Dark Matter Detection with Quantum Sensors

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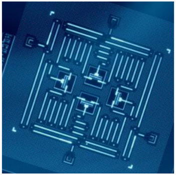
Outline:

1. Introduction
2. Qubits
3. DM Detection with Superconducting Qubits
4. DM Detection with Rydberg Atoms
5. Quantum Enhancement
6. Summary

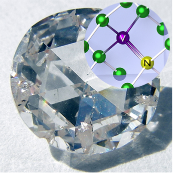
1. Introduction

Quantum technologies are rapidly developing

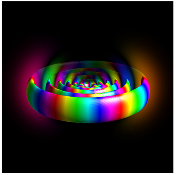
- Quantum computer is (probably) a primary driving force
- Many quantum devices (particularly, qubits) are excellent sensors, sensitive to external fields



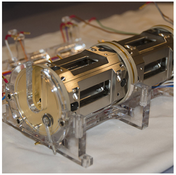
(Transmon) Qubit



NV Center



Rydberg Atom



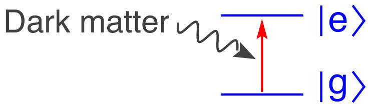
Ion Trap

and more ...

[All the pictures are from Wikipedia]

$\Rightarrow$ They can be (potentially) used to detect BSM physics

What I discuss today: Wave-like DM search with qubits

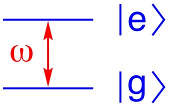


Qubit: Two-level quantum system

- Various types of qubits have been proposed and realized
- Many of them are sensitive to external EM field
- Qubits can be used for DM detection
  [Dixit et al. ('21); Chen, Fukuda, Inada, TM, Nitta, Sichanugrist ('22, '23, '24); Engelhardt, Bhoonah, Liu ('23); Chigusa, Hazumi, Herbschleb, Mizuochi, Nakayama ('23); Agrawal et al. ('23); Ito, Kitano, Nakano, Takai ('23); Braggio et al. ('24)]

2. Qubit

Qubit: Quantum two-level system



$$\Leftrightarrow \quad |\psi(t)\rangle = \psi_g(t)\,|g\rangle + \psi_e(t)\,|e\rangle$$

Many of existing qubits are sensitive to external EM field

$\Rightarrow$ Qubits may be excited by DM-induced EM field

Qubit state evolves unitarily (if no decoherence)

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle \quad \Rightarrow \quad |\psi(t)\rangle = U_{\rm DM}(t)\,|\psi(0)\rangle$$

$U_{\rm DM}$: Unitary operator

Qubit coupled with electric field:

$$\Rightarrow \; H = \omega \, |e\rangle\langle e| - 2\eta \left(|g\rangle\langle e| + |e\rangle\langle g| \right) \cos(mt - \alpha)$$

1st term: Energy gap of the qubit

2nd term: $|g\rangle \leftrightarrow |e\rangle$ transition by DM

η: Driving strength (model-dependent)

- η is proportional to DM induced electric field
- Confirmation of $\eta \neq 0$ is the purpose of detection experiments

Evolution of the state: $|\psi(t)\rangle = f_g(t)\,|g\rangle + f_e(t)e^{-i\omega t}\,|e\rangle$

$$\begin{cases} i\dot{f}_g \simeq -\eta\, e^{-i(\omega-m)t-i\alpha} f_e \\ i\dot{f}_e \simeq -\eta\, e^{i(\omega-m)t+i\alpha} f_g \end{cases} \quad \Rightarrow \quad \begin{pmatrix} f_g(t) \\ f_e(t) \end{pmatrix} = U_{\mathrm{DM}}(t) \begin{pmatrix} f_g(0) \\ f_e(t) \end{pmatrix}$$

In the resonant case ($\omega \simeq m$):

$$U_{\rm DM}(t) \simeq \begin{pmatrix} \cos\eta t & ie^{-i\alpha}\sin\eta t \\ ie^{i\alpha}\sin\eta t & \cos\eta t \end{pmatrix} \xrightarrow{\eta t \ll 1} \begin{pmatrix} 1 & ie^{-i\alpha}\eta t \\ ie^{i\alpha}\eta t & 1 \end{pmatrix}$$

$\Rightarrow$ DM-induced EM field can induce the excitation process

In the non-resonant case ($\omega \ll m$ or $\omega \gg m$)

$$U_{\rm DM}(t) \simeq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\Rightarrow$ Excitation is suppressed in the non-resonance case

DM-induced electric field can excite qubits

$$P_{g\to e}(t) \simeq \begin{cases} (\eta t)^2 & : \omega = m_X \\ 0 & : \omega \neq m_X \end{cases}$$

t should be shorter than the coherence time of the system τ

Example of the target: dark photon X_μ

$$\mathcal{L} \ni -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}\epsilon F_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2 X_\mu X^\mu$$

$F_{\mu\nu}$: EM field

Dark photon DM induces (effective) electric field

$$\vec{X} \simeq \bar{X}\vec{n}\sin(m_X t - \alpha) \;\; \text{with} \;\; \rho_{\rm DM} = \frac{1}{2}m_X^2\bar{X}^2$$

$$\vec{E}^{(\rm DM)} = -\epsilon\dot{\vec{X}} = -\epsilon\, m_X \bar{X}\,\vec{n}\cos(m_X t - \alpha)$$

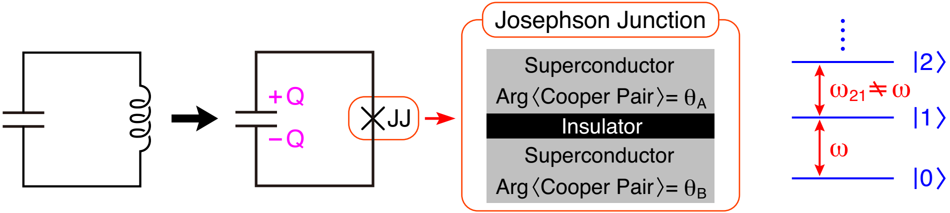
$$|\vec{E}^{(\rm DM)}| = \epsilon\sqrt{2\rho_{\rm DM}}$$

3. DM Detection with Superconducting Qubits

Chen, Fukuda, Inada, TM, Nitta, Sichanugrist

PRL 131 (2023) 211001 [arXiv 2212.03884]

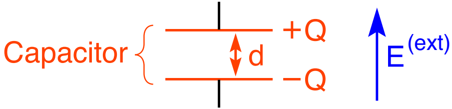
Superconducting qubit: Capacitor + Josephson junction (JJ)



The system has discrete energy levels

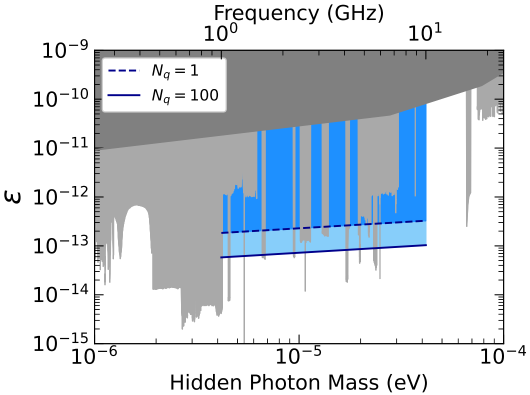
$\Rightarrow$ $|0\rangle$ and $|1\rangle$ are usually used as $|g\rangle$ and $|e\rangle$, respectively

Dipole interaction of superconducting qubit with electric field



$\Leftrightarrow \; H_{\text{int}} = QdE^{(\text{ext})}$

Dark photon DM: 1 year frequency scan ($1 \leq f \leq 10$ GHz)



- $d = 100\ \mu$m
- $C = 0.1$ pF
- $\mathcal{Q} = 10^6$
- Error / qubit $= 0.1$ %

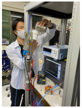
$\Leftrightarrow$ For $C = 0.1$ pF and $d = 100\ \mu$m:

$$p_{g\to e} \simeq 0.1 \times \left(\frac{\epsilon}{10^{-11}}\right)^2 \left(\frac{f}{1\ \text{GHz}}\right) \left(\frac{\tau}{100\ \mu\text{s}}\right)^2$$

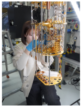
Actual experiment has been started in Japan

***DarQ* experiment** "Dark matter search using Qubits"

UTokyo ICEPP


Toshiaki Inada


Yuya Mino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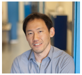
Kan Nakazono


Chikara Kawai


Yutaro Iiyama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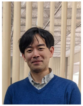
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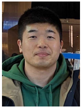
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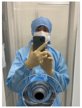
Shion Chen


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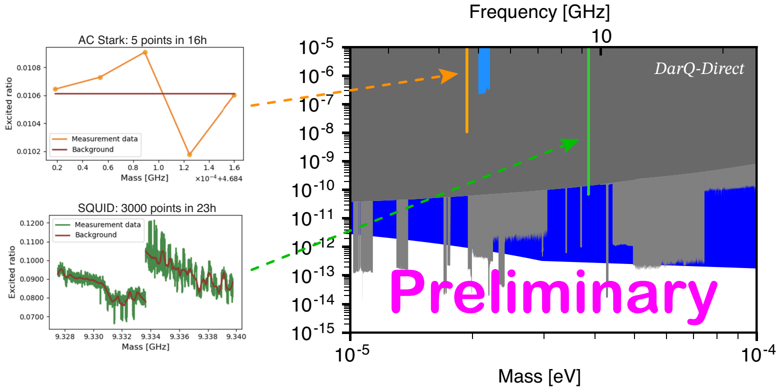
Preliminary result

[Watanabe et al. (DarQ Collaboration)]



⇒ More (and better) results will (hopefully) come soon

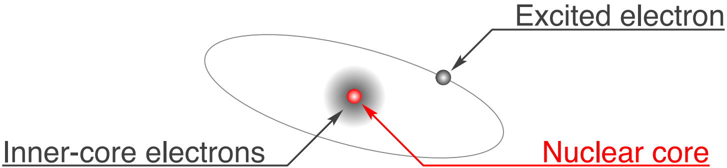
⇒ We hope to cover the blue-shaded region

4. DM Detection with Rydberg Atoms

Chigusa, Kasamaki, Kusano, TM, Nakayama, Ozawa, Takahashi, Umemoto, Vutha

PRL 136 (2026) 151801 [arXiv 2507.12860]

Rydberg atom: inner core + excited electron



$$E_{n,\ell} \simeq -\frac{13.6\,\mathrm{eV}}{(n-\delta_\ell)^2} \equiv -\frac{13.6\,\mathrm{eV}}{\nu^2}$$ $\delta_\ell \sim O(1)$: "quantum defect"

Rydberg atom as a qubit

$\Rightarrow$ Use two of highly-excited energy eigenstates as $|g\rangle$ and $|e\rangle$

$\Rightarrow$ Dynamics of the most-outer electron is the key

Transition occurs with external electric field

$\hat{H}_{\text{int}} = e\vec{E}\,\hat{\vec{r}} = eE\vec{n}\,\hat{\vec{r}}\cos m_{\text{DM}}t$ $\hat{\vec{r}}$: position operator

$\Rightarrow$ DM detection

[See also Matsuki, Yamamoto ('91); Ogawa, Matsuki, Yamamoto ('96); Gúe et al. ('23); Graham et al. ('24); Engelhardt, Bhoonah, Liu ('24)]

Advantages of Rydberg atoms

- $\langle f|\,\hat{\vec{r}}\,|i\rangle \sim r_{\text{Ryd}} \sim n^2 a_{\text{Bohr}}$

  $\Leftrightarrow$ Rydberg state has enhanced dipole interaction

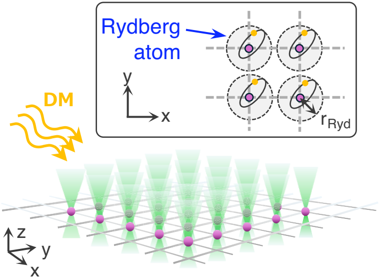
- Rydberg state has (relatively) long lifetime

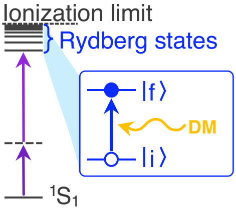
  $\Leftrightarrow$ $\tau_{\text{Ryd}} \sim O(0.1-1)$ msec

- Technologies for trapping, state control, and readout are established

DM detection using Rydberg atom in optical tweezer array (OTA)

$\Rightarrow$ Signal: transition between Rydberg states



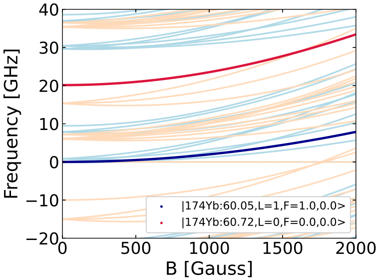


Example: Yb

- Ground state: [Xe] $4f^{14}\,6s^2$
- Rydberg state can be realized with exciting one of electrons in 6s orbit

Energy eigenvalues depend on external magnetic field

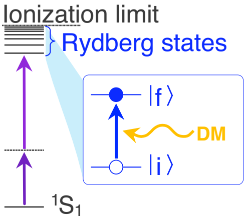
⇔ Zeeman / diamagnetic effects

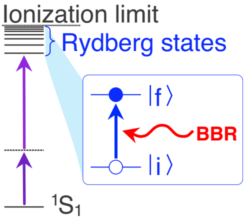


⇒ We may scan the DM mass with applying external (static) magnetic field

Background

Excitation due to blackbody radiation (BBR)

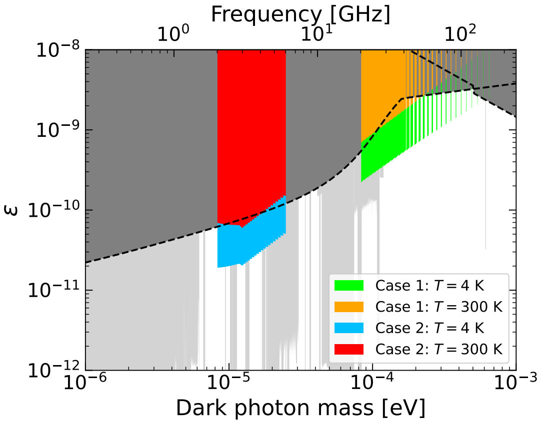




$$\gamma^{(\mathrm{rad})}_{i\to f} = \frac{4}{3}\alpha\omega_{fi}^3 \left|\langle f|\,\hat{\vec{r}}\,|i\rangle\right|^2 \times \begin{cases} f_\gamma(\omega_{fi}) & : \; E_i < E_f \\ 1 + f_\gamma(\omega_{fi}) & : \; E_i > E_f \end{cases}$$

$$f_\gamma(\omega) = \frac{1}{e^{\omega/T} - 1}$$

Expected sensitivity to dark photon DM (10 sec per frequency bin)



- Case 1: $(\nu_i, \nu_f) = (30.05, 30.72), \cdots, (60.05, 60.72);\ 0 \leq B \leq 2\,\mathrm{kG}$
- Case 2: $(\nu_i, \nu_f) = (70.72, 71.05), \cdots, (88.72, 89.05);\ 0 \leq B \leq 0.5\,\mathrm{kG}$

5. Quantum Enhancement

Chen, Fukuda, Inada, TM, Nitta, Sichanugrist

PRL 133 (2023) 021801 [arXiv 2311.10413]

Fukuda, TM, Sichanugrist

[arXiv 2511.03253]

Case of $N_{\rm Q} \gg 1$ $(N_{\rm Q} = \#\text{ of qubits})$

$\Rightarrow$ Case of individual readout: (# of signal) $\propto N_{\rm Q}$

Signal rate can be $O(N_{\rm Q}^2)$ with entangled states

$\Rightarrow$ DM detection using quantum computers!

$\Rightarrow$ Assumption: Any unitary operation is applicable to the system

$U_{\rm DM}$ induces phase-rotation to its eigenstates

$$U_{\rm DM} \simeq \begin{pmatrix} \cos\delta & i\sin\delta \\ i\sin\delta & \cos\delta \end{pmatrix} \quad \text{with} \quad \delta \equiv \eta\tau$$

$$\Rightarrow \; U_{\rm DM}|\pm\rangle = e^{\pm i\delta}|\pm\rangle \quad \text{with } |\pm\rangle \equiv \frac{1}{\sqrt{2}}\left(|g\rangle \pm |e\rangle\right)$$

$$\Rightarrow \; U_{\rm DM}^{\otimes N_{\rm Q}}|+\rangle^{\otimes N_{\rm Q}} = e^{iN_{\rm Q}\delta}|+\rangle^{\otimes N_{\rm Q}}$$

We can design the following quantum (unitary) operation

$$U_{\text{GHZ}} : \begin{pmatrix} |g\rangle^{\otimes N_{\text{q}}} \\ |e\rangle^{\otimes N_{\text{q}}} \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} |+\rangle^{\otimes N_{\text{q}}} + |-\rangle^{\otimes N_{\text{q}}} \\ |+\rangle^{\otimes N_{\text{q}}} - |-\rangle^{\otimes N_{\text{q}}} \end{pmatrix}$$

Starting with $|g\rangle^{\otimes N_{\text{q}}}$:

1. Apply U_{GHZ}: $\frac{1}{\sqrt{2}} \left(|+\rangle^{\otimes N_{\text{q}}} + |-\rangle^{\otimes N_{\text{q}}} \right)$
2. Evolution with DM: $\frac{1}{\sqrt{2}} \left(e^{iN_{\text{q}}\delta} |+\rangle^{\otimes N_{\text{q}}} + e^{-iN_{\text{q}}\delta} |-\rangle^{\otimes N_{\text{q}}} \right)$
3. Apply U_{GHZ}^{-1}: $\cos N_{\text{q}}\delta \, |g\rangle^{\otimes N_{\text{q}}} + i \sin N_{\text{q}}\delta \, |e\rangle^{\otimes N_{\text{q}}}$

Transition probability:

$$p_{g \to e} = \sin^2 N_{\text{q}}\delta \simeq N_{\text{q}}^2 \delta^2 \quad \Rightarrow \quad \frac{S}{\sqrt{B}} \propto N_{\text{q}}^{3/2} \delta^2$$

We may enhance the signal rate using entanglement, but ...

- Need to suppress the noise rate
  [Escher, de Matos, Davidovich ('12); Demkowicz-Dobrzański, Kołodyński, Guţă ('12); Kołodyński, Demkowicz-Dobrzański ('13)]

- $\cdots$

Coherence time of a single qubit: τ_{Q}

$\Rightarrow$ For entangled state: $\tau_{\mathrm{Entangled}} \sim N_{\mathrm{Q}}^{-1} \tau_{\mathrm{Q}}$

$\Rightarrow$ Protocol with GHZ state has an advantage when $\tau_{\mathrm{Q}} \gtrsim \tau_{\mathrm{DM}}$
(Sensitivity is better than the individual readout)

$\Rightarrow$ We need high-quality qubits (with long coherence time)

Another idea: Noise reduction with quantum error correction (QEC)

Without the noise, state is symmetric under exchanges of qubits

$$[U_{\rm DM}\,|0\rangle]^{\otimes N_{\rm Q}} \simeq \left[e^{-i\eta X t}\,|0\rangle\right]^{\otimes N_{\rm Q}} \simeq |0\rangle^{\otimes N_{\rm Q}} - i\eta t \sum_i X_i\,|0\rangle^{\otimes N_{\rm Q}} + \cdots$$

$$X = |1\rangle\langle 0| + |0\rangle\langle 1|$$

If no noise, the state stays in "Code space" neglecting terms of $O(\eta^2)$:

Code space: $\{|0\rangle^{\otimes N_{\rm Q}}, |W\rangle\}$

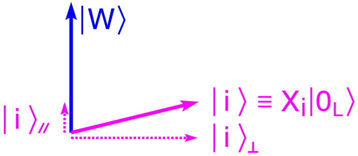
$$|W\rangle \equiv \frac{1}{\sqrt{N_{\rm Q}}}\sum_i X_i\,|0\rangle^{\otimes N_{\rm Q}} = \frac{1}{\sqrt{N_{\rm Q}}}\big(\,|100\cdots\rangle + |010\cdots\rangle + \cdots\big)$$

Noises act on individual qubit

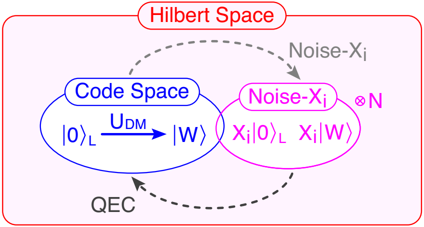
$$|0\rangle^{\otimes N_{\rm Q}} \xrightarrow{\text{Noise on } i\text{-th qubit}} |i\rangle = |0\cdots 010\cdots 0\rangle$$

Noises kick out the state from the code space

$$\langle i|W\rangle = \frac{1}{\sqrt{N_{\mathrm{Q}}}} \quad \Rightarrow$$



QEC may bring the state back to the code space



We propose to perform the following protocol

1. Specify the “subspace” to which the state is likely to belong

   $\Rightarrow$ Information about the error (noise) location, if error occurs

2. Correct error without destroying logical information

   $$\Rightarrow \; aX_i |0_L\rangle + bX_i |W\rangle \xrightarrow{\text{QEC}} a|0_L\rangle + b|W\rangle$$

$\Rightarrow$ The noise rate can be reduced to $O(\gamma_Q)$, not $O(N_Q\gamma_Q)$

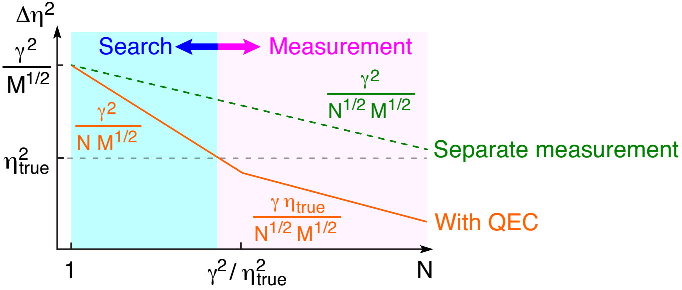
Sensitivity after M measurement cycles (taking optimal $t \sim \gamma_{\mathrm{Q}}^{-1}$):

$$\frac{S}{\sqrt{B}} \sim N_{\mathrm{Q}} \times \frac{\sqrt{M}\eta^2 t^2}{\sqrt{\gamma_{\mathrm{Q}} t}} \quad \Leftrightarrow \quad \frac{S_{\mathrm{indiv}}}{\sqrt{B_{\mathrm{indiv}}}} \sim \sqrt{N_{\mathrm{Q}}} \times \frac{\sqrt{M}\eta^2 t^2}{\sqrt{\gamma_{\mathrm{Q}} t}}$$

For $N_{\mathrm{Q}} \to \infty$, we do not beat the standard quantum limit (SQL)

$\Leftrightarrow$ QEC helps to reach the discovery threshold earlier

Expected sensitivity (with M measurement cycle)



Our protocol is useful particularly for the DM detection

$\Leftrightarrow$ Because of the random reset of the phase α, observable should be proportional to η^2

6. Summary

Qubits are prominent candidates of DM detector

- We may reach parameter region which has not been unexplored
- New experiments have started

We may use quantum properties of qubits for signal enhancement

- Entanglement is the key
- We may use QEC for noise reduction

More progresses in quantum technologies should happen

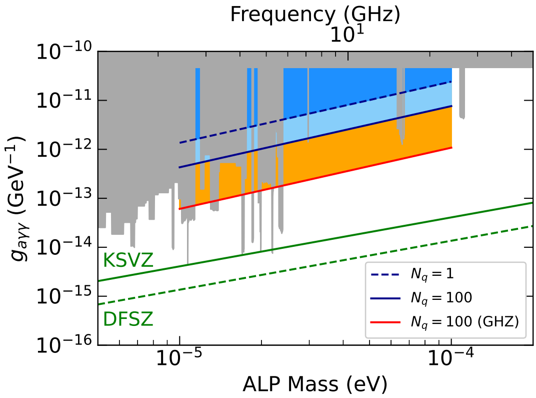
$\Rightarrow$ It is interesting to think about their applications

Backups

Backup: Axion Search

Axion DM search: 1-year scan with the entangled state

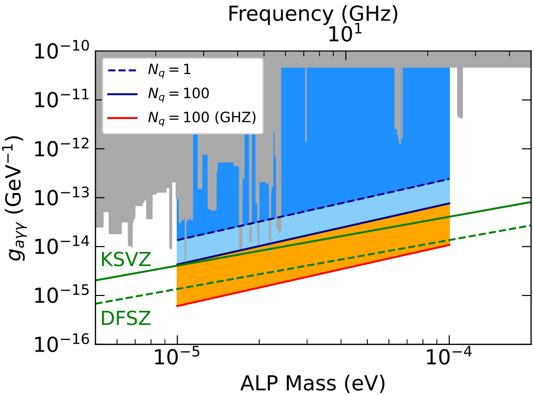
⇒ If we can use GHZ state:



- $\kappa = 1$
- $B = 5\ \mathrm{T}$
- Error / qubit $= 0.1\ \%$

Axion DM search: 1-year scan with the entangled state

$\Rightarrow$ We may be able to use the cavity effect



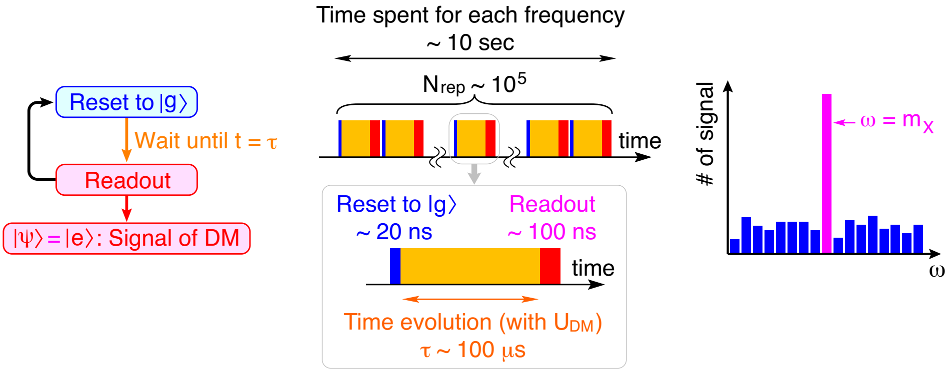
- $\kappa = 100$
- $B = 5$ T
- Error / qubit $= 0.1$ %

Backup: Frequency Scan

Search strategy (with frequency-tunable SQUID qubits)

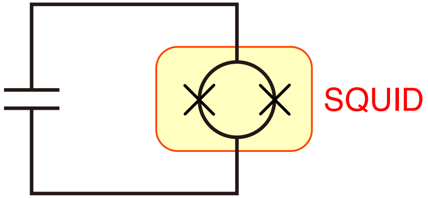
- For fixed ω, repeat the measurement cycle (reset, wait, and readout) as many time as possible

- Scan the qubit frequency ω



Frequency scan

Frequency scan is possible with qubit consisting of SQUID and capacitor

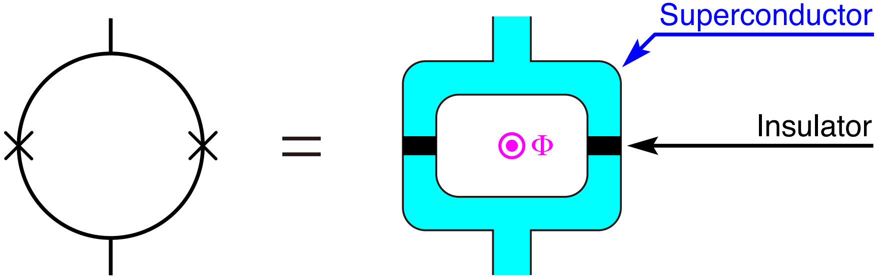


SQUID: superconducting quantum interference device

- Quantum device sensitive to magnetic flux
- With SQUID, the qubit frequency ω can be changed

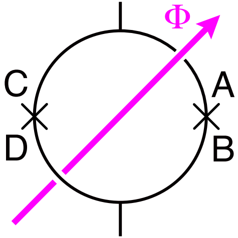
SQUID

- Loop-shaped superconductors separated by insulating layers



- We consider the case with external magnetic flux Φ going through the loop

Phases in the presence of magnetic flux



$$\theta_C - \theta_A = (2e) \int_{A \to C} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_B - \theta_D = (2e) \int_{D \to B} \vec{A}(\vec{x})\, d\vec{x}$$

$$\theta_{BA} - \theta_{DC} = (2e) \oint \vec{A}(\vec{x})\, d\vec{x} = (2e)\, \Phi = \frac{2\pi}{\Phi_0} \Phi$$

$$\theta_{YX} = \theta_Y - \theta_X$$

$\Phi_0 = \dfrac{h}{2e}$: magnetic flux quantum

Define: $\theta \equiv (\theta_{BA} + \theta_{DC})/2$

$$H_{\text{SQUID}} \simeq -J\,(\cos\theta_{BA} + \cos\theta_{DC}) = -2J\cos(e\Phi)\cos\theta$$

Based on the previous analysis with $J \to 2J\cos(e\Phi)$

$$\omega \simeq \sqrt{\frac{2J}{Z}\cos(e\Phi)}$$

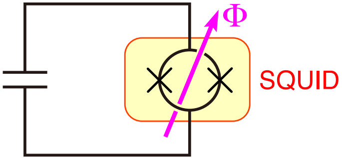
$$Z = (2e)^{-2}C$$

The excitation energy depends on Φ

$\Rightarrow$ Frequency scan is possible with varying the external magnetic field

Frequency tunability with SQUID

SQUID: superconducting quantum interference device



$$\Rightarrow \quad H_{\rm SQUID} \simeq -2J\cos(e\Phi)\cos\theta \simeq J\cos(e\Phi)\theta^2 + \cdots$$

$$\Rightarrow \quad \omega \simeq \sqrt{\frac{2J}{(2e)^{-2}C}\cos(e\Phi)}$$

Φ: magnetic flux going through the SQUID loop

Backup: Rydberg Atoms

Rydberg atom: Trapping

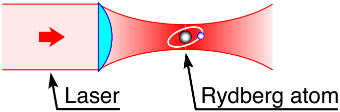
- Atoms in electric field are polarized

$$U(\vec{r}) \simeq -\frac{1}{2}\alpha(\omega)|\vec{E}(\vec{r})|^2$$

$\alpha(\omega)$: Atomic polarizability

- Atoms are attracted to high intensity (for small enough ω)

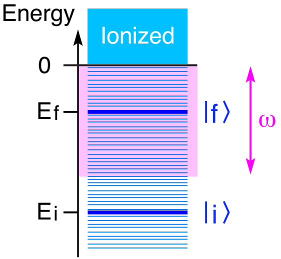
Optical tweezers for trapping Rydberg atoms



$\Rightarrow$ Now, optical tweezer array (OTA) is available

Rydberg atom: Readout (via selective ionization)

1. Apply laser pulse with frequency $|E_f| < \omega < |E_i|$

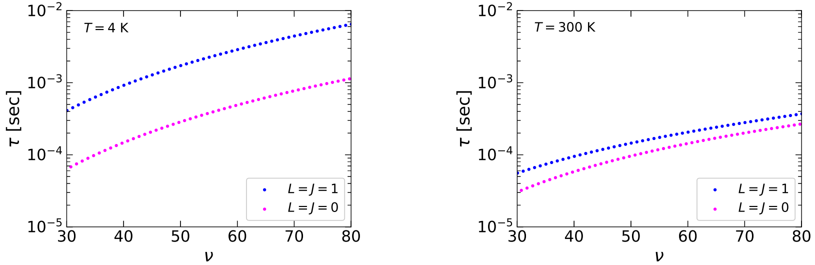


$\Rightarrow$ Rydberg atoms in $|f\rangle$ state are ionized

2. Count the number of ions

$\Rightarrow$ The number of atoms in $|f\rangle$ state is known

Lifetime of the Rydberg states of ^{174}Yb

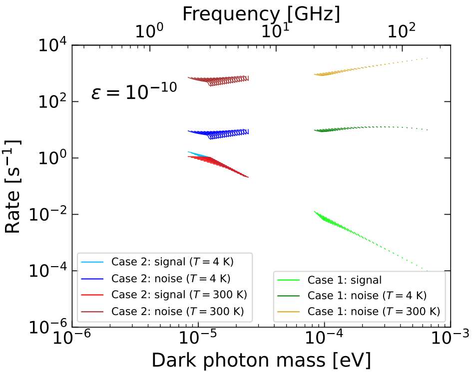


[based on PairInteractions]

- $\tau_{\rm Ryd} \sim O(0.1 - 1)$ msec is possible
- Coherence time of DM:

$$\tau_{\rm DM} \sim \frac{1}{m_{\rm DM} v_{\rm DM}^2} \sim 0.4\,{\rm msec} \times \left(\frac{m_{\rm DM}}{10\,\mu{\rm eV}} \right)^{-1}$$

Event rate (with $\epsilon = 10^{-10}$) and noise rate



[based on `rydcalc`]

Backup: Quantum Circuit for GHZ

$U_{\rm DM}$ induces pure phase rotations of its eigenstates

$$U_{\rm DM} \simeq \begin{pmatrix} \cos\delta & -i\sin\delta \\ -i\sin\delta & \cos\delta \end{pmatrix} \quad \text{with} \quad \delta \equiv \eta t \ll 1$$

$$\Rightarrow \; U_{\rm DM}\,|\pm\rangle = e^{\mp i\delta}\,|\pm\rangle \quad \text{with} \quad |\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(\,|0\rangle \pm |1\rangle\,\big)$$

$$\Rightarrow \; U_{\rm DM}^{\otimes N}\,|\pm\rangle^{\otimes N} = e^{\mp iN\delta}\,|\pm\rangle^{\otimes N}$$

We want to make the accumulated phase physical

$\Rightarrow$ We consider the following quantum (unitary) operation

$$U_{\rm GHZ} : \begin{pmatrix} |0\rangle^{\otimes N} \\ |1\rangle^{\otimes N} \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} |+\rangle^{\otimes N} + |-\rangle^{\otimes N} \\ |+\rangle^{\otimes N} - |-\rangle^{\otimes N} \end{pmatrix}$$

Basic unitary operations (quantum gates)

- Z gate

$$Z = |g\rangle\langle g| - |e\rangle\langle e| \;\; \Rightarrow \; |+\rangle \xrightarrow{Z} |-\rangle \;\; \text{with} \;\; |\pm\rangle \equiv \frac{1}{\sqrt{2}}\big(|g\rangle \pm |e\rangle\big)$$

- Hadamard gate

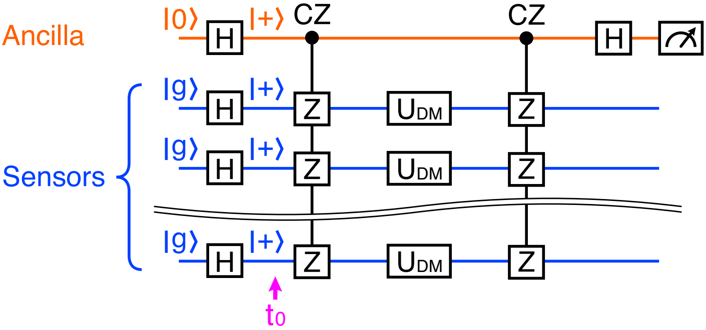
$$H = |+\rangle\langle g| + |-\rangle\langle e| \;\; \Rightarrow \; |g\rangle \xrightarrow{H} |+\rangle, \;\; |e\rangle \xrightarrow{H} |-\rangle$$

- Controlled Z gate

$$CZ = |0\rangle\langle 0| \otimes \mathbf{1} + |1\rangle\langle 1| \otimes Z$$

$$\Rightarrow \;\; \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |+\rangle \xrightarrow{CZ} \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle$$

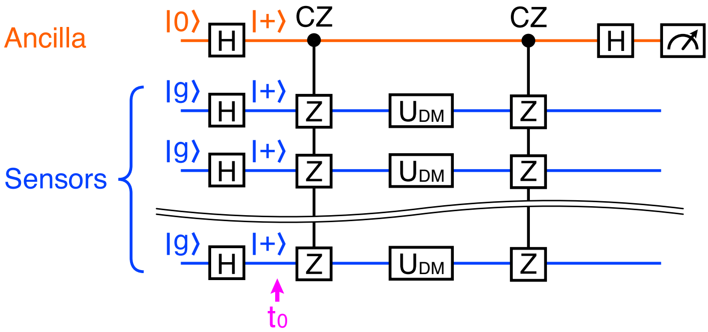
One measurement cycle for the signal enhancement



The above is an example of the quantum circuit

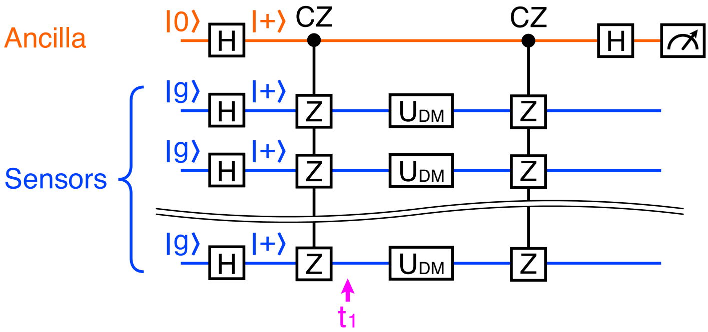
$\Rightarrow$ Let us first see how it works when $\alpha = 0$

One measurement cycle for the signal enhancement



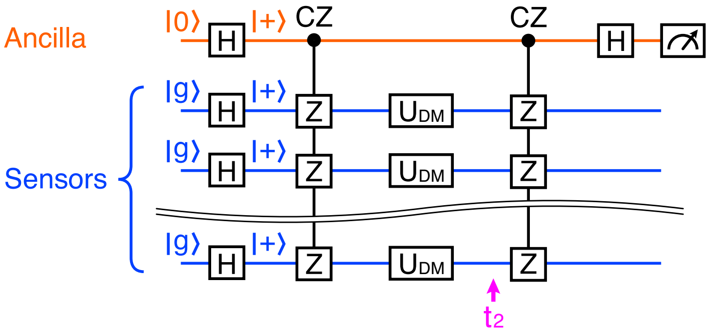
$$|\Psi(t_0)\rangle = |+\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}} = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}} + \frac{1}{\sqrt{2}}|1\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}}$$

One measurement cycle for the signal enhancement



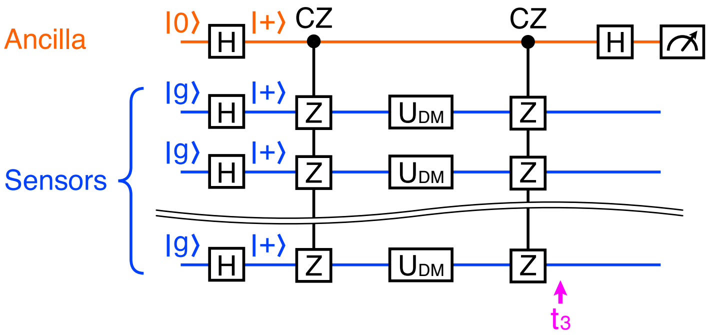
$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}} + \frac{1}{\sqrt{2}}|1\rangle \otimes |-\rangle^{\otimes N_{\mathrm{Q}}}$$

One measurement cycle for the signal enhancement



$$|\Psi(t_2)\rangle = \frac{1}{\sqrt{2}} e^{iN_{\rm Q}\delta} |0\rangle \otimes |+\rangle^{\otimes N_{\rm Q}} + \frac{1}{\sqrt{2}} e^{-iN_{\rm Q}\delta} |1\rangle \otimes |-\rangle^{\otimes N_{\rm Q}}$$

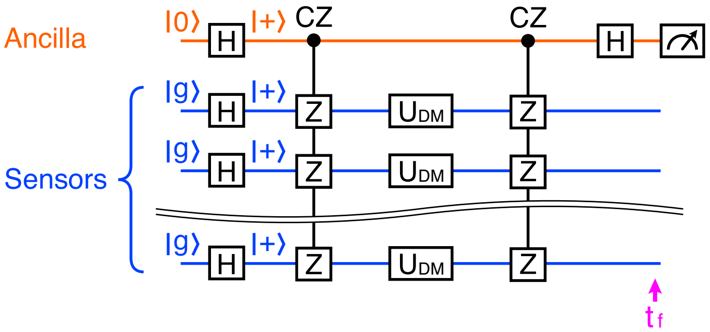
One measurement cycle for the signal enhancement



$$|\Psi(t_3)\rangle = \frac{1}{\sqrt{2}} e^{iN_{\mathrm{Q}}\delta} |0\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}} + \frac{1}{\sqrt{2}} e^{-iN_{\mathrm{Q}}\delta} |1\rangle \otimes |+\rangle^{\otimes N_{\mathrm{Q}}}$$

$$= \left(\cos\, N_{\mathrm{Q}}\delta\, |+\rangle + i \sin\, N_{\mathrm{Q}}\delta\, |-\rangle\right) \otimes |+\rangle^{\otimes N_{\mathrm{Q}}}$$

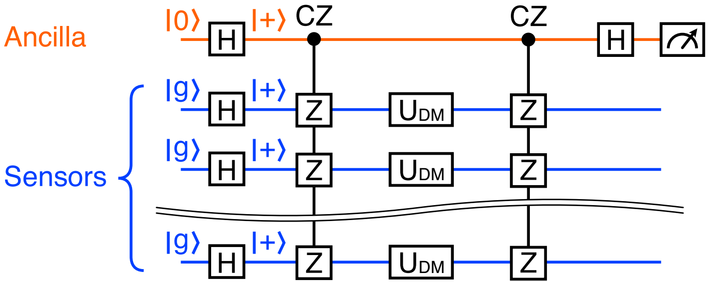
One measurement cycle for the signal enhancement



$$|\Psi(t_{\rm f})\rangle = \big(\cos\ N_{\rm Q}\delta\,|0\rangle + i\sin\ N_{\rm Q}\delta\,|1\rangle\big) \otimes |+\rangle^{\otimes N_{\rm Q}}$$

$\Rightarrow$ Ancilla qubit can be excited: $P_{0\to 1} \simeq \sin^2 N_{\rm Q}\delta \simeq N_{\rm Q}^2\delta^2$

The phase α is unknown in the actual search, but...



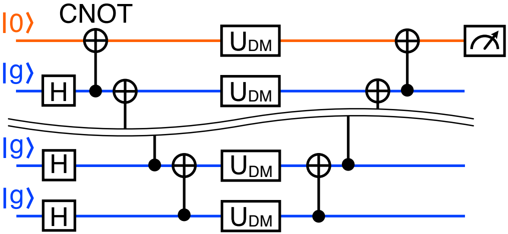
$$P_{0\to 1} \simeq N_{\mathrm{Q}}^2 \delta^2 \cos^2 \alpha \to \frac{1}{2} N_{\mathrm{Q}}^2 \delta^2$$

$\Rightarrow$ Signal rate can be of $O(N_{\mathrm{Q}}^2)$

$\Rightarrow$ The number of gate operation can be $O(N_{\mathrm{Q}})$

Circuit only with nearest neighbor interactions

$\Rightarrow$ (# of gates) $\sim O(N_{\rm Q})$



$\Rightarrow P_{0\to 1} \simeq \frac{1}{2} N_{\rm Q}^2 \delta^2$

CNOT (Controlled-NOT) $= |g\rangle\langle g| \otimes \mathbf{1} + |e\rangle\langle e| \otimes X$

$\Rightarrow$ (# of signals) $\sim O(N_{\rm Q}^2)$

$\Rightarrow$ (# of errors & noises) $\sim O(N_{\rm Q}) \ll$ (# of signals), for $N_{\rm Q} \gg 1$