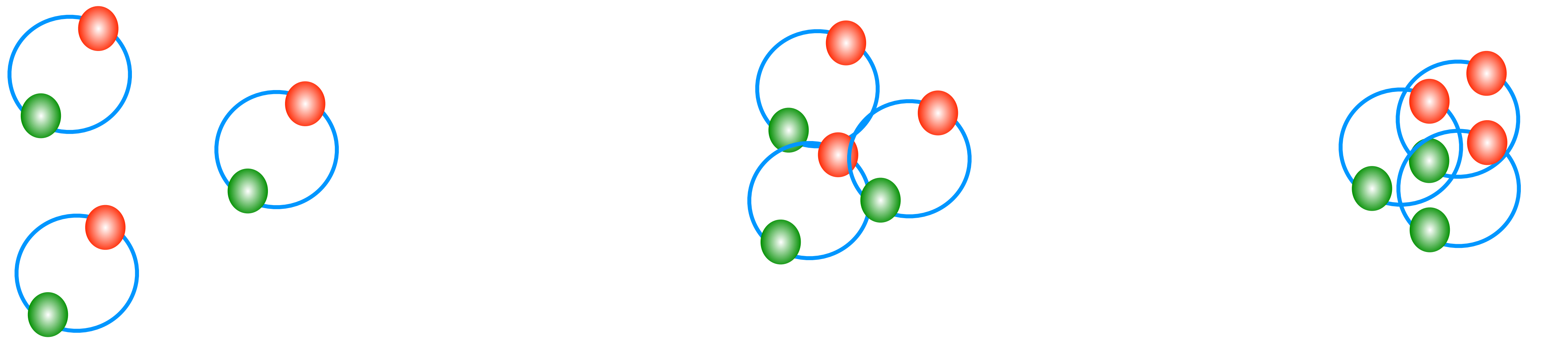


K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

- We mainly investigated T=40MeV
- Hadronic / Superfluidity phase transition around $\mu \approx m_{\mathrm{PS}}/2$
- BEC/BCS crossover in SF phase

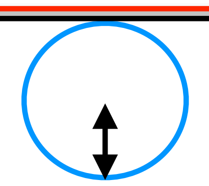
BEC phase
strong coupled
(well-described by ChPT)

BCS phase
weakly coupled



Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$
Quarks behave free particles

 Δ^{-1} Δ: gap energy

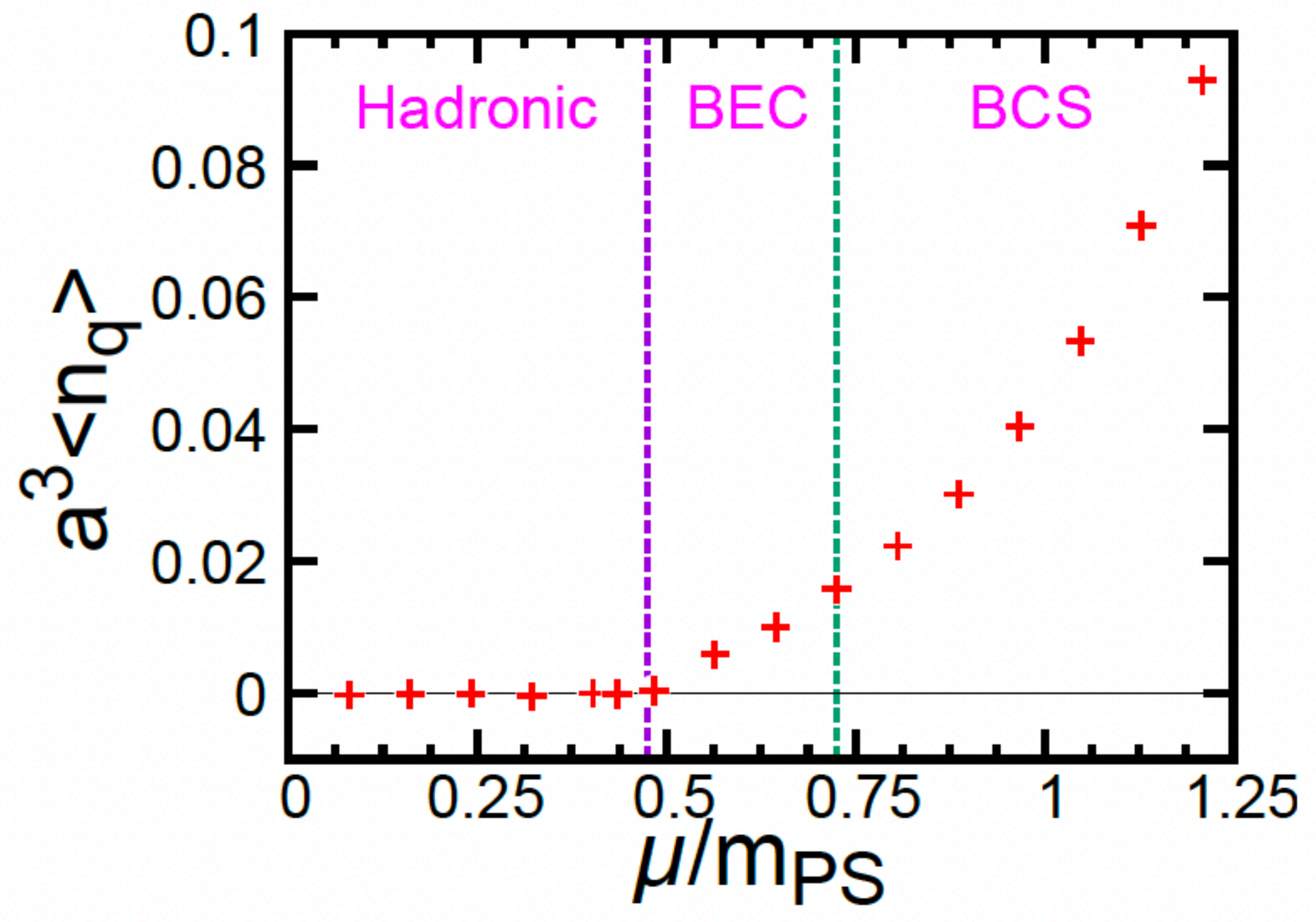
density

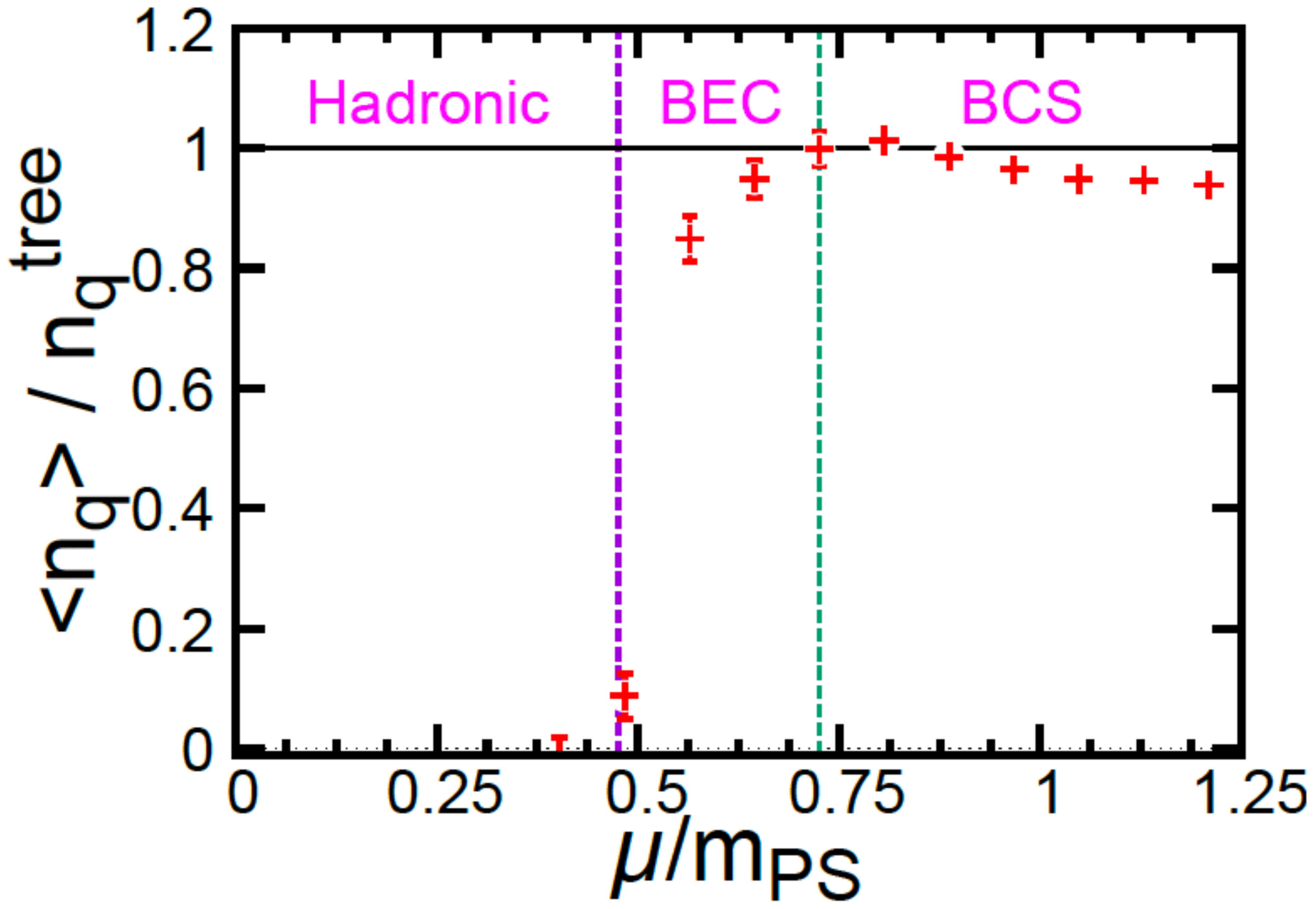
“definition” of BEC/BCS crossover

S. Hands, S. Kim and J.-I. Skullerud(2006)

- If $\langle n_q \rangle \approx n_q^{\text{tree}}$, then the region is identified as BCS phase

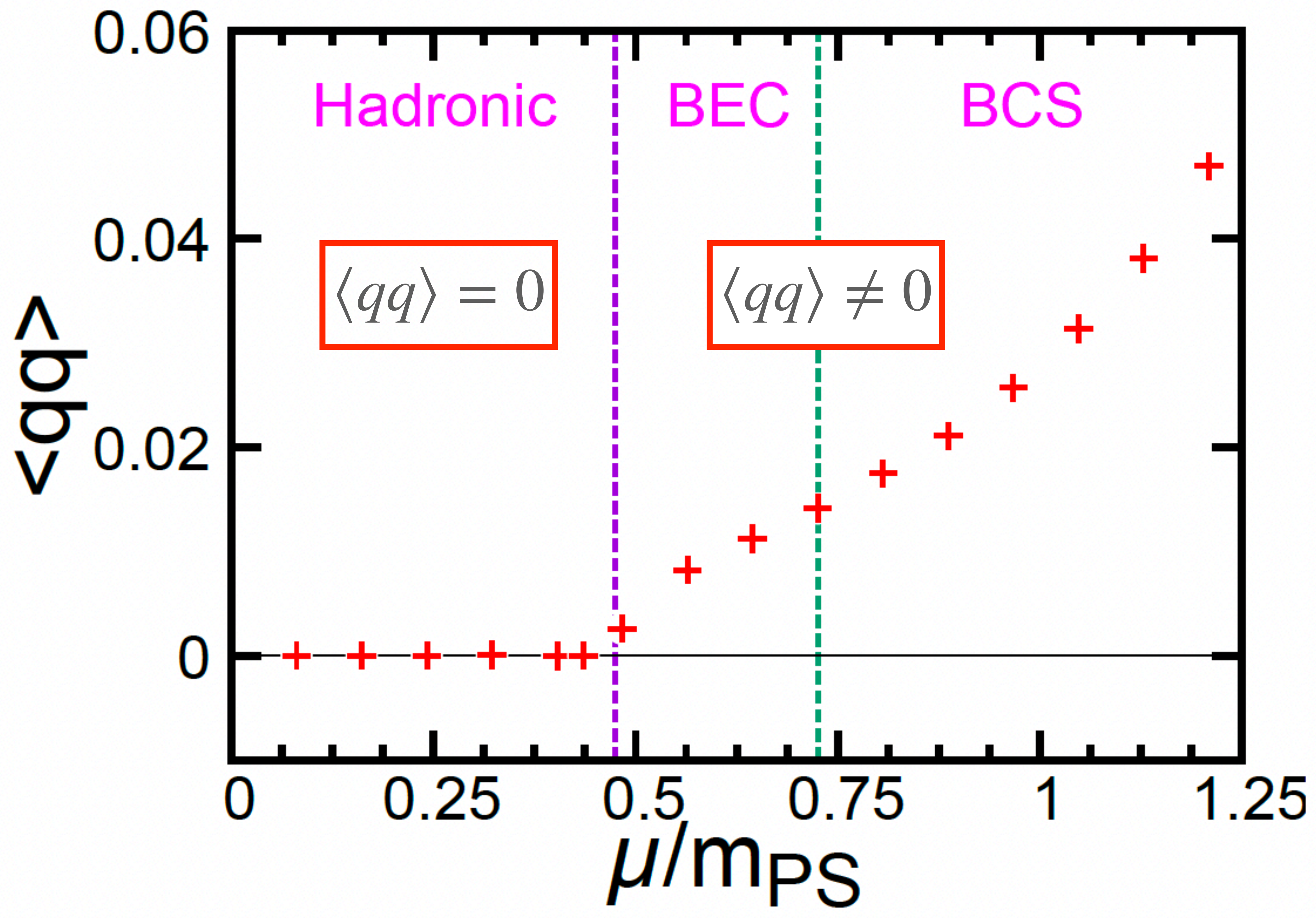
  $\langle n_q \rangle$ denotes a net quark number density



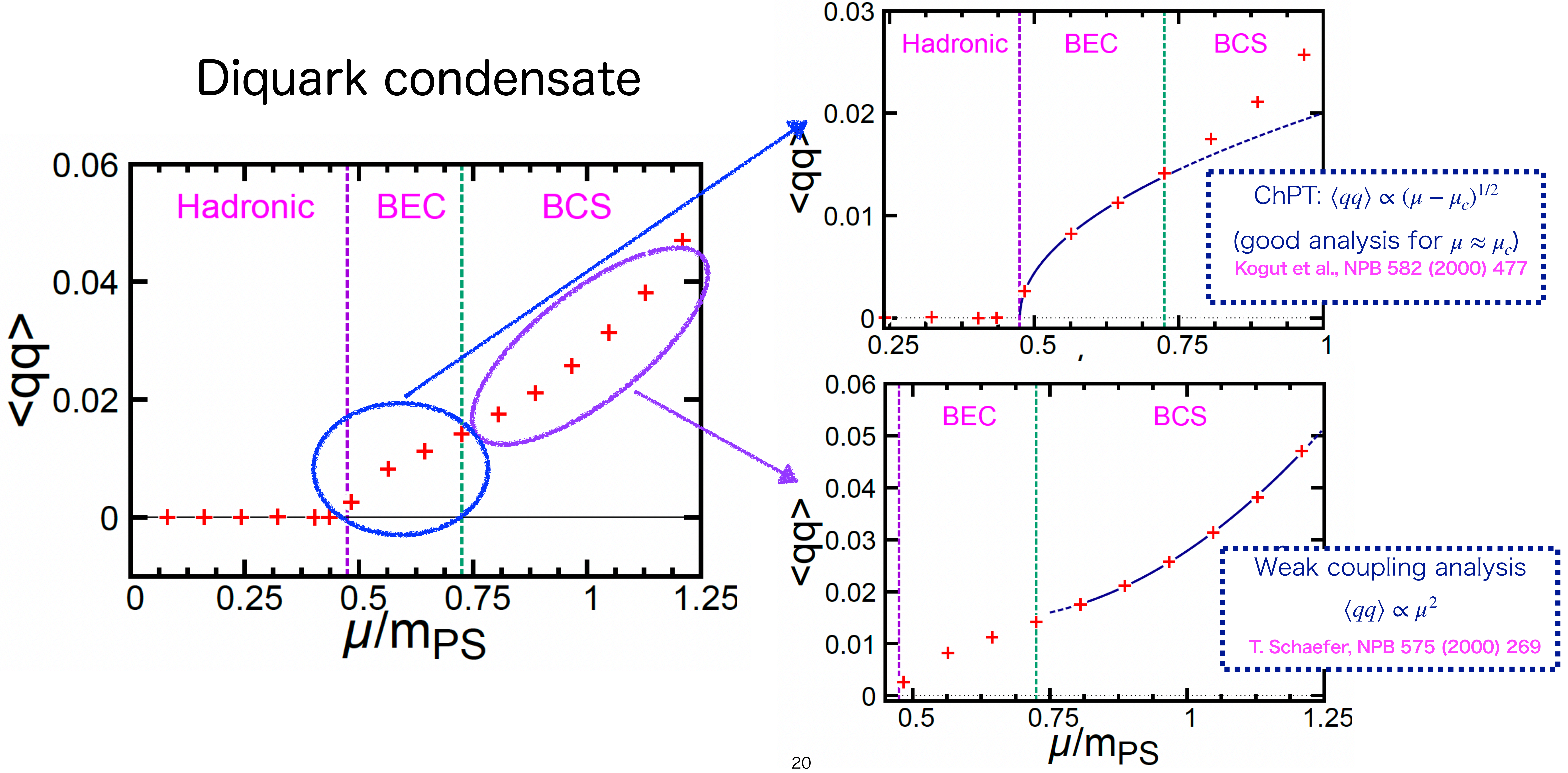


Order parameter of Superfluidity : $\langle qq \rangle$

Diquark condensate



Order parameter of Superfluidity : $\langle qq \rangle$



Equation of State

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, JHEP 10 (2024) 022

Equation of state

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle \frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle \frac{\partial S}{\partial \kappa}\rangle_{sub.} + \cancel{a\frac{\partial j}{\partial a}\langle \frac{\partial S}{\partial j}\rangle} \right)$

No renormalization for μ $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu,T} - \langle \cdot \rangle_{\mu=0,T}$

Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

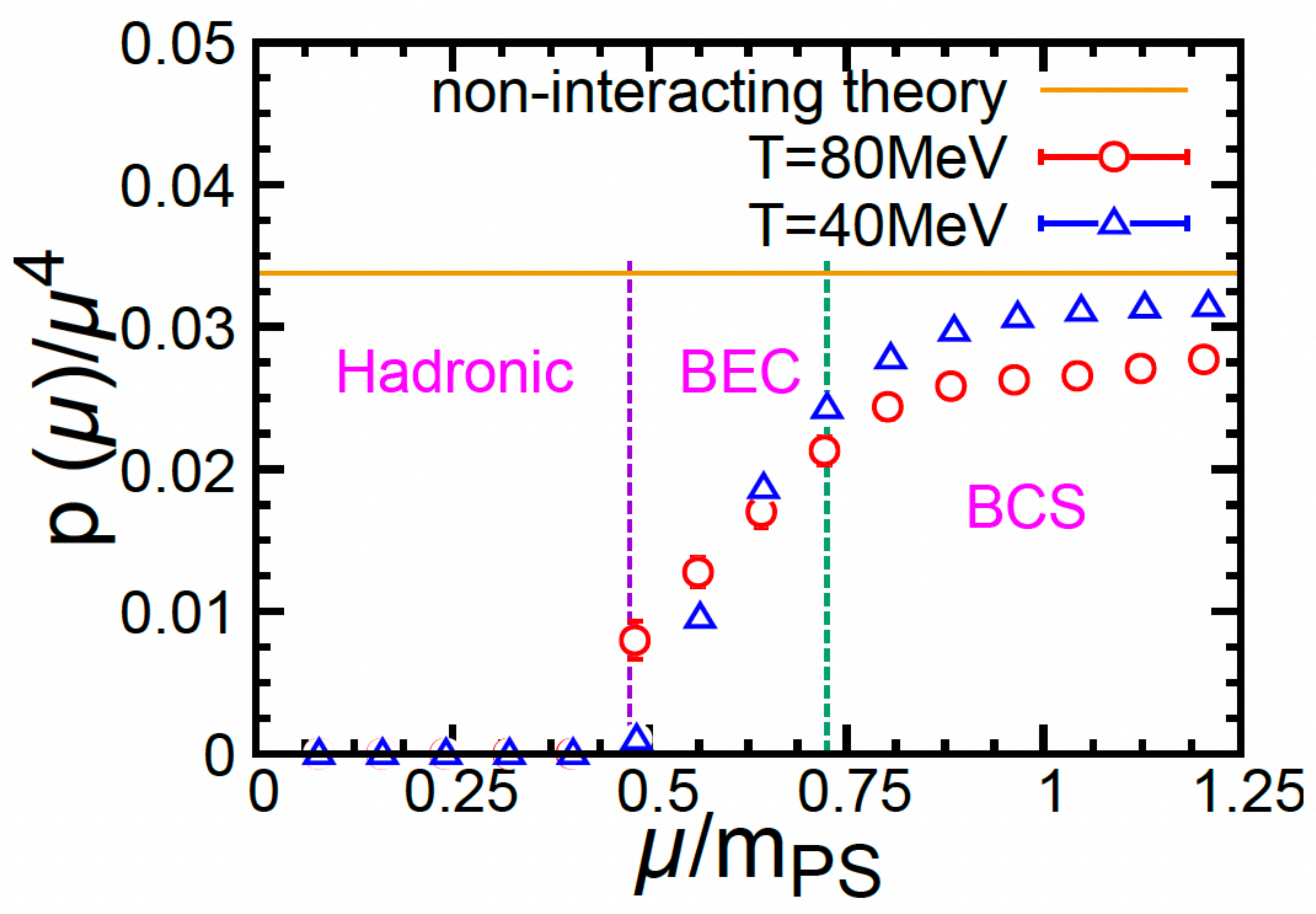
Early works for EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

Our work
Nonperturbatively calculate beta fn.

$$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$$

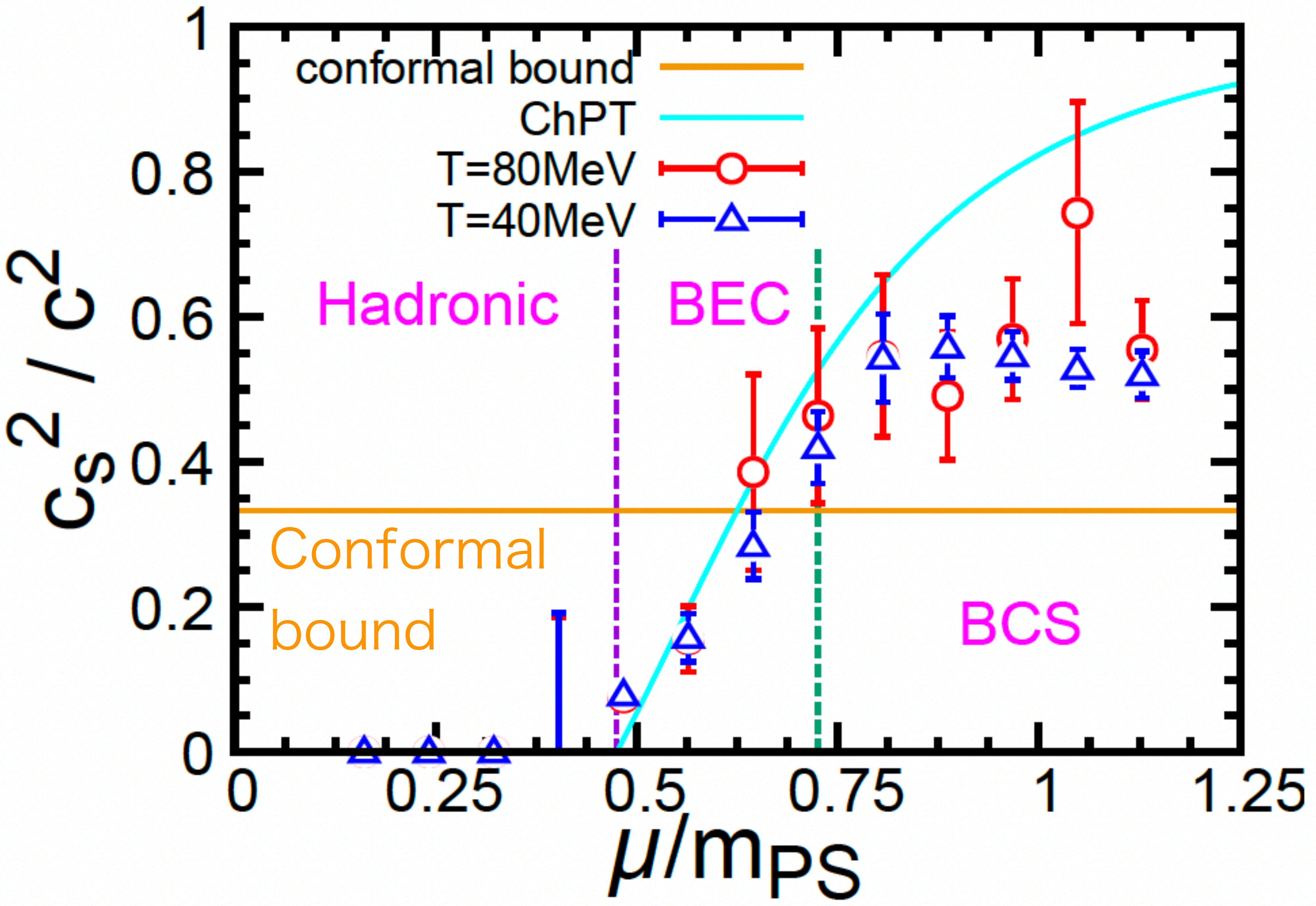
K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

EoS (pressure as a function of μ)



- p increases more rapidly near the critical point at lower-T
- In high-μ, the data approaches the Stefan-Boltzmann limit (=non-interacting reletivistic theory)
  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$
- Our largest data of p at T=40MeV reaches at 93% of p_{SB}

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!

- In BEC phase, result is consistent with ChPT

  Chiral Perturbation Theory (ChPT)

  $$c_s^2/c^2 = \frac{1 - \mu_c^4/\mu^4}{1 + 3\mu_c^4/\mu^4}$$: no free parameter!

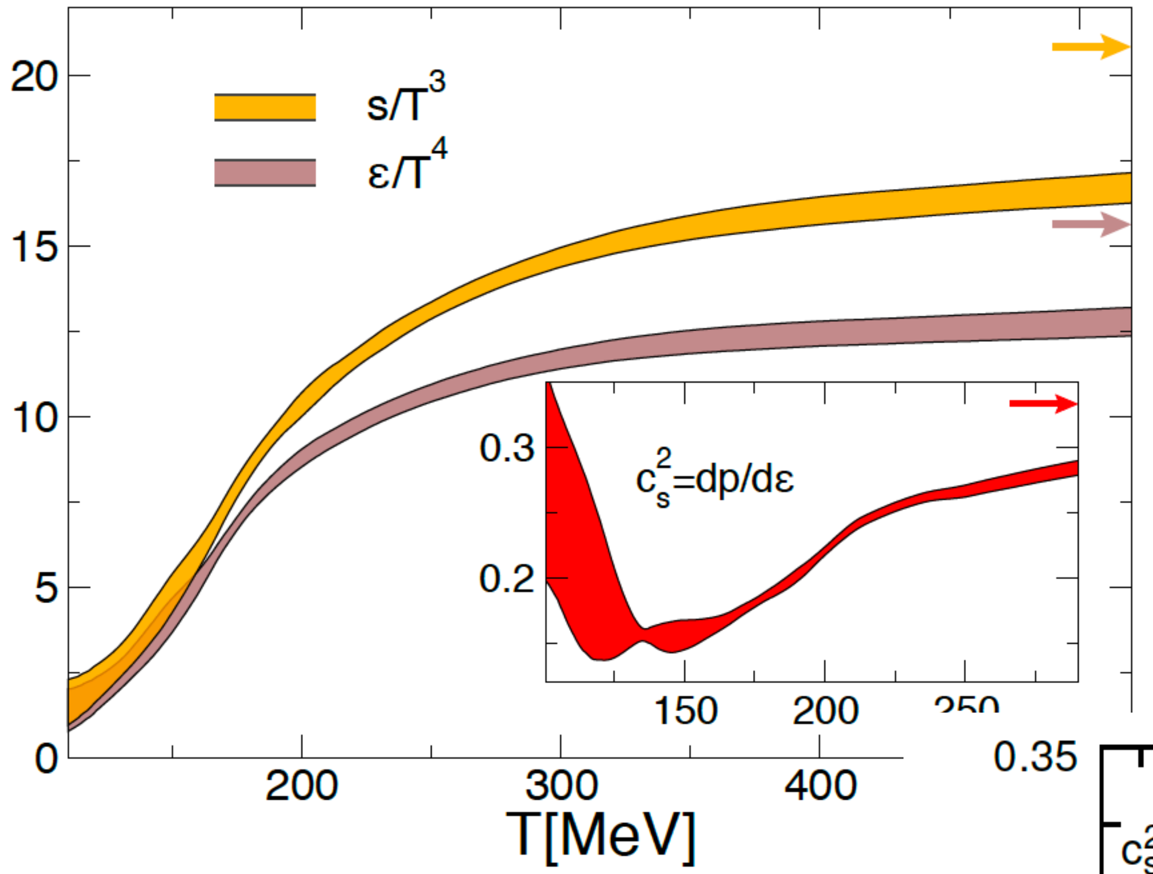
  Son and Stephanov (2001) : 3color QCD with isospin μ
  Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- c_s^2/c^2 exceeds the conformal bound

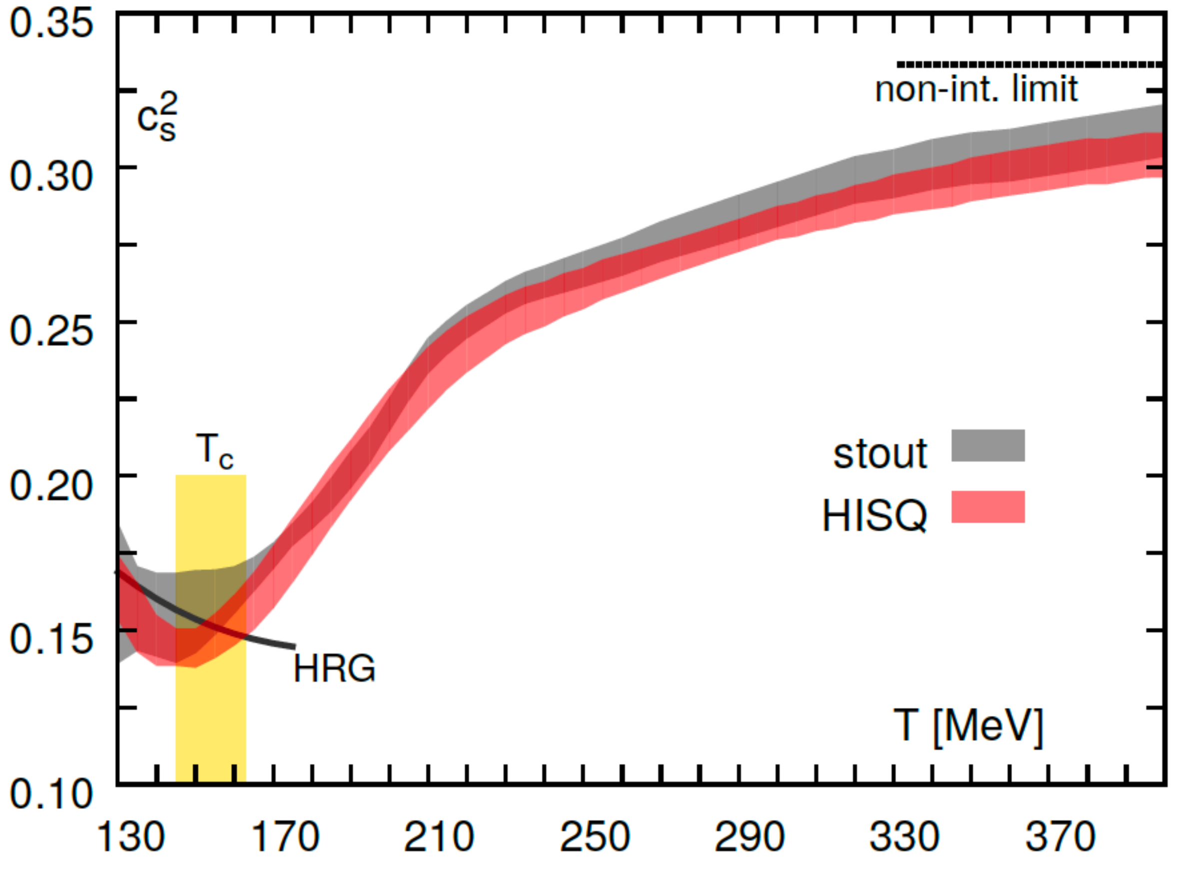
Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)

Borsanyi et al. (2013)



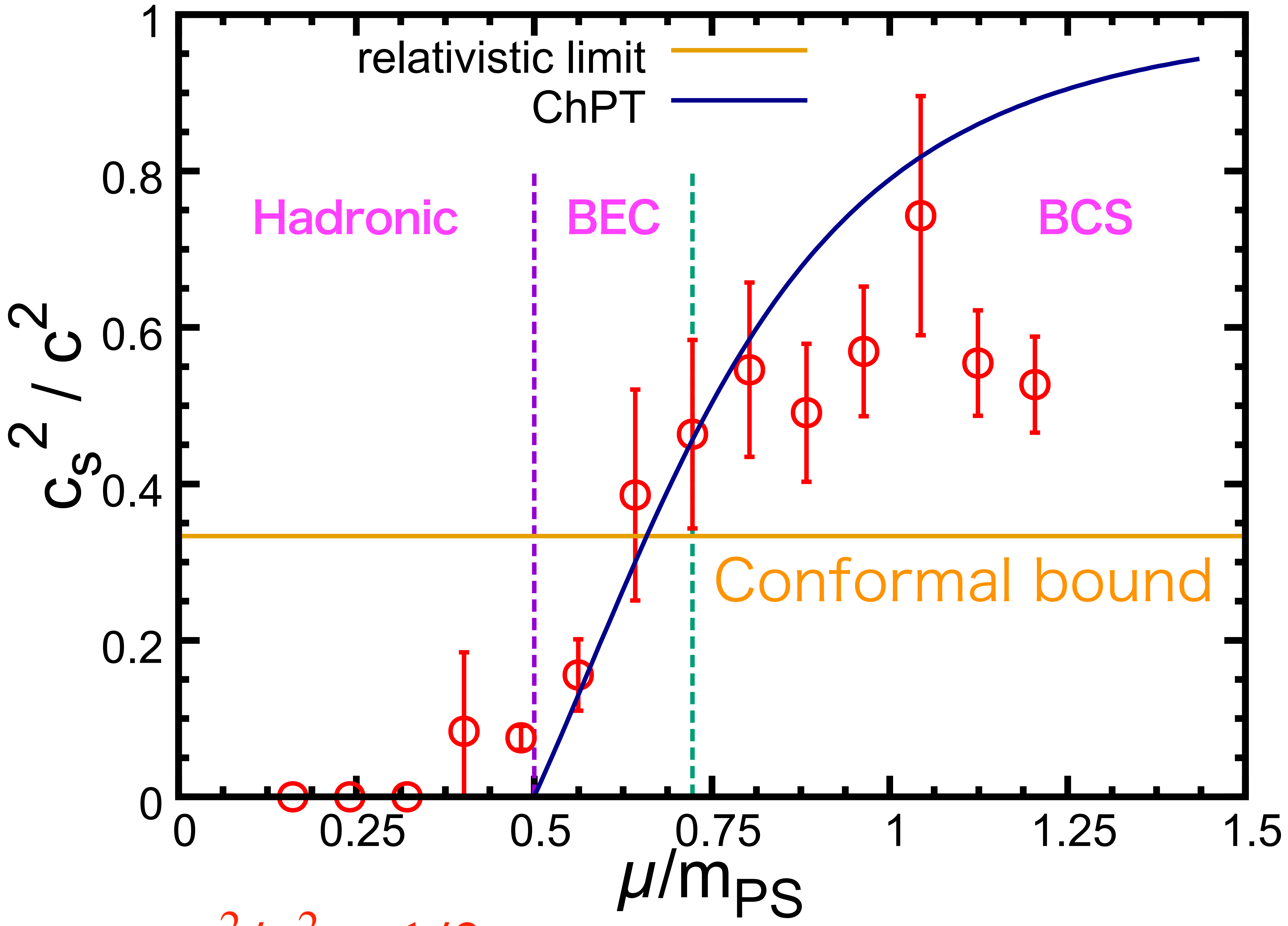
HotQCD (2014)



- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

Iida and El arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories

Conformal bound (Holography bound)

Conformal bound: A conjecture proposed by A.Cherman et al., 2009

"maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories"

A bound on the speed of sound from holography

Aleksey Cherman[*] and Thomas D. Cohen[†]
Center for Fundamental Physics, Department of Physics,
University of Maryland, College Park, MD 20742-4111

Abhinav Nellore[‡]
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

All Lattice Monte Carlo results have satisfied this bound for 40years!

Counterexamples of conformal bound

N=4 SYM at finite density

PHYSICAL REVIEW D **94**, 106008 (2016)

Breaking the sound barrier in holography

Carlos Hoyos,[1,*] Niko Jokela,[2,†] David Rodríguez Fernández,[1,‡] and Aleksi Vuorinen[2,§]
[1]*Department of Physics, Universidad de Oviedo, Avda. Calvo Sotelo 18, ES-33007 Oviedo, Spain*
[2]*Department of Physics and Helsinki Institute of Physics, P.O. Box 64, FI-00014 University of Helsinki, Finland*
(Received 20 September 2016; published 15 November 2016)

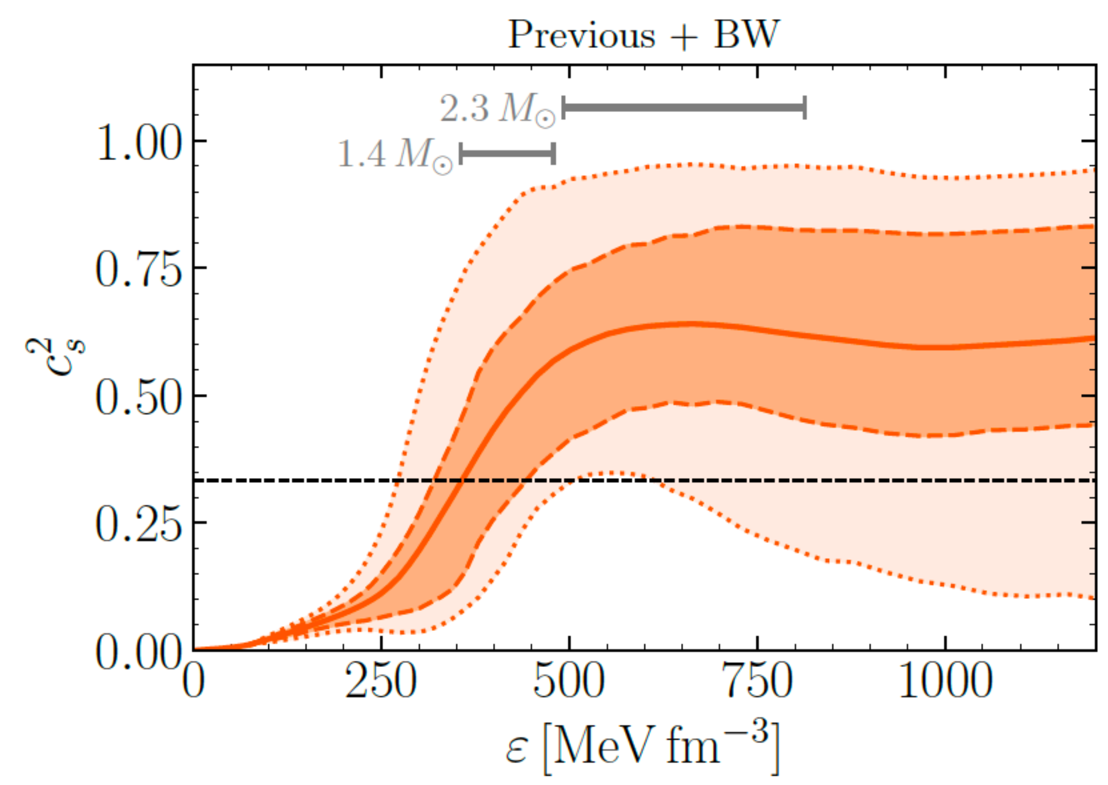
It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

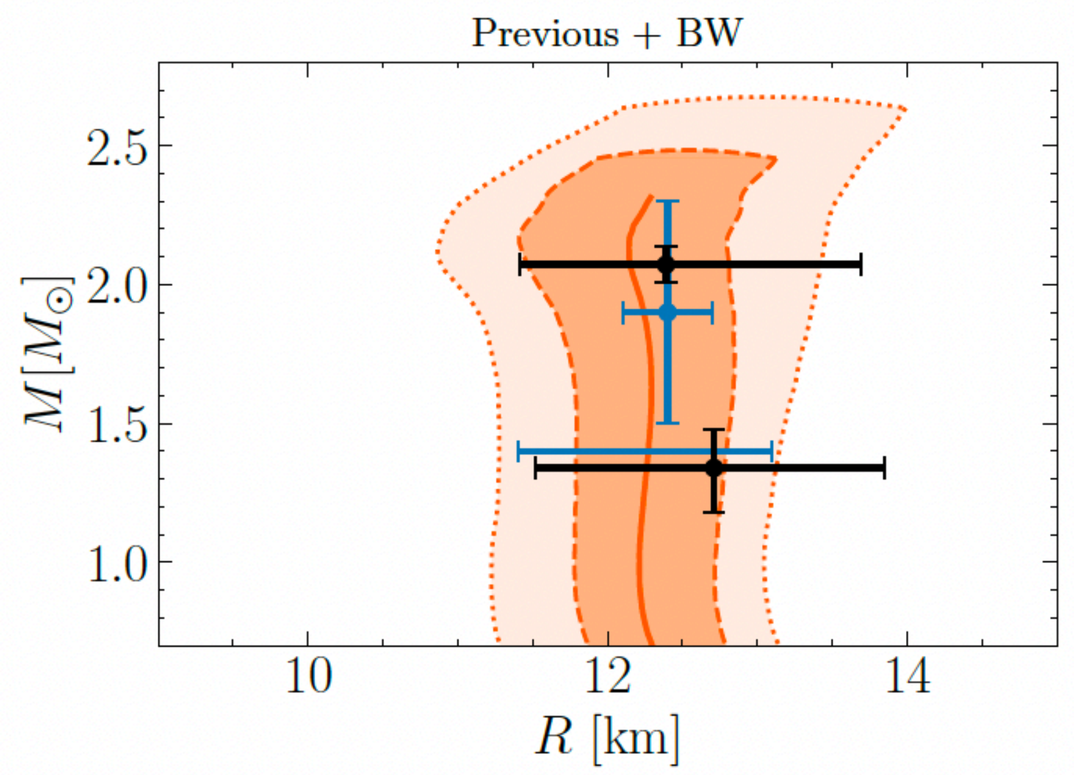
Evidence against a first-order phase transition in neutron star cores: impact of new data

Len Brandes,[*] Wolfram Weise,[†] and Norbert Kaiser[‡]
Technical University of Munich, TUM School of Natural Sciences, Physics Department, 85747 Garching, Germany
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

Bayesian analyses of recent observation data of neutron star





arXiv:2306.06218

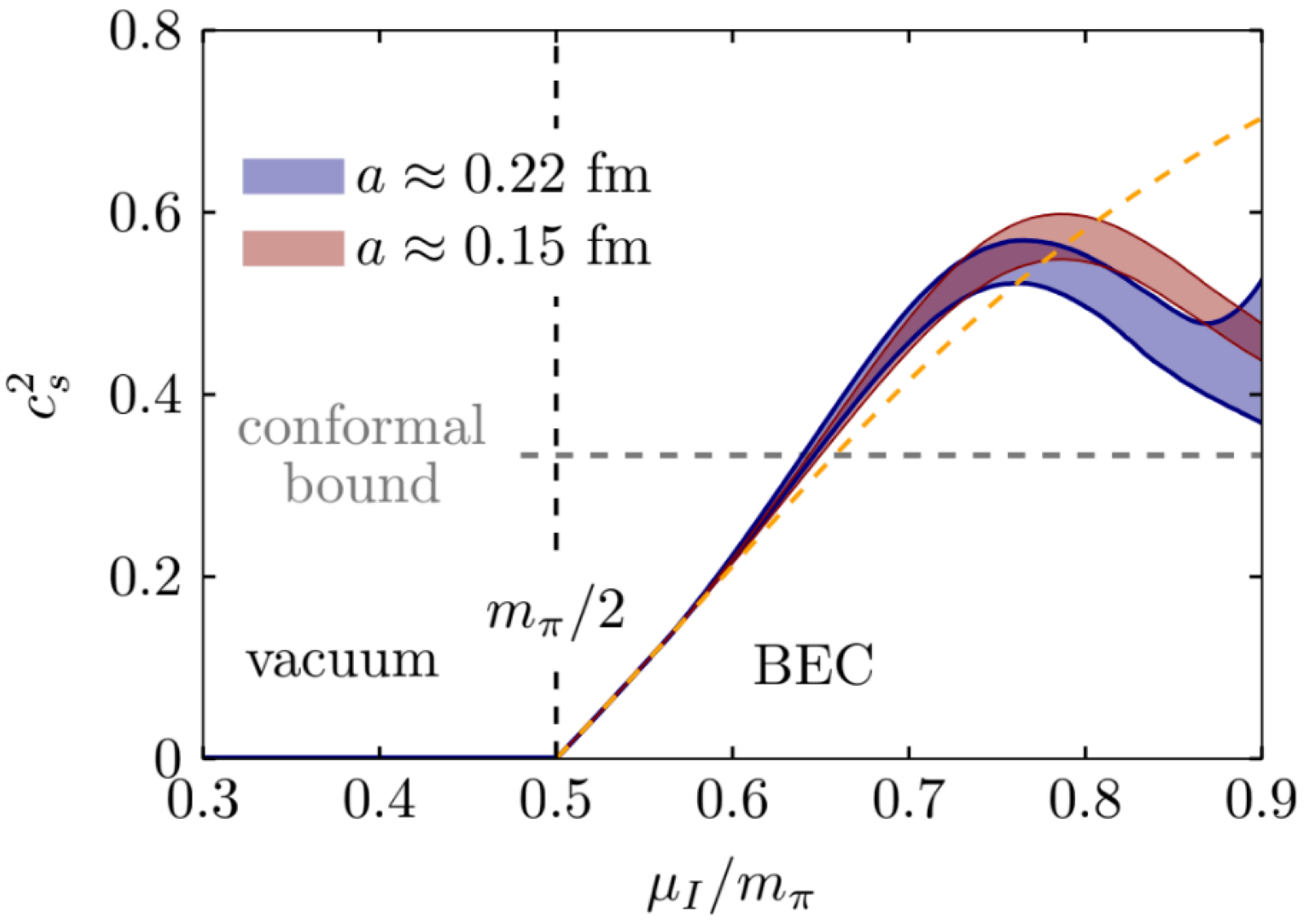
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

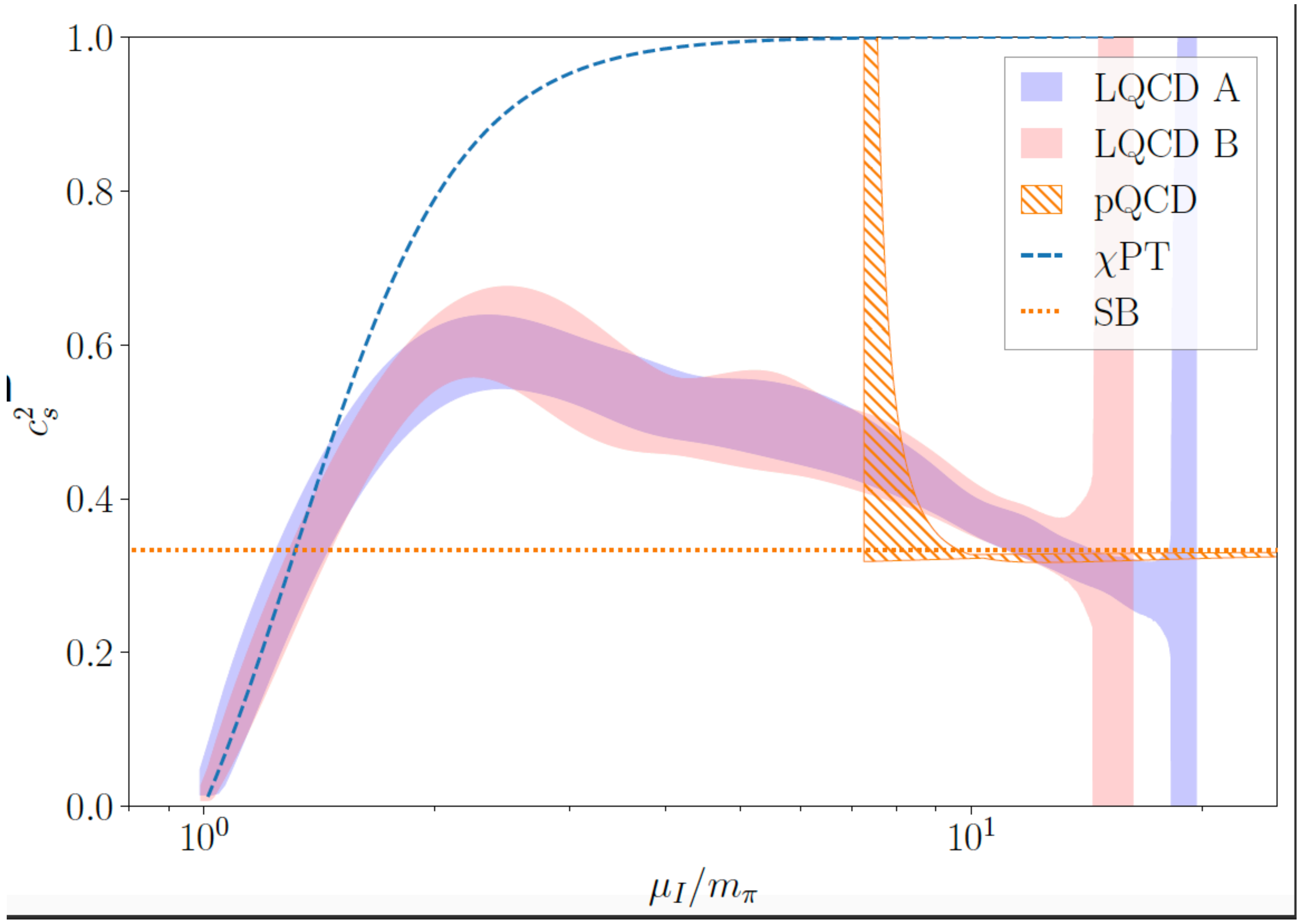
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014

Result with spline interpolation



New algorithm for n-point fn. calc.



Mass spectra

K. Iida, EI, K. Murakami, D. Suenaga, arXiv:2606.13974 (Yesterday!!)

Other interesting phenomena in superfluid phase

- At $\mu \neq 0$, (γ_5-)Hermiticity is lost.

  The pion is not guaranteed to be the lightest state.

- In the superfluid phase, the chiral condensate decreases.

  Some hadron masses should change. Brown-Rho (1991)

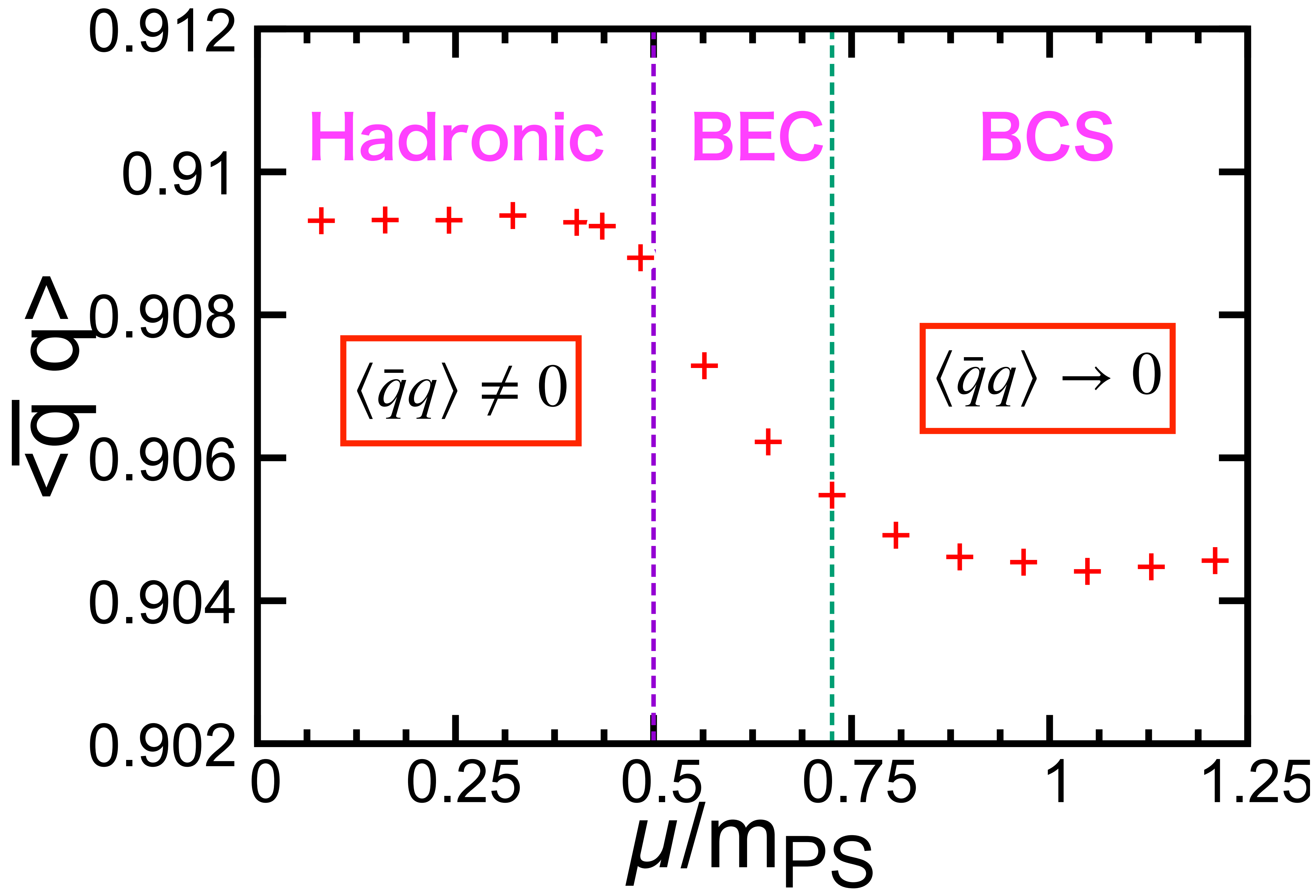
- Vector mesons becomes lighter? Hatsuda-Lee(1992)

  But, in QCD sum rule, ρ and ωmasses do not necessarily become lighter. (ϕ can be said to do so). S. Zschocke, O.P. Pavlenko, B. Kämpfer, Eur. Phys. J. A 15 (2002) 529–534

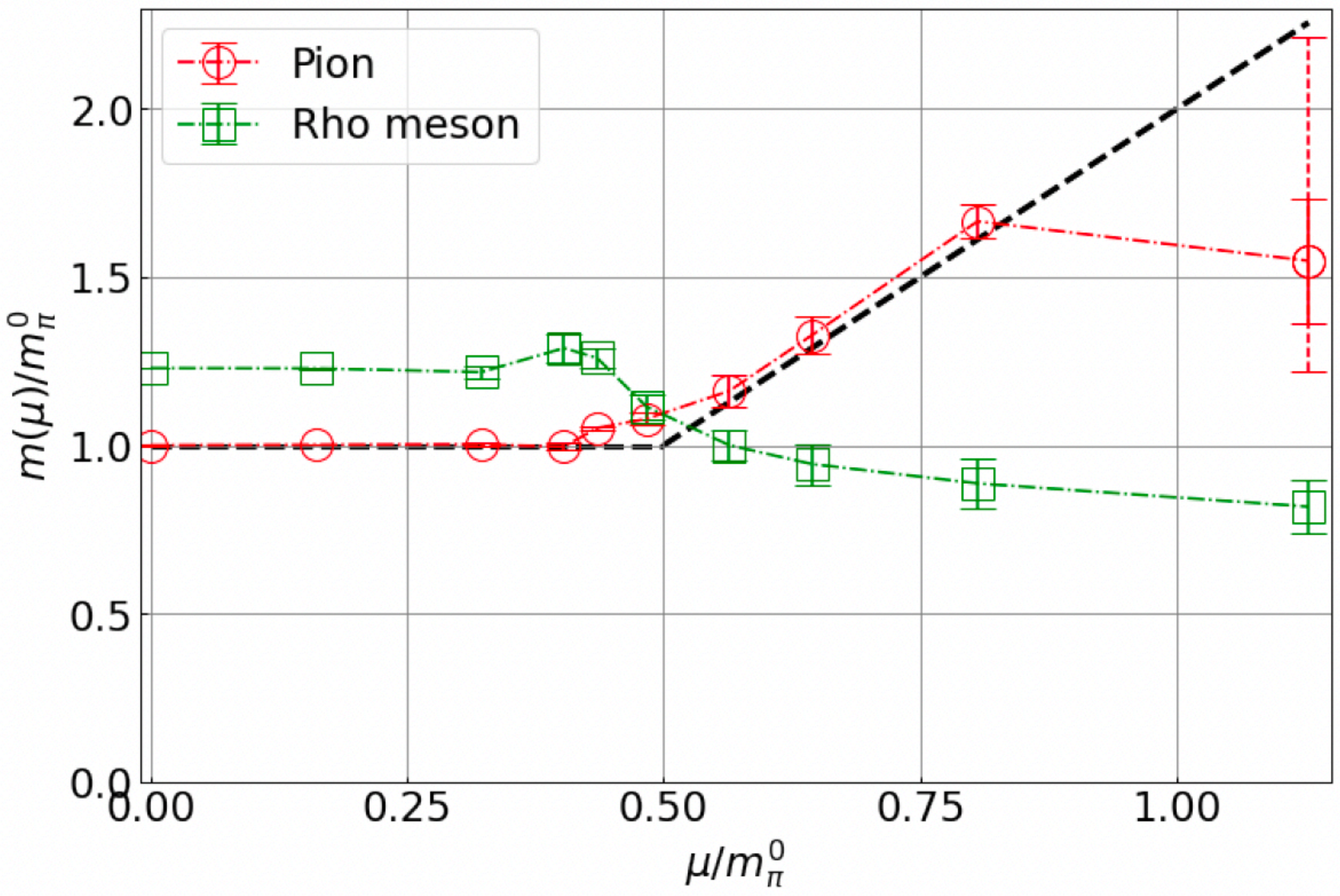
- Some experiments are ongoing E16 experiment at J-PARC

Our results (2color QCD)

Chiral condensate at T=40MeV



2color Mass spectrum

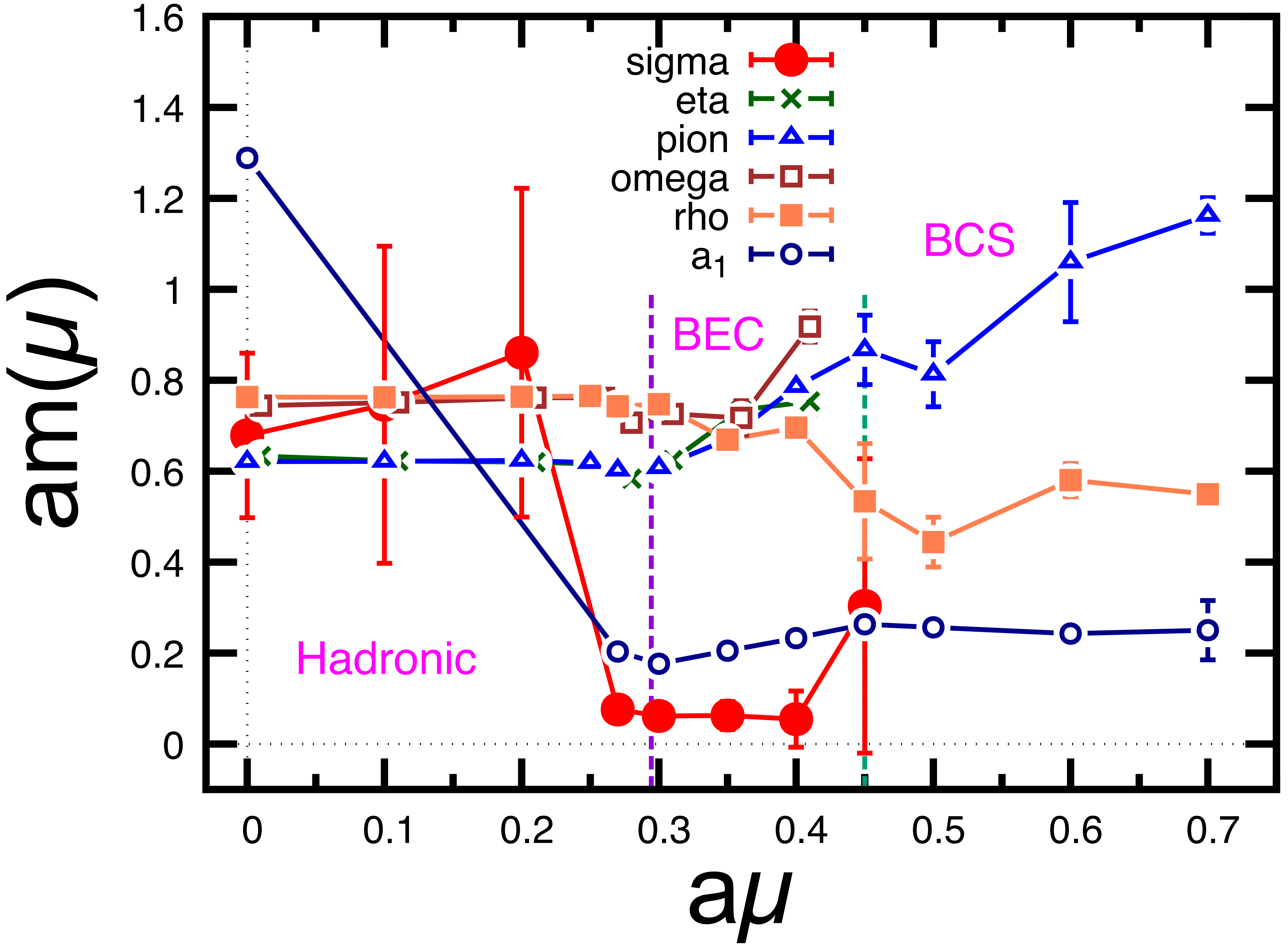


K.Murakami, D.Suenaga, K.Iida, EI,
PoS LATTICE2022 (2023) 154

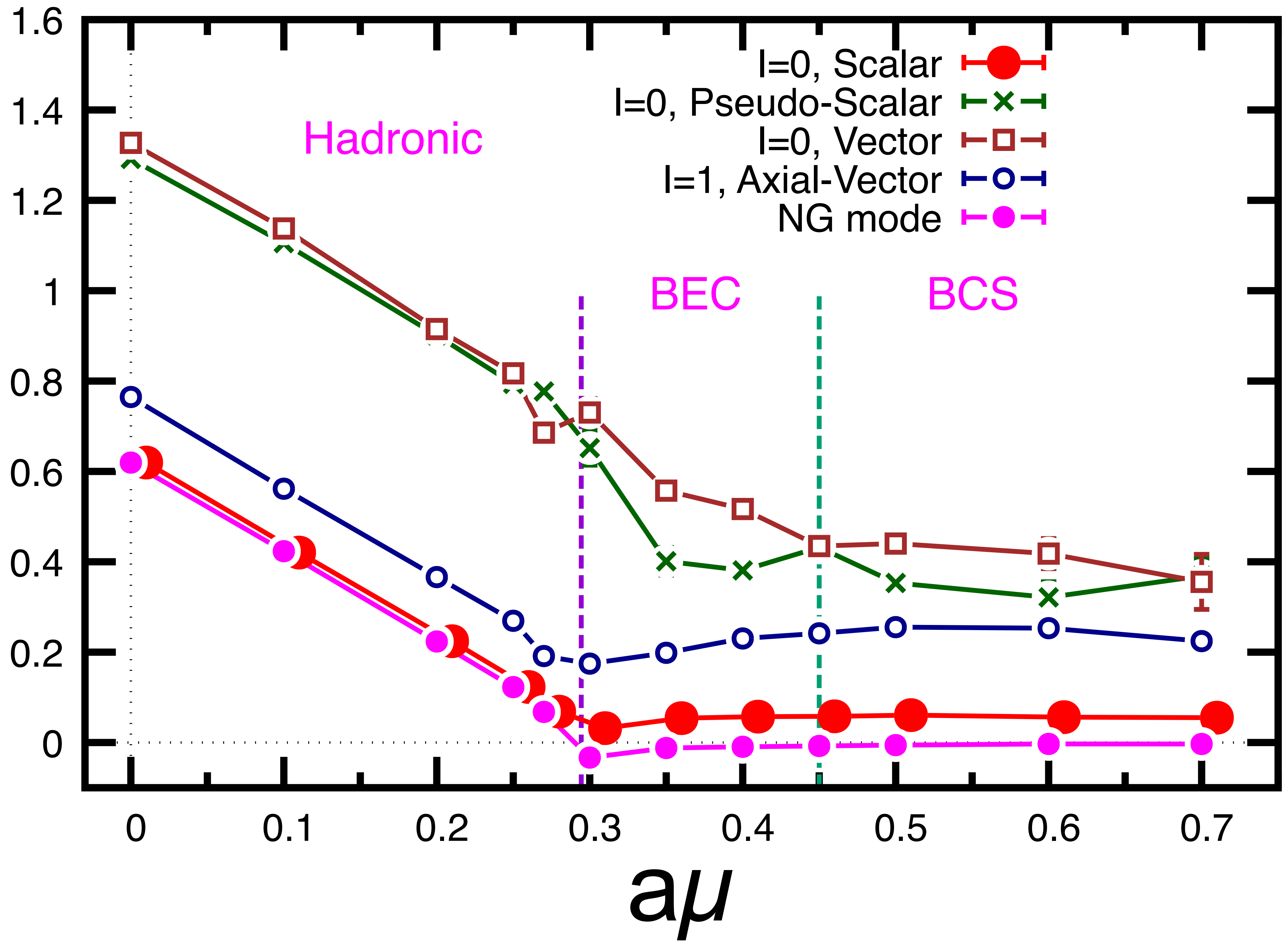
Summary of mass spectra

K. Iida, EI, K. Murakami, D. Suenaga, arXiv:2606.13974 (Yesterday!!)

Mesons



Diquarks



In Hadronic phase, $m_\pi \lesssim m_\eta < m_\sigma(\text{noisy}) < m_\rho \sim m_\omega \ll m_{a_1}$

(same with 3color QCD vacua)

In SF phase, $m_\sigma(\text{noisy}) < m_{a_1} < m_\rho < m_\pi \sim m_\eta(\text{noisy}) \ll m_\omega(\text{noisy})$

In both phases, $m_{NG} \lesssim m_{I=0,S} < m_{I=1,AV} < m_{I=0,PS} \lesssim m_{I=0,V}$

(In our notation($\psi_1^T K \Gamma \psi_2$, $K \equiv C\gamma_5\tau_2$), at $\mu = 0$

$\pi \leftrightarrow D_{I=0,S}$, $\rho \leftrightarrow D_{I=1,AV}$ are same multiplets)

Summary

Summary and future work

- Lattice numerical simulation for QCD-like theory w/o the sign problem has been ongoing
  2color finite-density QCD and 3color w/ isospin chemical potential are doable using exact algorithm

- Sound velocity exceeds the conformal bound in finite-μ QCD-like theory
  (All Lattice Monte Carlo results have satisfied $c_s^2/c^2 \leq 1/3$ for 40years!)

- Large sound velocity suggests stiffer-than-conventional picture of QCD matter (Is it real in neutron star?)

- The ordering of the mass spectra is also changed in the superfluid phase.
  This may change the nature of the NN potential.

- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), negative trace anomaly? (McLerran, Fukushima, Fujimoto et al.), others?
  - Effective model analyses combined with the lattice results are also ongoing

- Ongoing Lattice studies:
  => whole region of QCD phase diagram (extend to finite-T)
  => extended HAL QCD method in finite density
  => Find an explicit evidence of superfluidity (Fermi surface...)

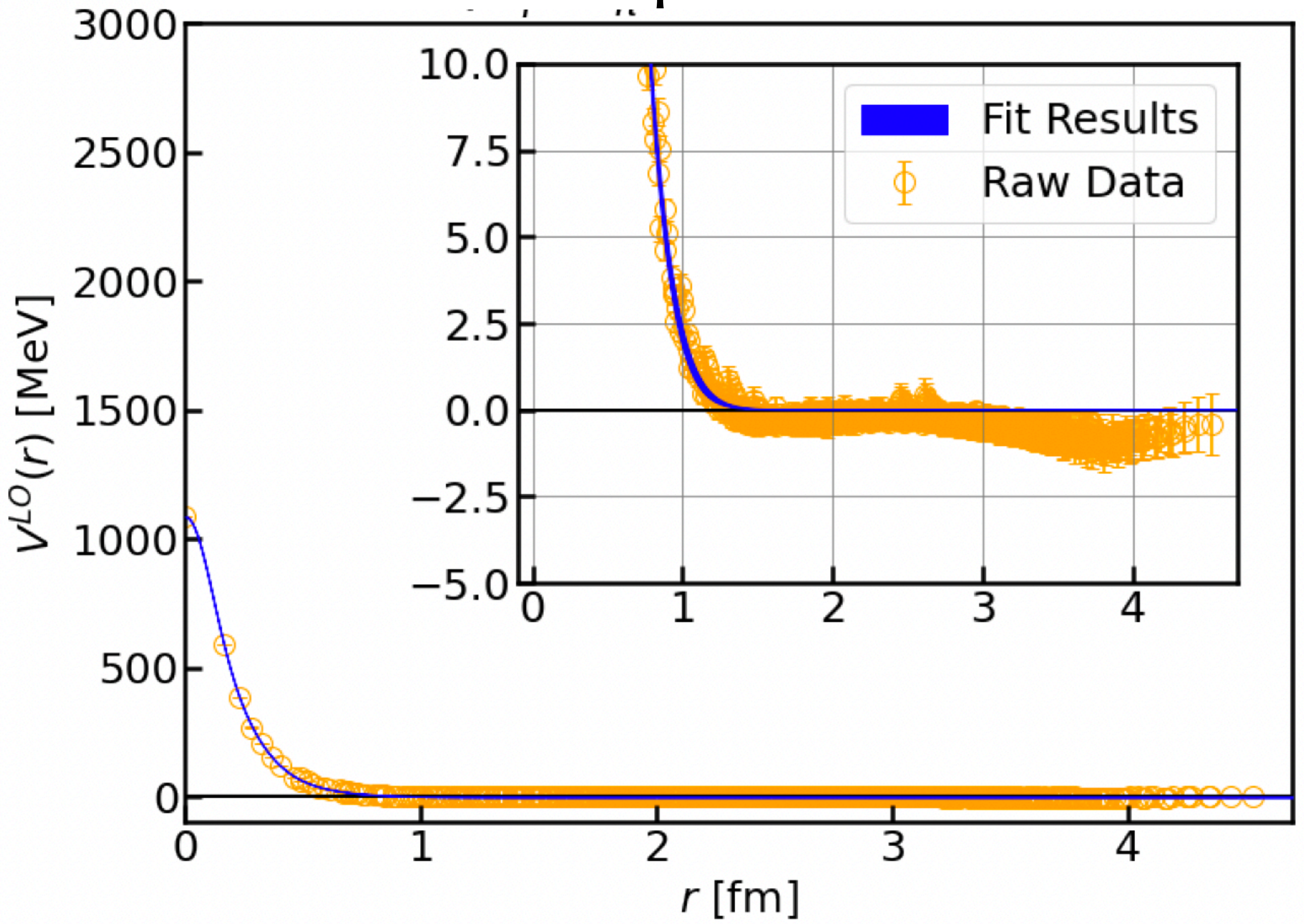
cf.) D.Suenaga, Y.Fujimoto
Minato and Fukushima...

Hadron potential by HAL QCD method

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

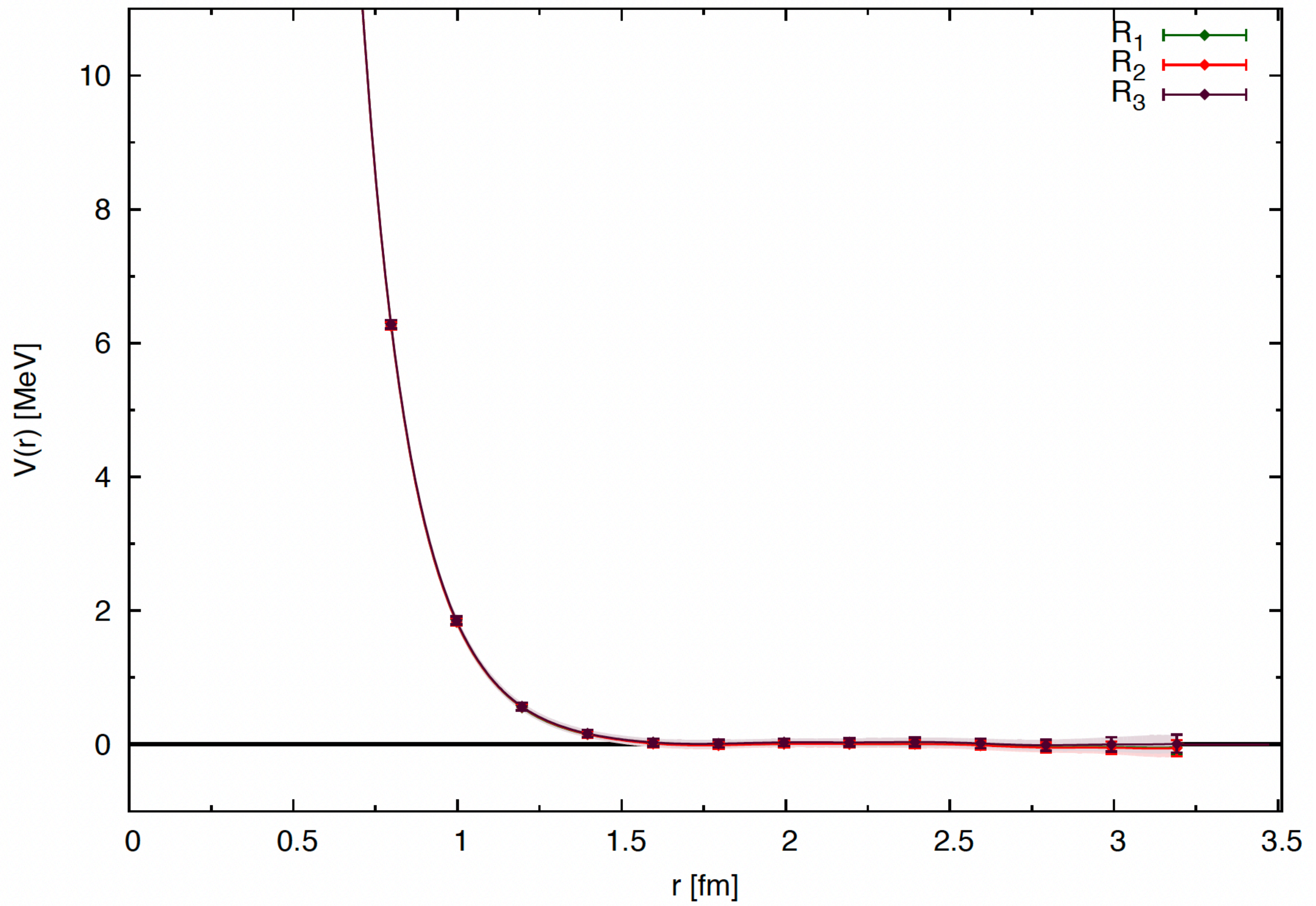
- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential
in hadronic phase of 2color QCD



K.Murakami, K.Iida, EI, JHEP 11 (2023) 231

I=2 $\pi\pi$ potential of 3color QCD

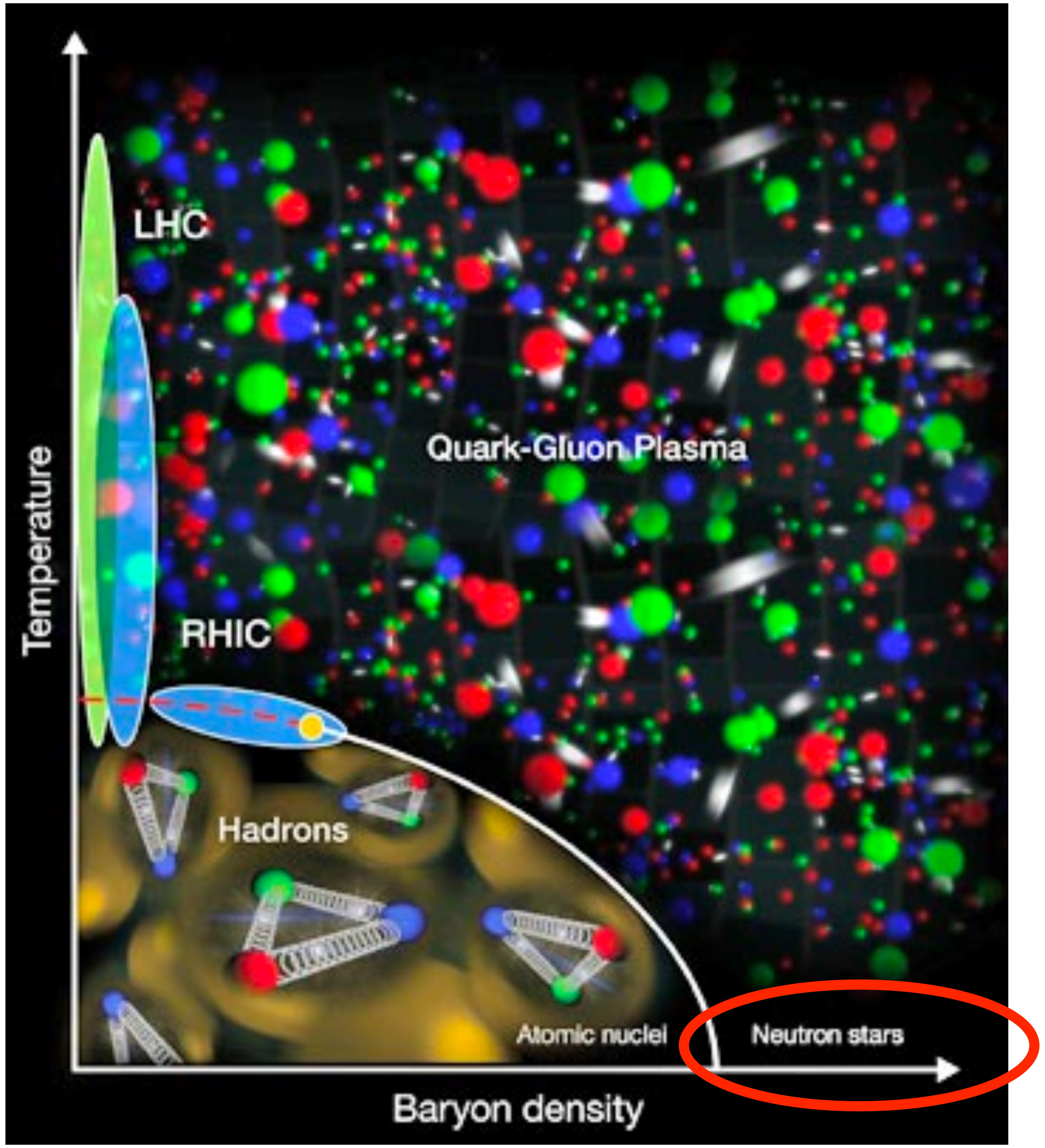


T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015 backup

To Neutron star (non-zero μ QCD)

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

Neutron star ~ Finite density QCD

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- μ : quark chemical potential

  ~ density of matter

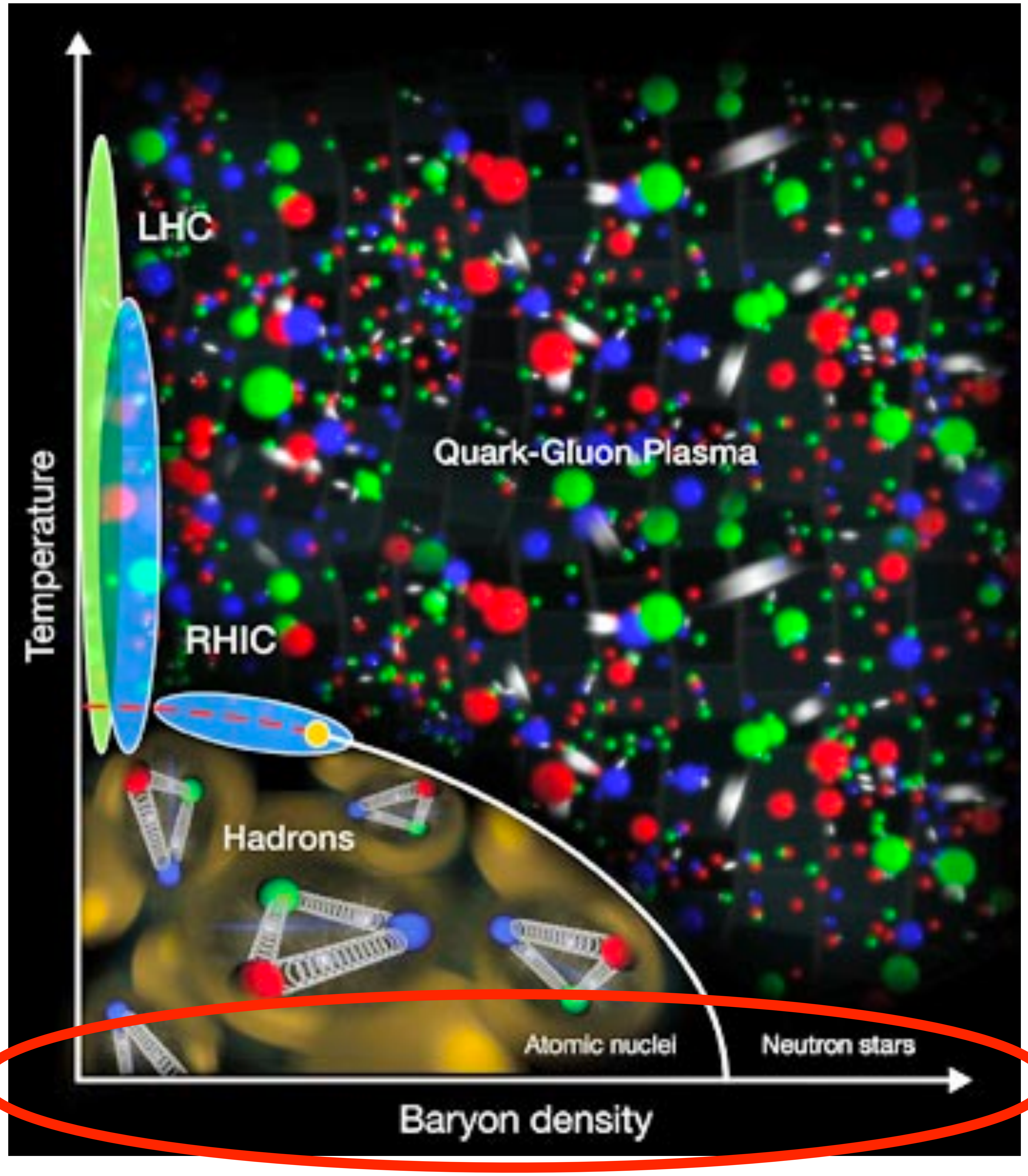
  ~ baryon chemical potential

  $(\mu_B = N_c \mu)$

Non-zero μ QCD is impossible in MC

expected QCD phase diagram

T



$\propto \mu$

©BNL/RHIC

Neutron star ~ Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem

K.Nagata, Finite-density lattice QCD and sign problem: Current status and open problems Prog.Part.Nucl.Phys. 127 (2022) 103991

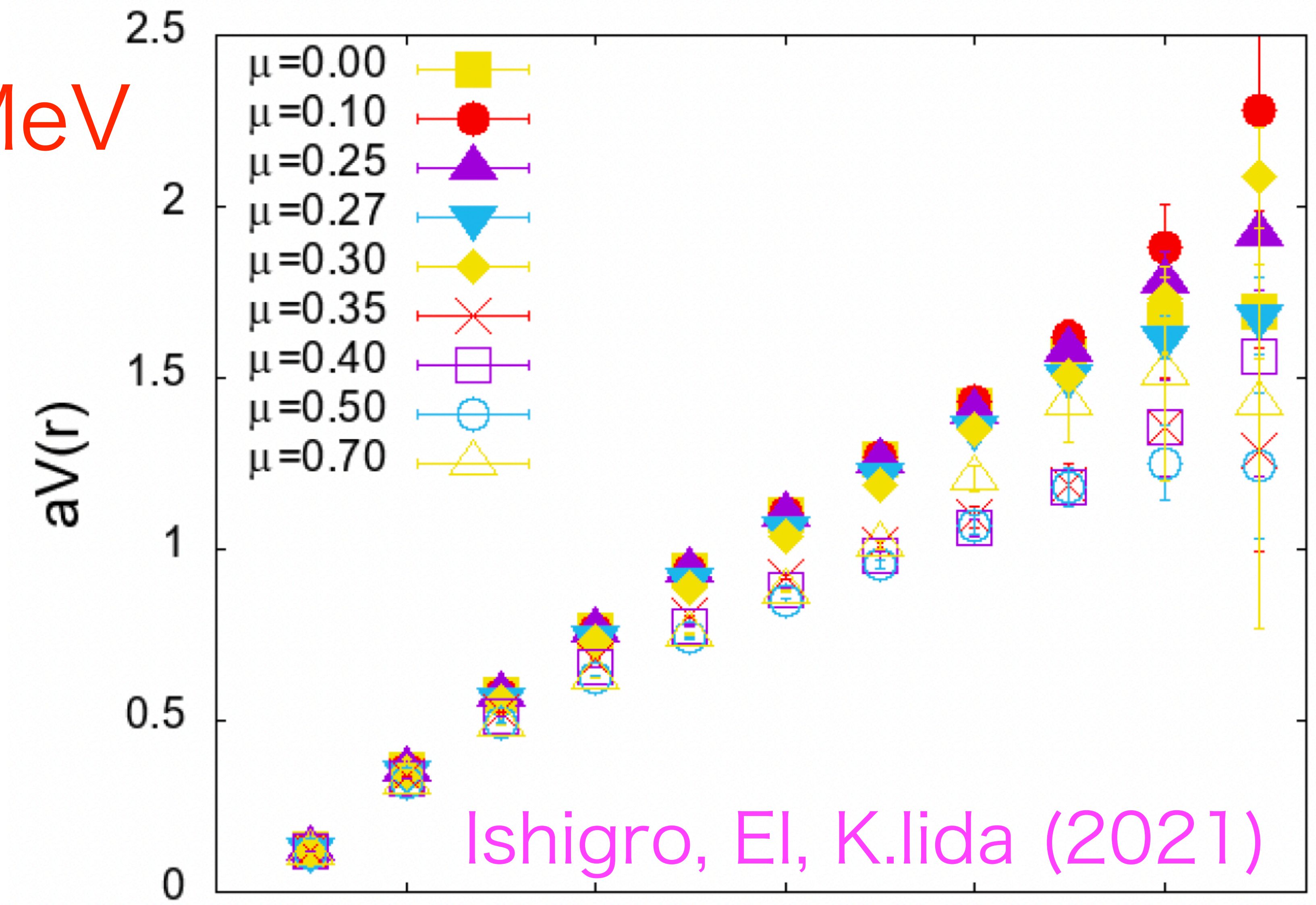
- Sign problem is NP-hard

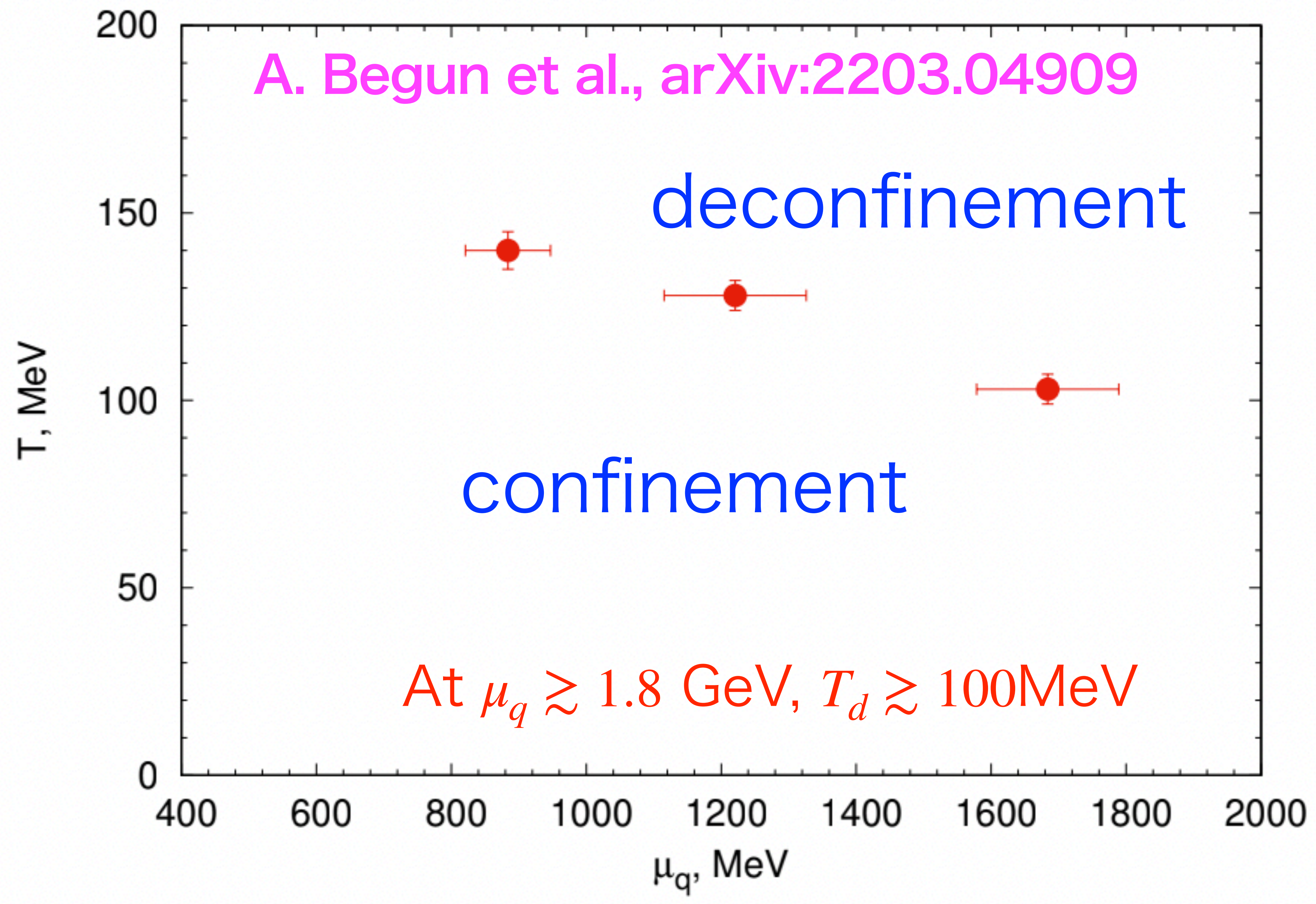
Troyer and Weise, 2005

Need to change the theory
or the algorithm (quantum computation)

Confinement remains even in high density

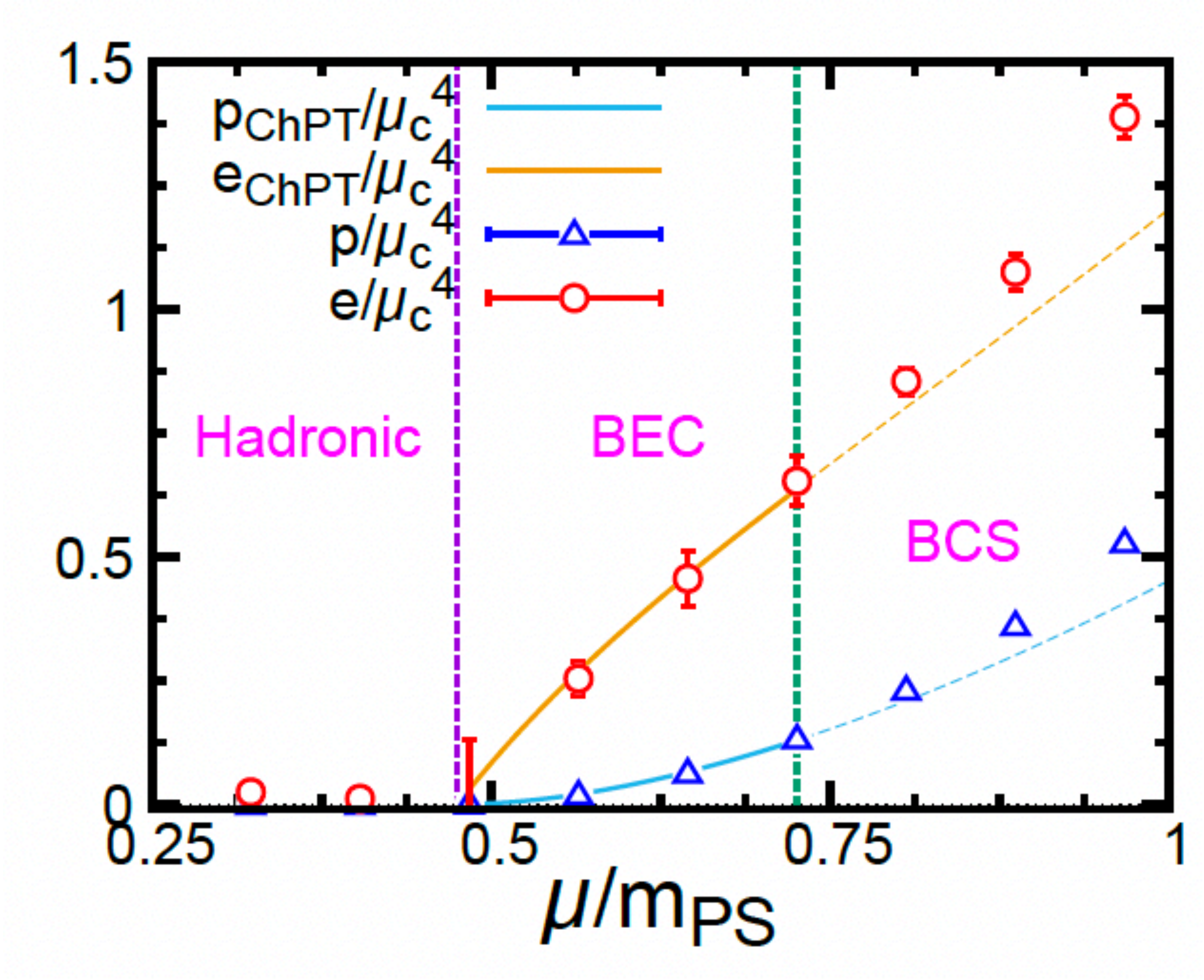
T=40MeV





- $q\bar{q}$ potential at T=40MeV also show a linear potential (=confinement)
- Other 2 groups also show:
  T~90-100MeV is the critical T for deconfinement
- In 2color QCD, the confinement occurs even at high-density.
  Hadronic superfluidity
- cf.) In real dense QCD, it is expected that quark dof would be relevant
  Condensate has color charge: $\langle (qq)^a \rangle$

EoS and consistency with ChPT result in BEC



- ChPT prediction (valid for near μ_c)

$$p_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)^2$$

$$e_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)\left(1 + 3\frac{\mu_c^2}{\mu^2}\right)$$

- We obtain the pion decay constant(F)
  from fit of p : F=51.1(5) MeV
  from fit of e : F=56.7(7) MeV
  cf.) F=60.8(1.6) by fitting of $\langle n_q \rangle$ at 140MeV
  (different mass, staggered fermion)

N. Astrakhantsev et al. (2020)

Conformal bound (Holography bound)

Conformal bound: A conjecture proposed by A.Cherman et al., 2009

"maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories"

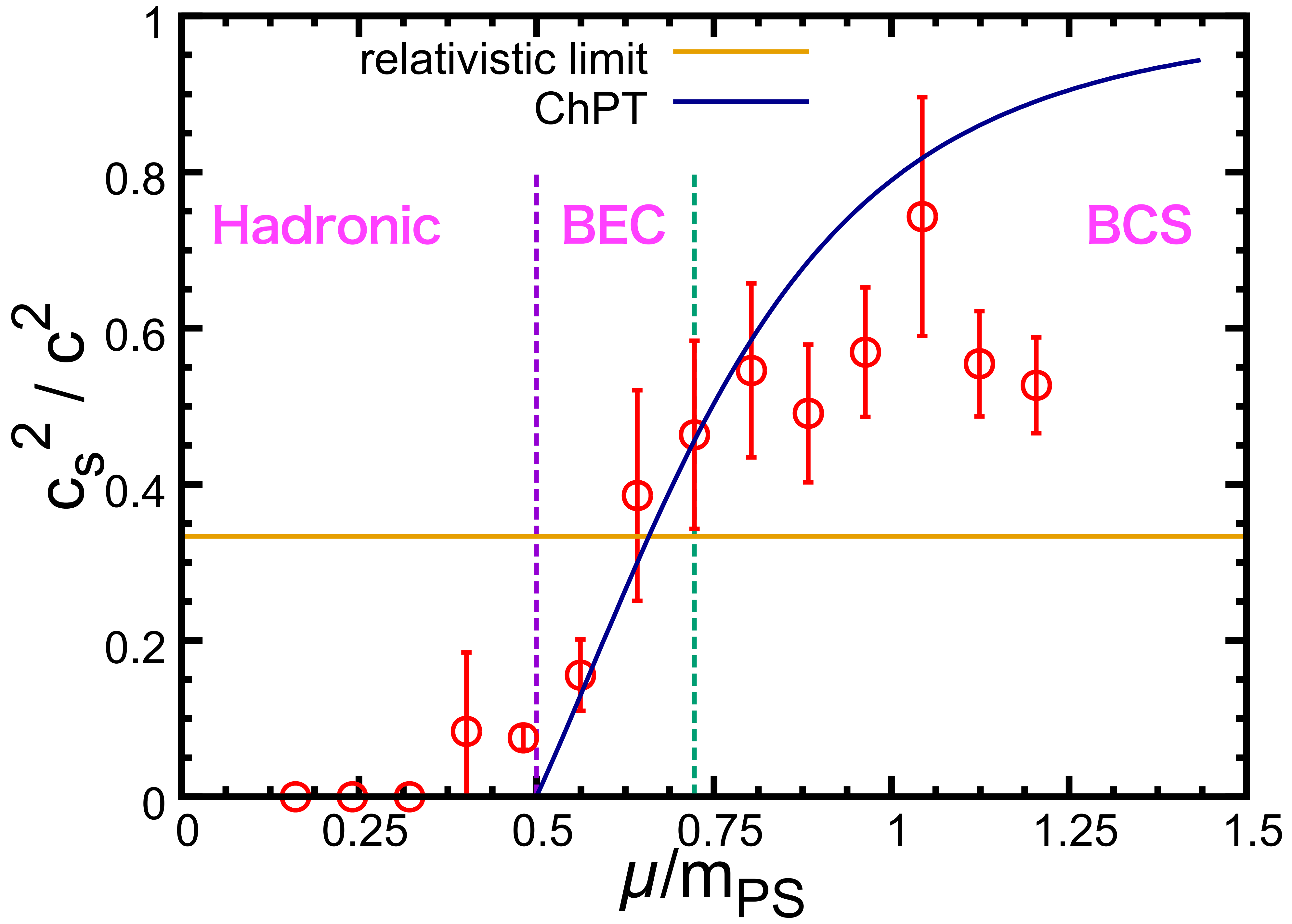
A bound on the speed of sound from holography

Aleksey Cherman* and Thomas D. Cohen†
Center for Fundamental Physics, Department of Physics,
University of Maryland, College Park, MD 20742-4111

The first counterexample using Lattice Monte Carlo is shown in finite-density QCD-like theory (K.Iida and EI, 2022)

v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

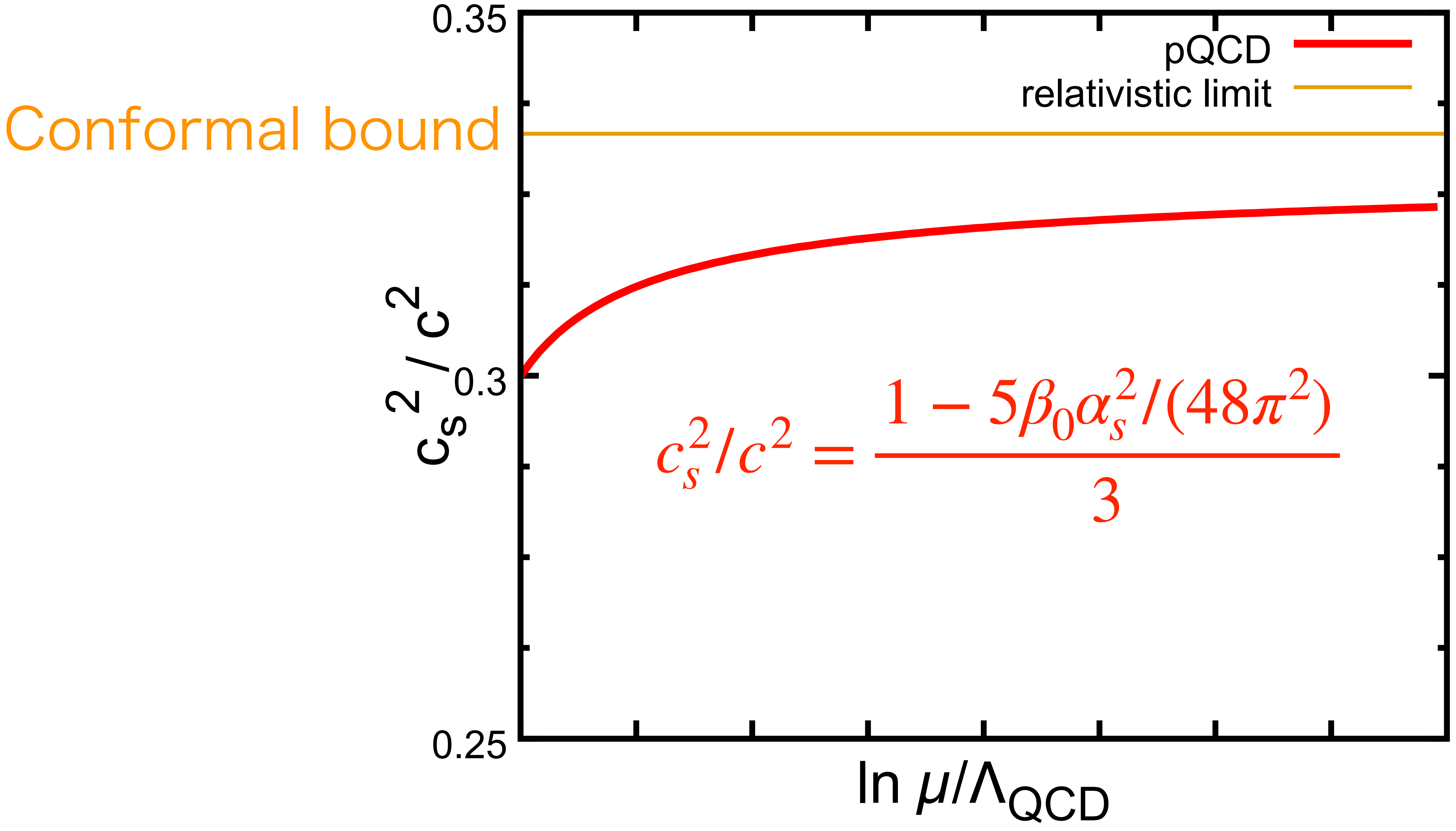
Further high density?



Kojo, Baym, Hatsuda (2021)

pQCD prediction

(Ultra high-density regime)

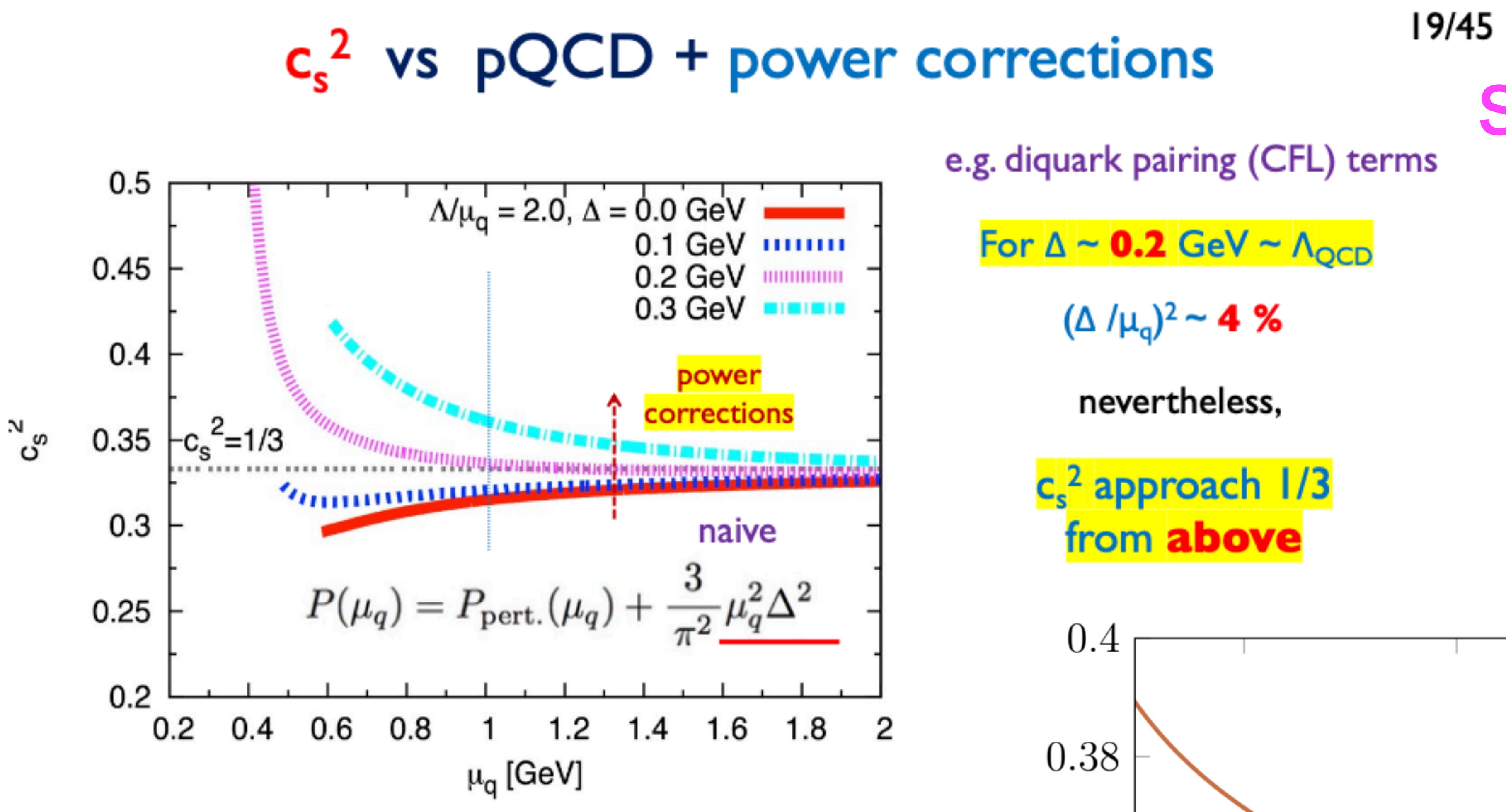


- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$
  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

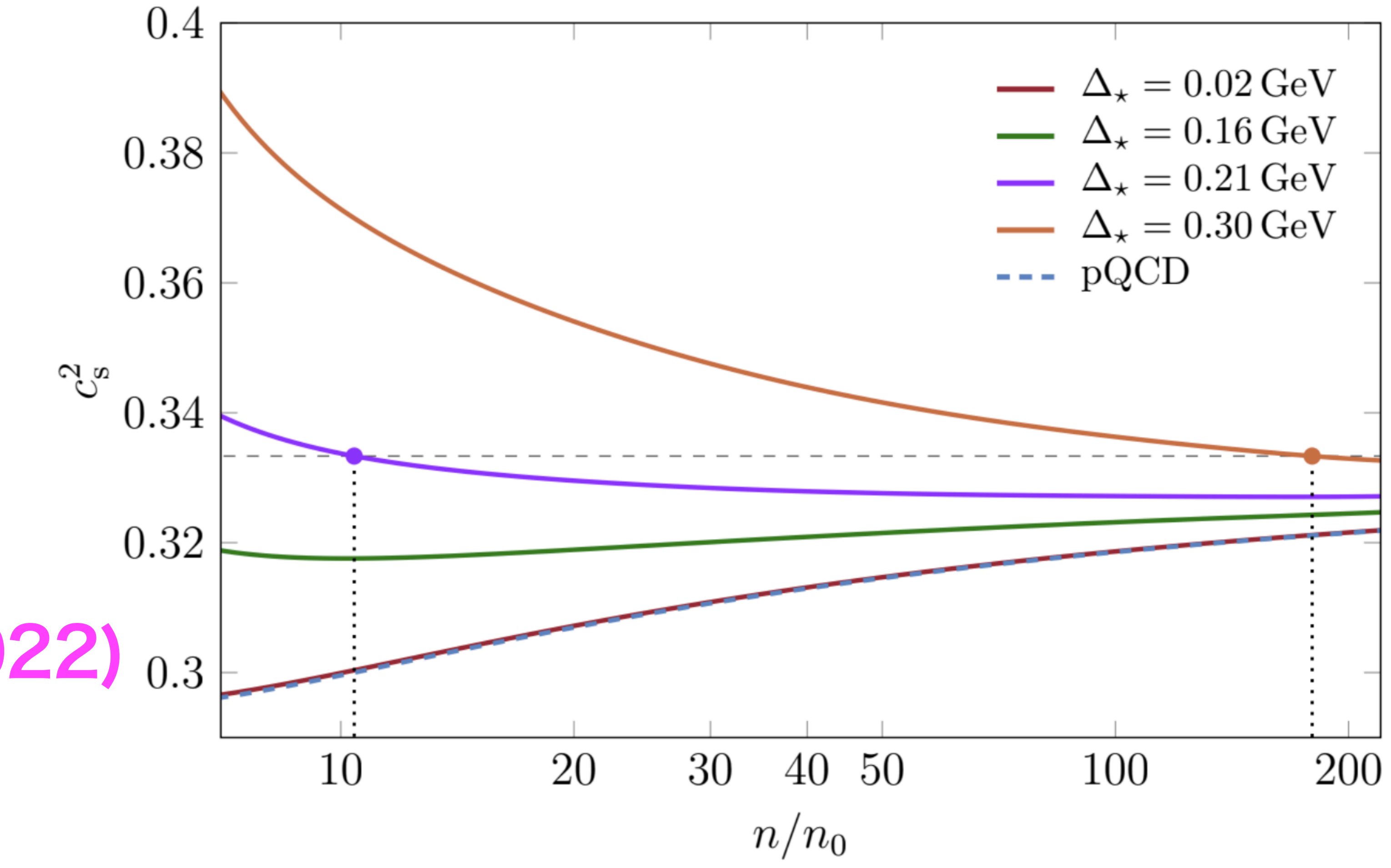
Further high density?

pQCD + power correction due to diquark gap

Slide by Kojo (2019)

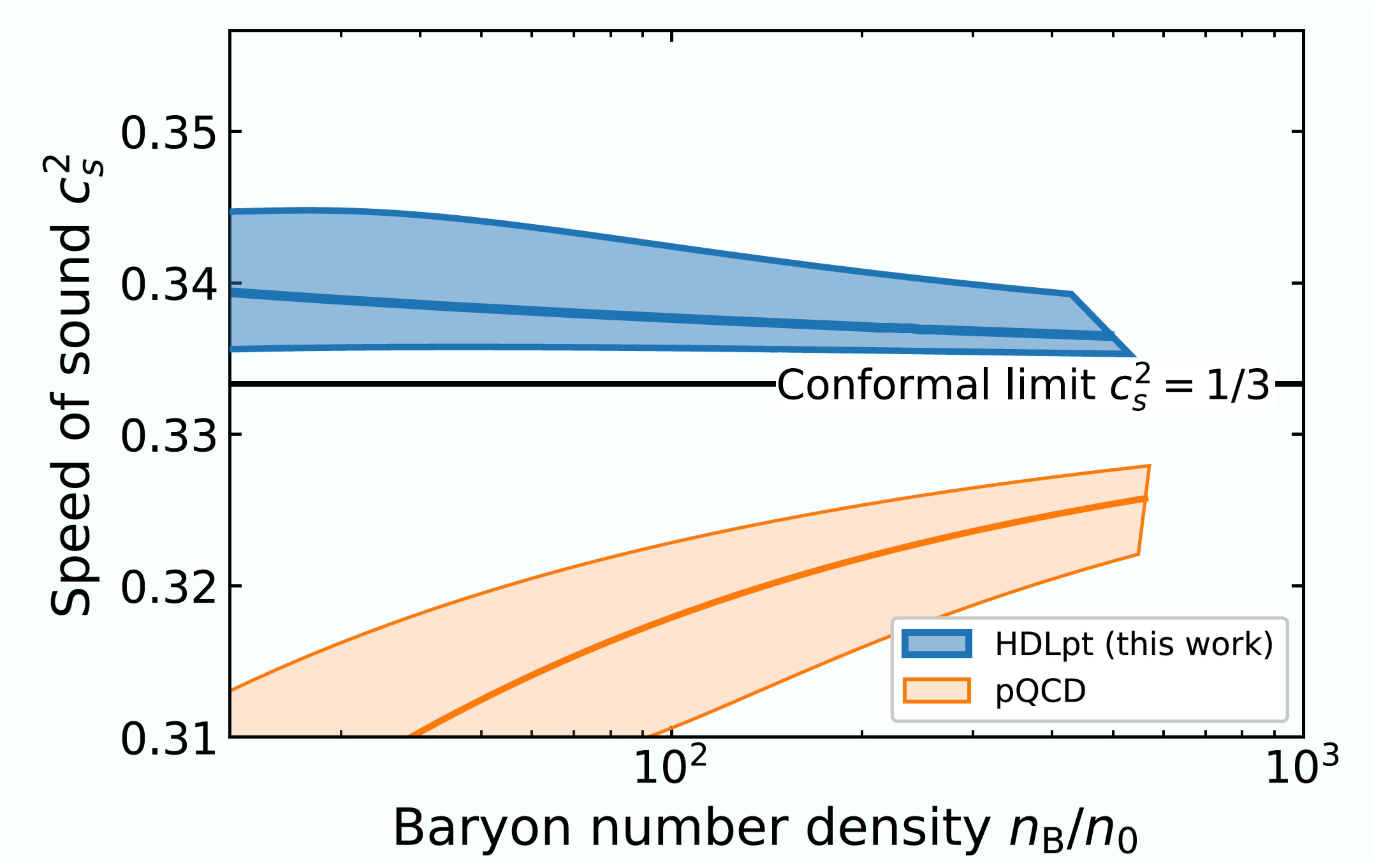


fRG analysis
Braun, Geissel, Schallmo(2022)



Hard thermal loop resummation

Fujimoto and Fukushima(2021)



Two problems in finite-density QCD simulations

K.Nagata, Finite-density lattice QCD and sign problem: Current status and open problems Prog.Part.Nucl.Phys. 127 (2022) 103991

(1) sign problem

$$Z = \int \mathcal{D}U [\det D] e^{-S_g[U]}$$

e^{-S_E} must be real-positive

$\mu = 0$ $\quad D^\dagger = \gamma_5 D \gamma_5$ $\quad \det D$ real

$\mu \neq 0$ $\quad \Delta(-\mu)^\dagger = \gamma_5 \Delta(\mu) \gamma_5$ $\quad \det \Delta(\mu)$ complex

In two-color QCD $\det \Delta(\mu)$ is real,

since the fundamental reps. of SU(2) takes a pseudo-real reps.

(2) numerical instability (onset problem)

in the low-T and high-density regime:

$\mu/m_{PS} \geq 1/2$ in low-T $\quad m_{PS}$: pseudo-scalar (pion) mass at $\mu = 0$

Dynamical pair-creation and annihilation frequently occur,

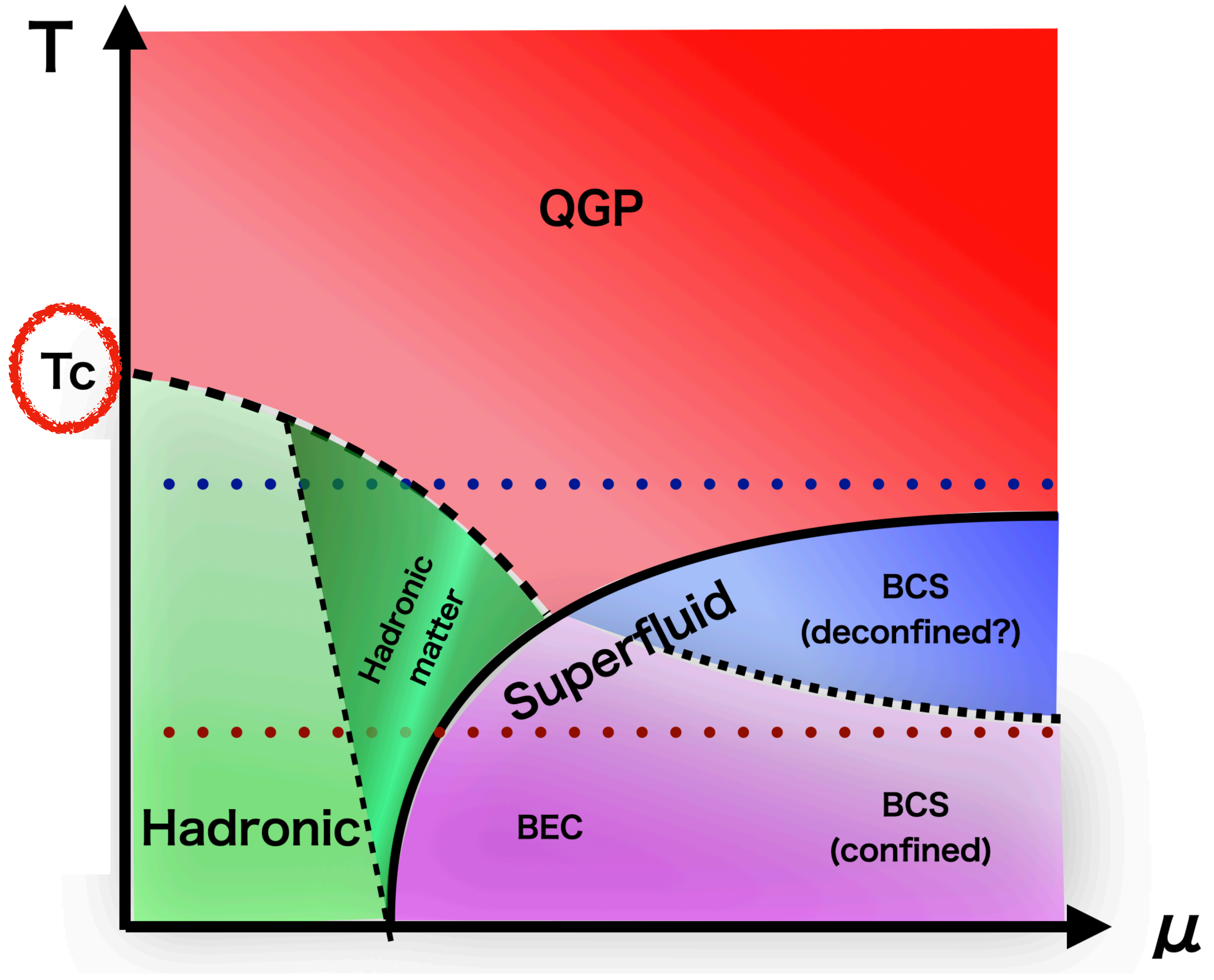
then system becomes unstable

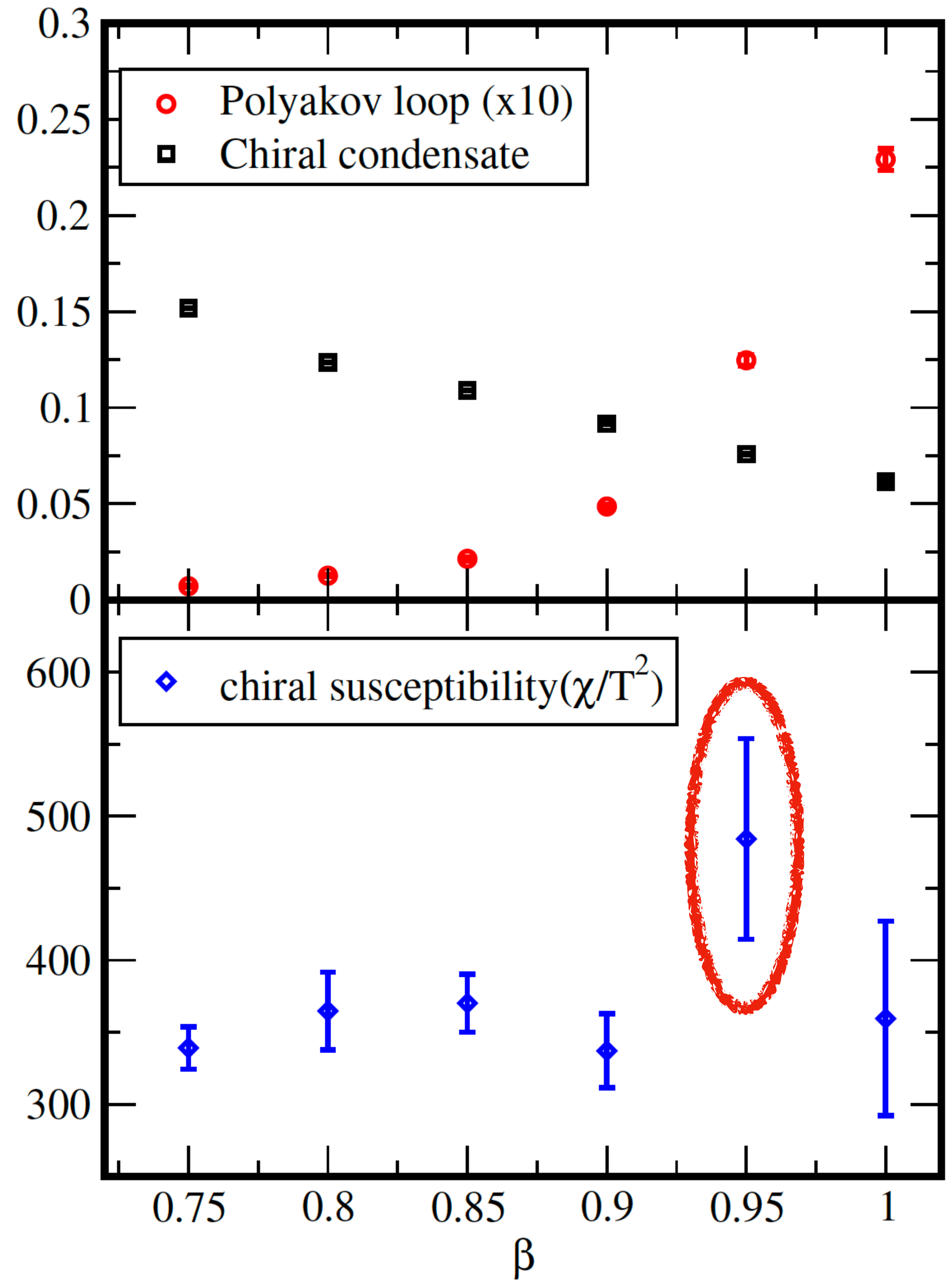
Phase diagram

Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

- Tc at $\mu = 0$ from chiral susceptibility





Scale setting at $\mu = 0$

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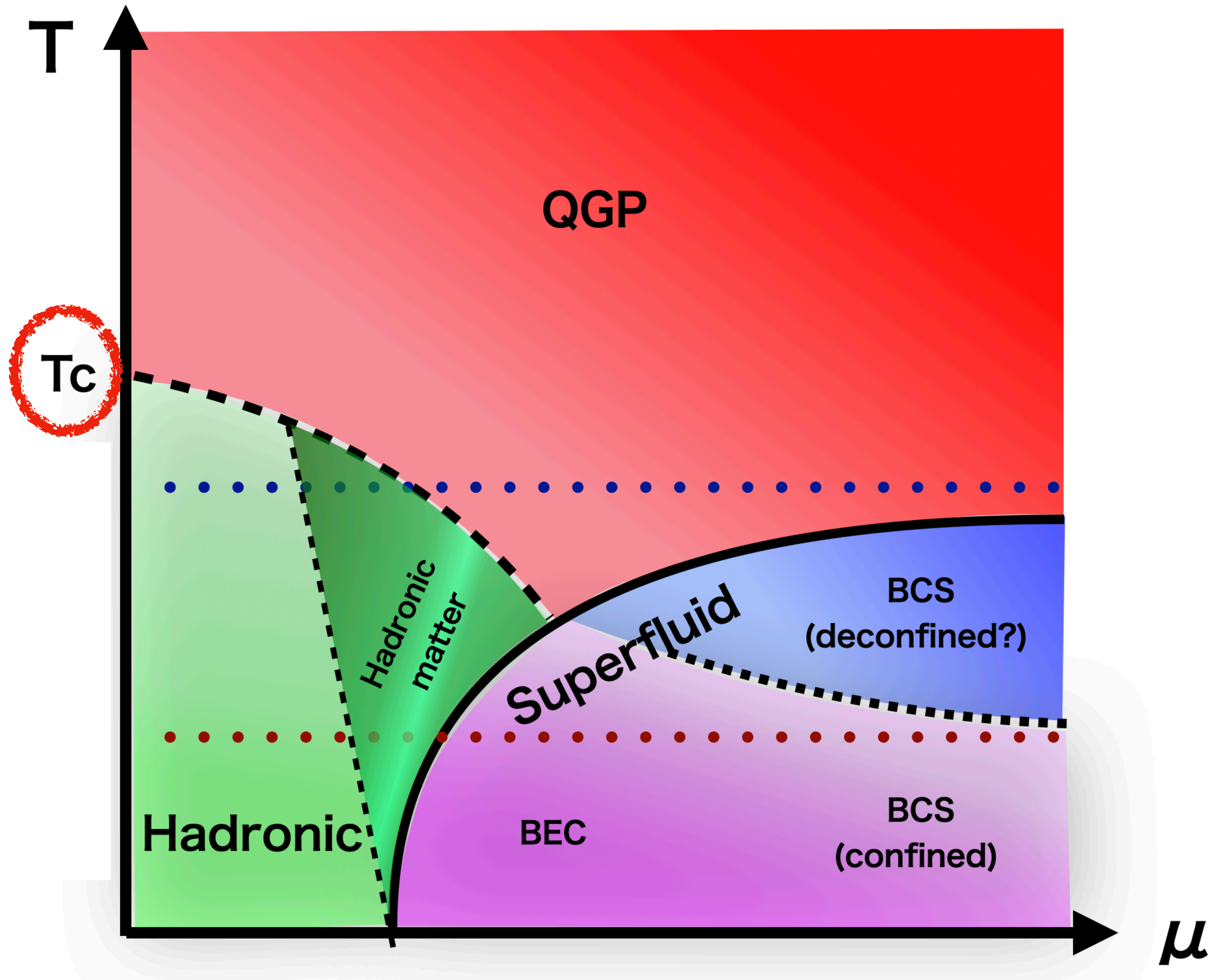
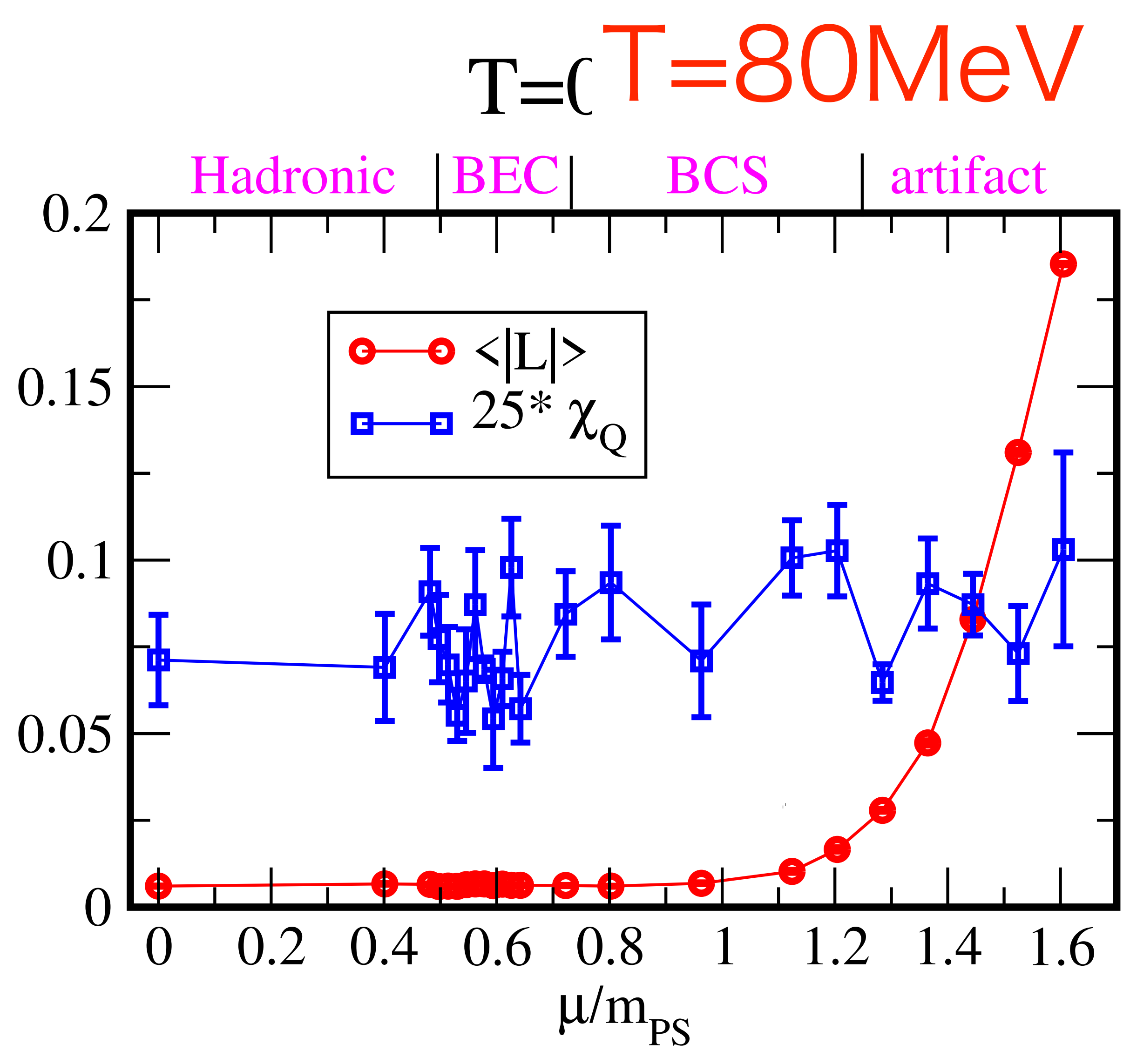


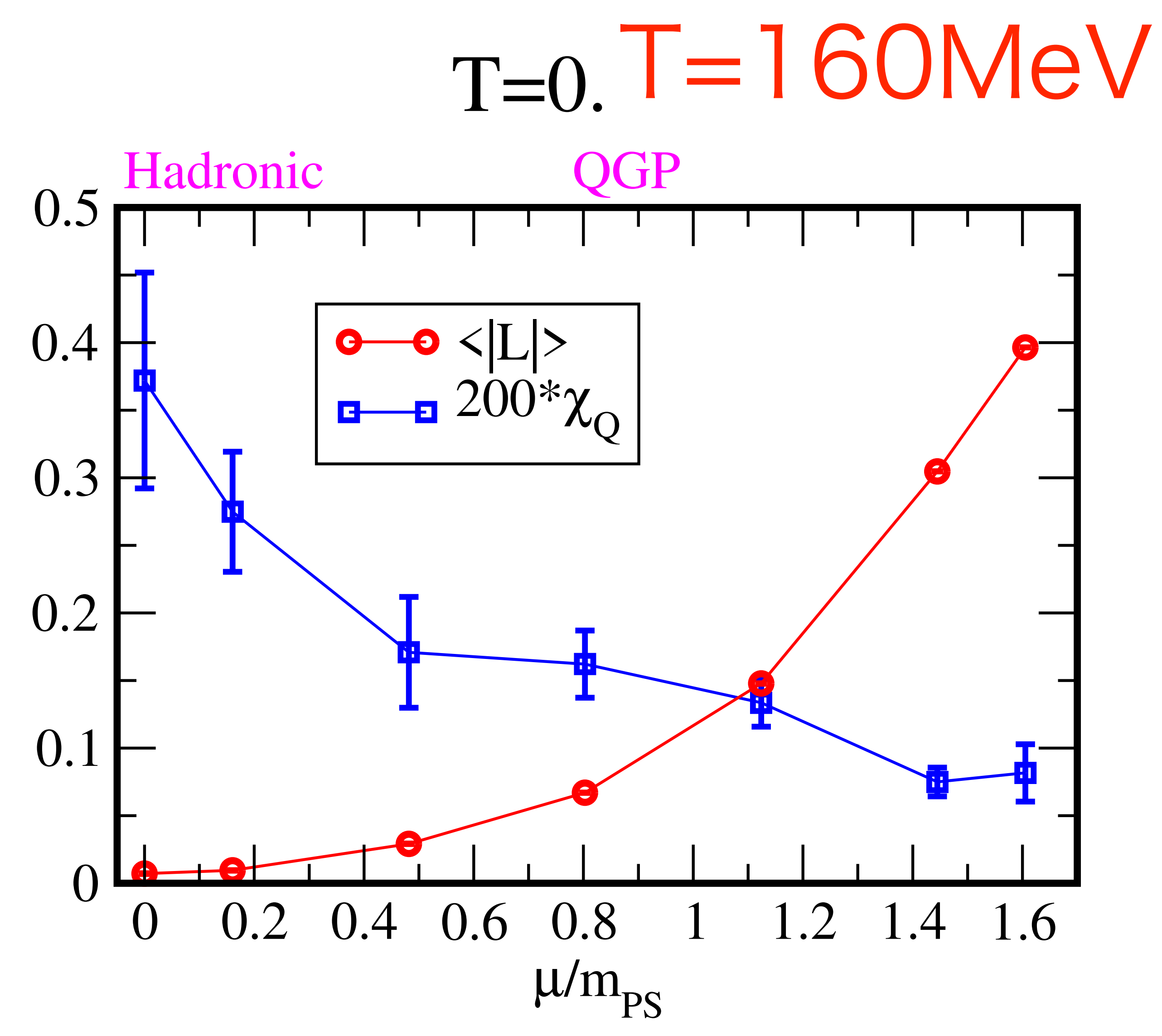
- Tc at $\mu = 0$ from chiral susceptibility

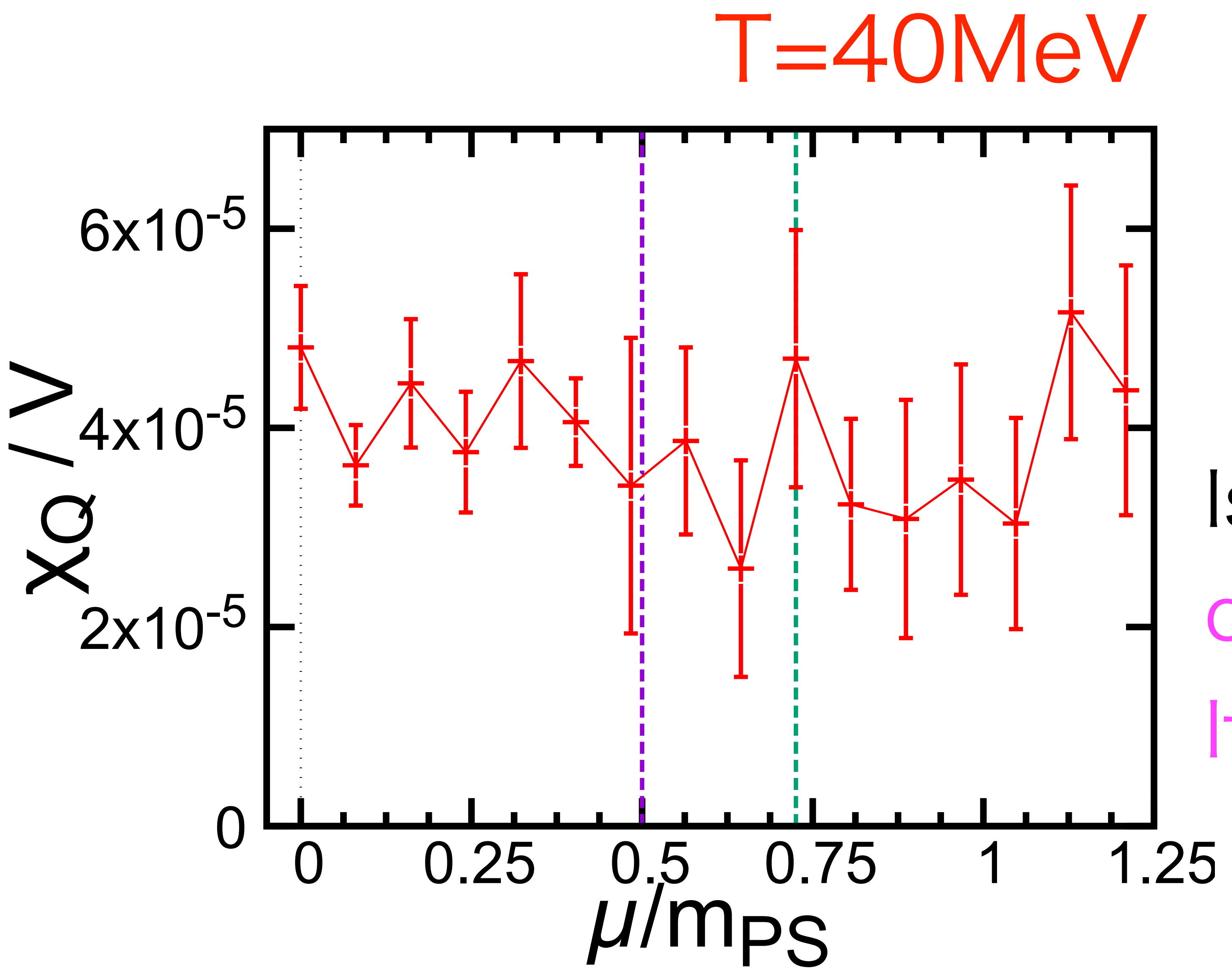
- Assume Tc=200MeV
  Tc is realize Nt=10, $\beta = 0.95$
  (a=0.1[fm])

- Find relationship between β (lattice bare coupling) and a (lattice spacing)
  In finite density simulation,
  a=0.1658[fm]

Topological susceptibility χ_Q







- j->0 extrapolation is done
- $\chi_Q > 0$ even in high density

Is it related with confinement?

cf.) Kawaguchi-Suenaga(2023)

If quark mass is heavy then $\chi_Q > 0$?