Neutrino Mixing and Leptonic CP-Violation – Theory and Tests in Future Experiments

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There have been remarkable discoveries in neutrino physics in the last 28 years.

Experimental Proofs for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of** $\mu-$**like events**

Dominant $\nu_\mu \rightarrow \nu_\tau$ K2K, T2K, MINOS; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO;

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ **BOREXINO**

$-\bar{\nu}_e$ (from reactors): **Daya Bay, RENO, Double Chooz**

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS, NOν**A (**ν_μ **from accelerators):** $\nu_\mu \rightarrow \nu_e$

T2K, NOν**A (**$\bar{\nu}_\mu$ **from accelerators):** $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

Compelling Evidences for $\nu-$Oscillations: ν mixing

$$|\nu_l> = \sum_{j=1}^{n} U^*_{lj}\,|\nu_j>\,, \quad \nu_j:\; m_j \neq 0\,; \quad l = e,\mu,\tau\,; \quad n \geq 3\,;$$

$$\nu_{l\mathrm{L}}(x) = \sum_{j=1}^{n} U_{lj}\,\nu_{j\mathrm{L}}(x)\,, \quad \nu_{j\mathrm{L}}(x):\; m_j \neq 0\,; \quad l = e,\mu,\tau\,.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

ν_j, $m_j \neq 0$: **Dirac or Majorana particles.**

Data: 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 0.45\ (0.20)$ **eV;**

KATRIN: $m_{\bar{\nu}_e} < 0.45$ **eV;**

Cosmology: $\sum_j m_j <$ **0.064 - 0.616 eV (95% CL; 2503.07752).**

$(\beta\beta)_{0\nu}-$**decay experiments** (**ν_j-Majorana):** $\min(m_j) < 0.38$ **eV.**

These data imply that

$$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q, \; q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ **eV:** $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^{2}$

This suggests that

• ν_j get their masses from a mechanism which differs from that generating the masses of $m_{e,\mu,\tau}, m_q$ in the SM;

• the smallness of m_{ν_j} is related to the existence of a new fundamental mass scale in particle physics, i.e., to the existence of New Physics beyond the SM.

Natural to assume ν_j "differ" from $m_{e,\mu,\tau}, m_q$ because they are Majorana particles (e, μ, τ, quarks are Dirac particles).

The observation of, e.g., $(\beta\beta)_{0\nu}-$ decay would be a proof.

These ideas are realised in many theoretical models which predict massive Majorana ν_j: seesaw (N_j, H, T), HTP (H^{--}, H^-), models of lepton flavour.

The observed patterns of $\nu-$mixing and of $\Delta m^2_{\rm atm}$ and $\Delta m^2_{\odot}$ can be related to Majorana ν_j and a new fundamental (approximate flavour) symmetry, e.g.,

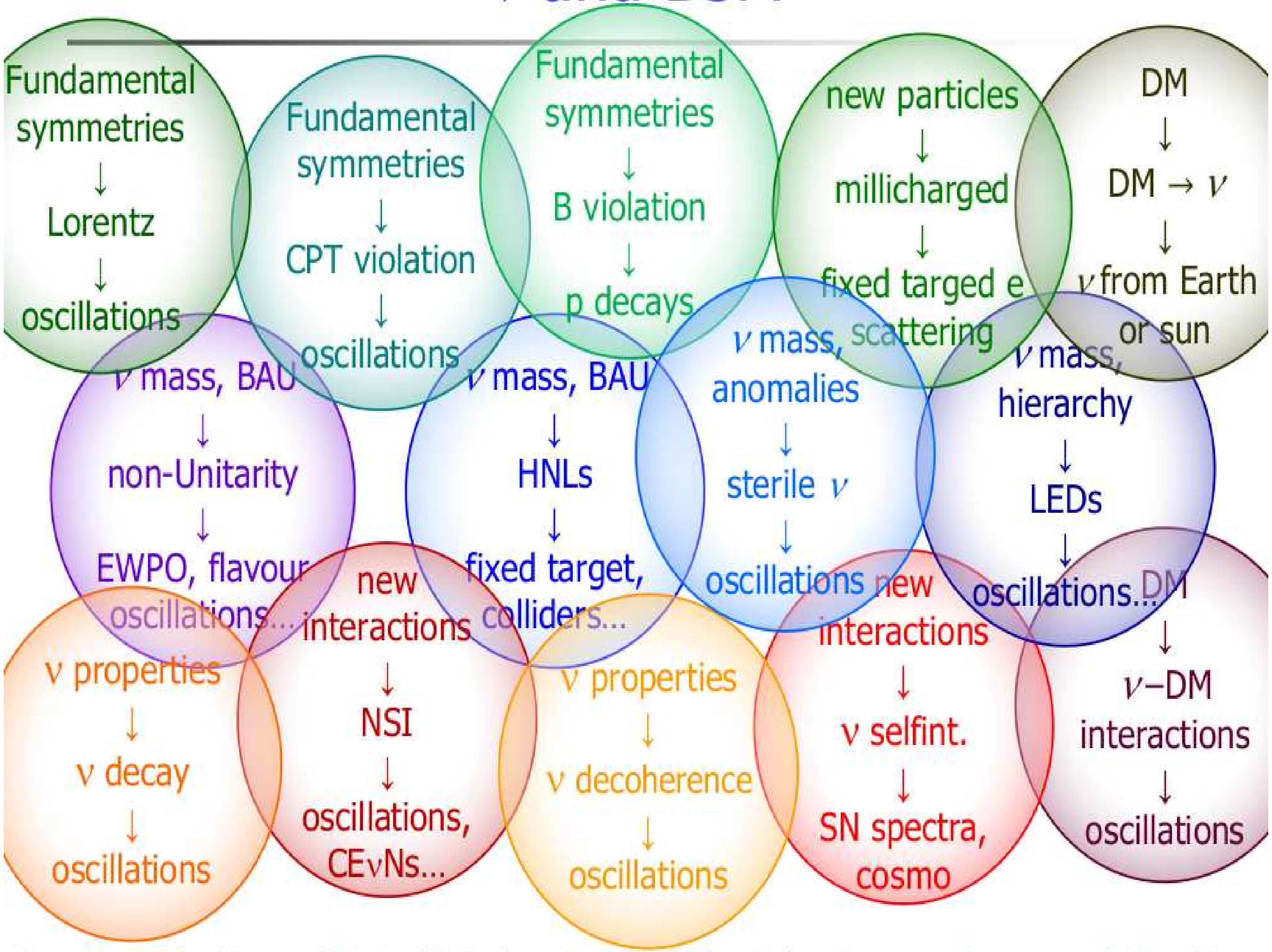
$$A_4\ (\sim \Gamma_3),\ S_4\ (\sim \Gamma_4), ...,\ U(1)_{L'}\ (L' = L_e - L_\mu - L_\tau), ...$$

These discoveries suggest the existence of New Physics beyond that of the SM.

The New Physics can manifest itself (can have a variety of different "flavours"):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...).
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the Majorana nature of massive neutrinos ($L \neq$ const.).
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j, doubly charged scalars,...
- In the existence of new (FChNC, FCFNSNC) neutrino interactions ($U(1)_X$, $M_X \lesssim 50$ MeV).
- In the existence of LFV processes: $\mu \to e + \gamma$, $\mu \to 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of "unknown unknowns"...

ν and BSM



See posters by Natsumi Taniuchi, Joshua Barrow, Daisy Kalra, Roxanne Guenette, Cailian Jiang

E. Fernandez-Martinez, talk at Neutrino 2024, June 17-22, Milano

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6$,...) if, e.g., sterile ν_R, $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,...} \sim 1$ eV;
in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analises of short baseline (SBL) reactor neutrino oscillation data ("reactor neutrino anomaly"), data of radioactive source callibration of the solar neutrino SAGE and GALLEX experiments ("Gallium anomaly"); tests (DANSS, NEOS, PROSPECT, STEREO, ICARUS (at Fermilab), ...):
M. Danilov (talk at Neutrino 2024): "The experimental evidence for eV ν_s is fading away."

ii) $M_{4,5,...} \sim (0.10 - 10^3)$ GeV, low scale seesaw models;
$M_{4,5,...} \sim (10^6 - 10^{14})$ GeV, high scale seesaw models.

Reference Model: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary.

ν_j, $m_j \neq 0$: **Dirac or Majorana particles.**

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky et al., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.34 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.306$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.448\ (2.502) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.545\ (0.551)$, NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0222\ (0.0223)$

F. Capozzi et al. (Bari Group), arXiv:2003.08511.

- $\mathrm{sgn}(\Delta m^2_{\rm atm}) = \mathrm{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\rm atm} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

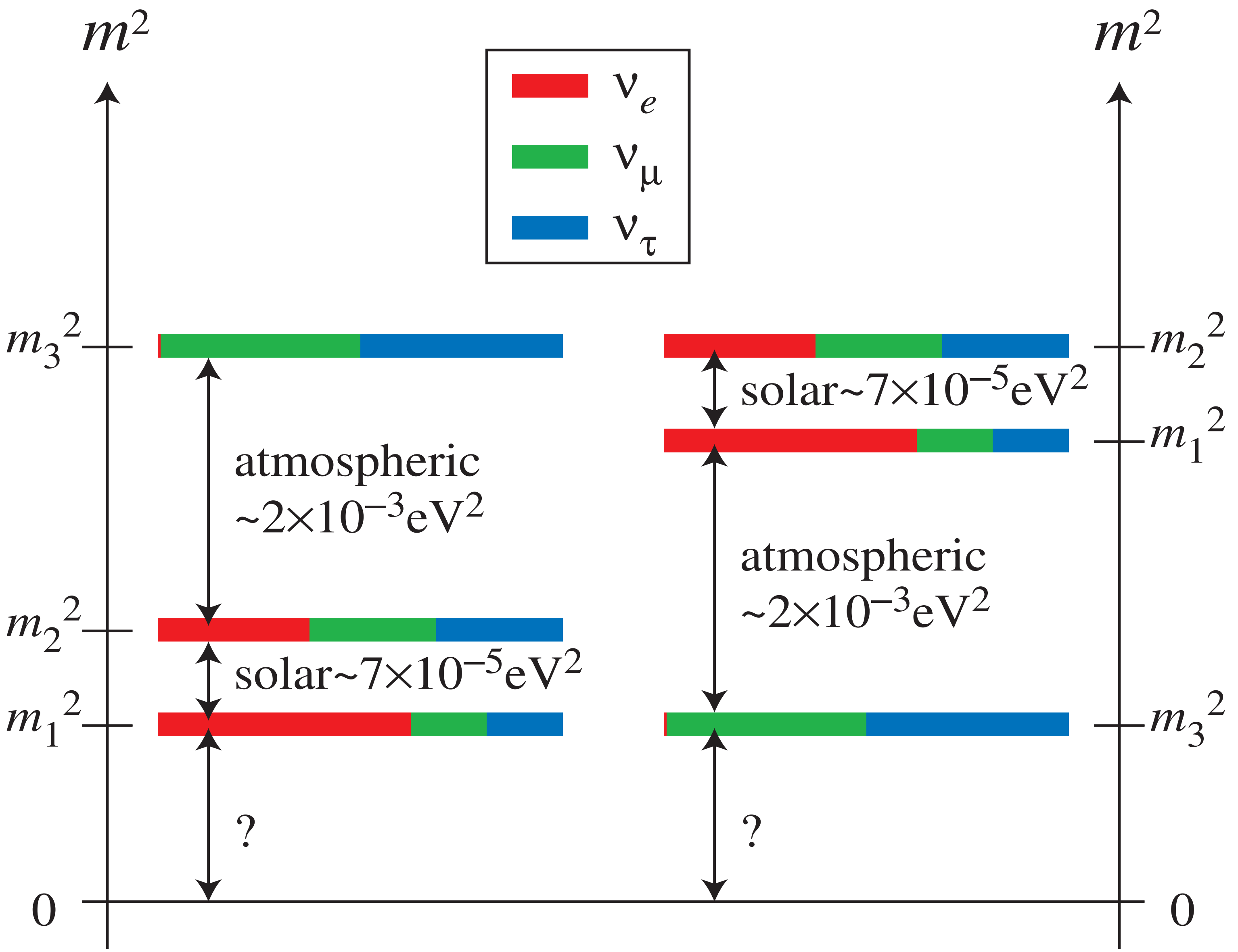
Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\; m^2_{1,2,3} >> |\Delta m^2_{31(32)}|,\; \text{QD};\; m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;


m^2
m^2
ν_e
ν_μ
ν_τ
$m_3{}^2$
atmospheric
$\sim 2\times10^{-3} eV^2$
$m_2{}^2$
solar$\sim 7\times10^{-5} eV^2$
$m_1{}^2$
?
0
$m_2{}^2$
solar$\sim 7\times10^{-5} eV^2$
$m_1{}^2$
atmospheric
$\sim 2\times10^{-3} eV^2$
$m_3{}^2$
?
0

• **Dirac phase** δ: $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{\rm CP}^{(l,l')} \propto J_{\rm CP} \propto \sin\theta_{13} \sin\delta$:

3ν—mixing: P.I. Krastev, S.T.P., 1988

$$A_{\rm CP}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \; , \quad l \neq l' = e, \mu, \tau$$

$$A_{\rm T}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \; l \neq l'$$

$$A_{\rm CP(T)}^{(e,\mu)} = A_{\rm CP(T)}^{(\mu,\tau)} = -A_{\rm CP(T)}^{(e,\tau)}$$

In vacuum: $A^{(e,\mu)}_{\rm CP(T)} = 4\, J_{\rm CP} F^{vac}_{osc}$ $\;(A^{(e,\mu)}_{\rm CP(T)} = A^{(\mu,\tau)}_{\rm CP(T)} = -\, A^{(e,\tau)}_{\rm CP(T)})$

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin(\frac{\Delta m^2_{21}}{2E} L) + \sin(\frac{\Delta m^2_{32}}{2E} L) + \sin(\frac{\Delta m^2_{13}}{2E} L)$$

P.I. Krastev, S.T.P., 1988

In matter: **Matter effects violate**

$$\text{CP}: \quad P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})$$

$$\text{CPT}: \quad P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (constant density or density profile symmetric relative to the middle point, e.g., Earth)

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

In matter with constant density (T2K, NOνA, T2HK, DUNE):

$$\text{J}^{\text{mat}}_{\text{CP}} = \text{J}^{\text{vac}}_{\text{CP}}\ \text{R}_{\text{CP}}, \quad \text{A}^{(\text{e},\mu)}_{\text{T}} = \text{J}^{\text{mat}}_{\text{CP}} \text{F}^{\text{mat}}_{\text{osc}}$$

R_{CP}- real, does not depend on θ_{23} and δ; $\;|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

2018:

$R_{CP} > 0$, $R_{CP} \leq 1.2$; **numerically** R_{CP}**(Krastev,Petcov;1988)=Naumov-HS factor (from 1991).**

S.T.P., Y.-L. Zhou, 1806.09112

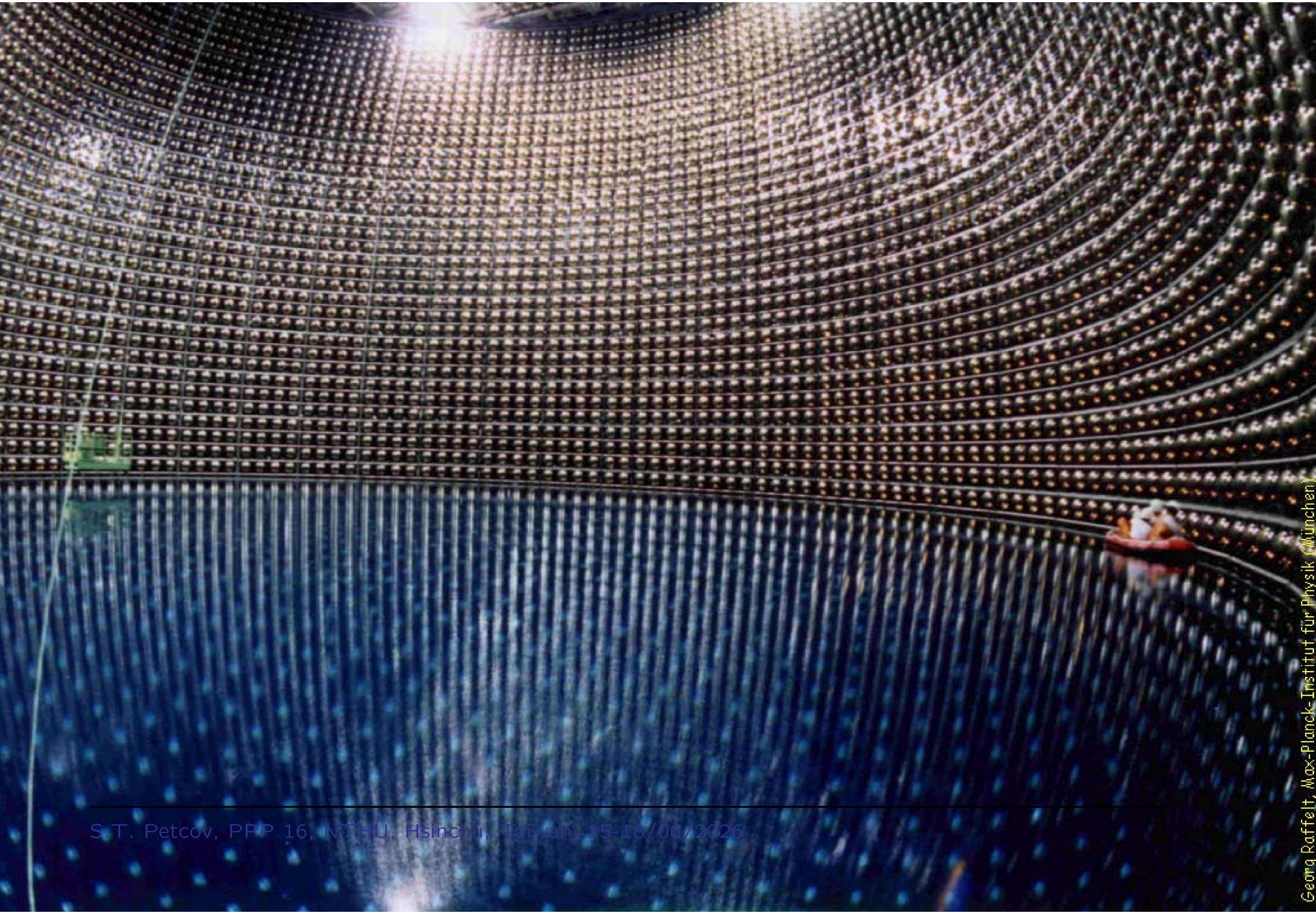
Current data: $|J_{CP}| \lesssim$ 0.040 (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- **Majorana phases** α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987
  - $|<m>|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

T2K: Tokai - Super Kamiokande

NOνA: Fermilab - site in Minnesota


Georg Raffelt, Max-Planck-Institut für Physik (München)

T2K: Tokai - Super Kamiokande; off-axis ν beam, $E = 0.6$ GeV, $L \cong 295$ km, 50 kt water Cherenkov detector.

NOνA: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator detector.

SK experiment studying atmospheric ν_μ, $\tilde{\nu}_\mu$, ν_e, $\tilde{\nu}_e$ ($E \cong 0.1 \div 100$ GeV), and solar ν_e ($E \cong 5 \div 14$ MeV) oscillations.

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\Delta m^2_{21}/10^{-5}$ eV^2	NO, IO	$7.49^{+0.19}_{-0.19}$	–	–	6.92 – 8.05	2.5
$\sin^2\theta_{12}/10^{-1}$	NO, IO	$3.08^{+0.012}_{-0.011}$	–	–	2.75 – 3.45	4.5
$\Delta m^2_{31}/10^{-3}$ eV^2	NO	$2.513^{+0.021}_{-0.019}$	–	–	2.451 – 2.578	1.1
$\Delta m^{32}/10^{-3}$ eV^2	IO	-2.484 ± 0.020	–	–	- 2.547 – (- 2.421)	1.1
$\sin^2\theta_{13}/10^{-2}$	NO	$2.215^{+0.056}_{-0.058}$	–	–	2.0300 – 2.388	3.0
	IO	2.231 ± 0.056	–	–	2.060 – 2.409	3.1
$\sin^2\theta_{23}/10^{-1}$	NO	$4.70^{+0.017}_{-0.013}$	–	–	**4.35 – 5.85**	5.5
	IO	$5.50^{+0.012}_{-0.015}$	–	–	**4.40 – 5.84**	4.4
δ	NO	212^{+26}_{-41}	–	–	124 – 364	19
	IO	274^{+22}_{-25}	–	–	**201 – 335**	8
$\Delta\chi^2_{\rm IO-NO}$	IO–NO	$+6.1$ (2.45σ)				

Global 3ν analysis of oscillation parameters: best-fit values and allowed ranges at $N_\sigma = 3$ for either NO or IO, including all data (also the SK atmospheric neutrino data). The latter column shows the formal "1σ fractional accuracy" for each parameter, defined as 1/6 of the 3σ range, divided by the best-fit value and expressed in percent. The last row reports the difference between the χ^2 minima in IO and NO. The results on the NO and CPV due to δ are inconclusive.

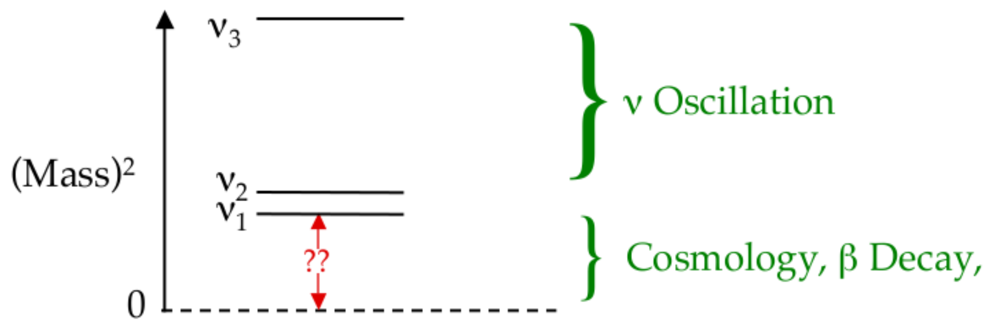
I. Esteban et al., arXiv:2410.05380 (NuFit-6.0 results).

θ_{12}, θ_{23} - **large**, θ_{13} - **small** **(very different from the quark mixing angles).**
$\sin^2\theta_{23}$ - **relatively large uncertainty.**

$$\Delta m^2_{21}/|\Delta m^2_{31}| \cong 1/30.$$

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

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Due to B. Kayser

Absolute Neutrino Mass Scale Measurements

Experiments on e^--spectrum in $^3\mathrm{H} \rightarrow ^3\mathrm{He} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$

(super-allowed $^3\mathrm{H} \rightarrow ^3\mathrm{He}$ transition, NME - constant, $E_0 \simeq 18574.3 \pm 1.7$ eV, half-life 12.3 years)

$$m_{\bar{\nu}_e} < 2.2 \text{ eV} \qquad (95\% \text{ C.L.}) \text{ Troitzk, Mainz.}$$

We have $m_{\bar{\nu}_e} \cong m_{1,2,3}$ in the case of QD spectrum.

The KATRIN experiment (11/06/2018) is planned to reach sensitivity

KATRIN: $m_{\bar{\nu}_e} \sim$ 0.2 eV

i.e., it will probe the region of the QD spectrum.

KATRIN data (2022):

$$m_{\bar{\nu}_e} < 0.81 \text{ eV} \qquad (90\% \text{ C.L.})$$

Latest KATRIN data (2024):

$$m_{\bar{\nu}_e} < 0.45 \text{ eV} \qquad (90\% \text{ C.L.})$$

E. Lokhov et al., talk at Neutrino 2024 (June 17-22, 2024, Milano)

$$\frac{d\,\Gamma(^3\mathrm{H} \rightarrow ^3\mathrm{He} + e^- + \bar{\nu}_e)}{d\,E_e} = \sum_i |U_{ei}|^2 \;\; \frac{d\,\Gamma(m_i)}{d\,E_e}\,,$$

$$\frac{d\,\Gamma(m_i)}{d\,E_e} = C\,p_e\,(E_e + m_e)\,(E_0 - E_e)\,\sqrt{(E_0 - E_e)^2 - m_i^2}\,F(E_e)\,\theta(E_0 - E_e - m_i)\;.$$

Here $E_e \leq E_0 - m_i$ is the kinetic energy of the electron, E_0 is the energy released in the decay, p_e is the electron momentum, m_e is the mass of the electron, $F(E_e)$ is the Fermi function which takes into account the Coulomb interaction of the final state particles, and C is a constant. $(E_0 - E_e)$ is the neutrino energy and $p_i = \sqrt{(E_0 - E_e)^2 - m_i^2}$ is the momentum of neutrino with mass m_i.

Usually

$$m_\beta \equiv m_{\bar{\nu}_e} = \sqrt{\sum_i |U_{ei}|^2 \; m_i^2} \;\; (\simeq m_{1,2,3}, \quad \mathrm{QD\;\;spectrum})$$

is considered as the neutrino mass related observable in β-decay experiments.

Improved β energy resolution requires a ***BIG*** β spectrometer.


KATRIN'S JOURNEY
UNITED KINGDOM
IRELAND
Dublin
Belfast
Leeds
Manchester
Liverpool
Birmingham
Cardiff
London
Celtic Sea
Irish Sea
DENMARK
Copenhagen
Malmö
Bornholm
Kaliningrad
Gdańsk
Vilnius
Minsk
Mahilyow
BELARUS
Hrodna
Homyel
Hamburg
Bremen
Berlin
Poznań
POLAND
Warsaw
Brest
Łódź
Wrocław
Kiev
Rivne
UKRAINE
L'viv
NETH.
Amsterdam
Rotterdam
Essen
Cologne
Leipzig
Brussels
Lille
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Bonn
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Frankfurt am Main
Prague
CZECH REPUBLIC
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LIECH.
Zürich
Bern
Vaduz
HUNGARY
ROMANIA
Bay of Biscay
FRANCE
Geneva
SWITZ.
Ljubljana
SLOVENIA
Zagreb
Bucharest
Constanța
Black Sea
Bordeaux
Lyon
Turin
Milan
Venice
CROATIA
BOSNIA AND HERZEGOVINA
Belgrade
Varna
Bilbao
Toulouse
Genoa
Sarajevo
Serbia
BULGARIA
Sofia
Porto
Andorra la Vella
Marseille
Florence
Istanbul
Skopje
PORTUGAL
Madrid
Barcelona
VATICAN CITY
Tirana
Bursa
Thessaloniki
TURKEY
Lisbon
SPAIN
Valencia
Balearic Sea
Sardinia
Tyrrhenian Sea
Naples
GREECE
Aegean Sea
İzmir
BALEARIC ISLANDS
Athens
Sevilla
Cagliari
Ionian Sea
Gibraltar
Málaga
Mediterranean Sea
Palermo
Sicily
Rhodes
Strait of Gibraltar
Ceuta (SPAIN)
Alborán Sea
Algiers
Melilla (SPAIN)
Oran
Tunis
Crete
Rabat
Casablanca
MOROCCO
ALGERIA
TUNISIA
Valletta
MALTA
Scale 1: 19,500,000
Lambert Conformal Conic Projection, standard parallels 40°N and 56°N
300 Kilometers
300 Miles

9

Future plans ($\gtrsim$ 2027): KATRIN++, 50 g of ^{3}H (use of atomic ^{3}H); goal: reach sensitivity to $m_\beta < 0.04$ eV.

In development: PROJECT 8 (CRES technique, B. Monreal, J. A. Formaggio, Phys.Rev.D 80(2009) 051301):

$$2\pi\, f(E_\beta) = \frac{eB}{E_\beta + m_e};\ \text{Energy resolution:}\ \frac{\Delta E}{m_e} = \frac{\Delta f}{f}$$

CRES technique demonstrated by Project 8 for e^- magnetically trapped inside a wave guide; best E-resolution $\Delta E_{FWHM} = 1.7$ eV at 18 keV.
Limit obtained $m_\beta < 152$ eV; goal: $m_\beta < 0.04$ eV.

In development: HOLMES and ECHo e^- capture calorimetric experiments,
$^{163}Ho \rightarrow ^{163}Dy[H_i] + \nu_e$, $^{163}Dy[H_i] \rightarrow ^{163}Dy + E_c$,
H_i =M1, M2, N1, N2, O1, O2, P1; EC from shell $\geq$ M1.
$Q = 2863.2 \pm 0.6$ eV; $\tau_{1/2} \cong 4570$ years; 2×10^{11} ^{163}Ho nuclei $\leftrightarrow$ 1 Bq.
End-point rate and m_β sensitivity depend on $Q - E_b(M1)$.
Current limits $m_{\nu_e} < 28$ eV (HOLMES), 19 eV (ECHo) (90% C.L.)

A. Nicciotti, talk at Neutrino 2024

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$ - the current most stringent constraints (Planck CMB + BAO data + lensing data + the Hubble constant datum [H0(R19)] + ΛCDM (6 parameter) model + assuming 3 light massive neutrinos):

$$\sum_j m_j \equiv \Sigma < 0.12 \text{ eV} \quad (95\% \text{ C.L.})$$

The upper limit depends on the data set and assumptions used. According to F. Capozzi et al., arXiv:2503.07752, it reads:

$$\sum_j m_j \equiv \Sigma < 0.064 - 0.616 \text{ eV} \quad (95\% \text{ C.L.})$$

where 0.616 eV corresponds to the data set used which leads to one of the most conservative result (Λ CDM+Σ+ A_{lens}+Planck+lensing data).

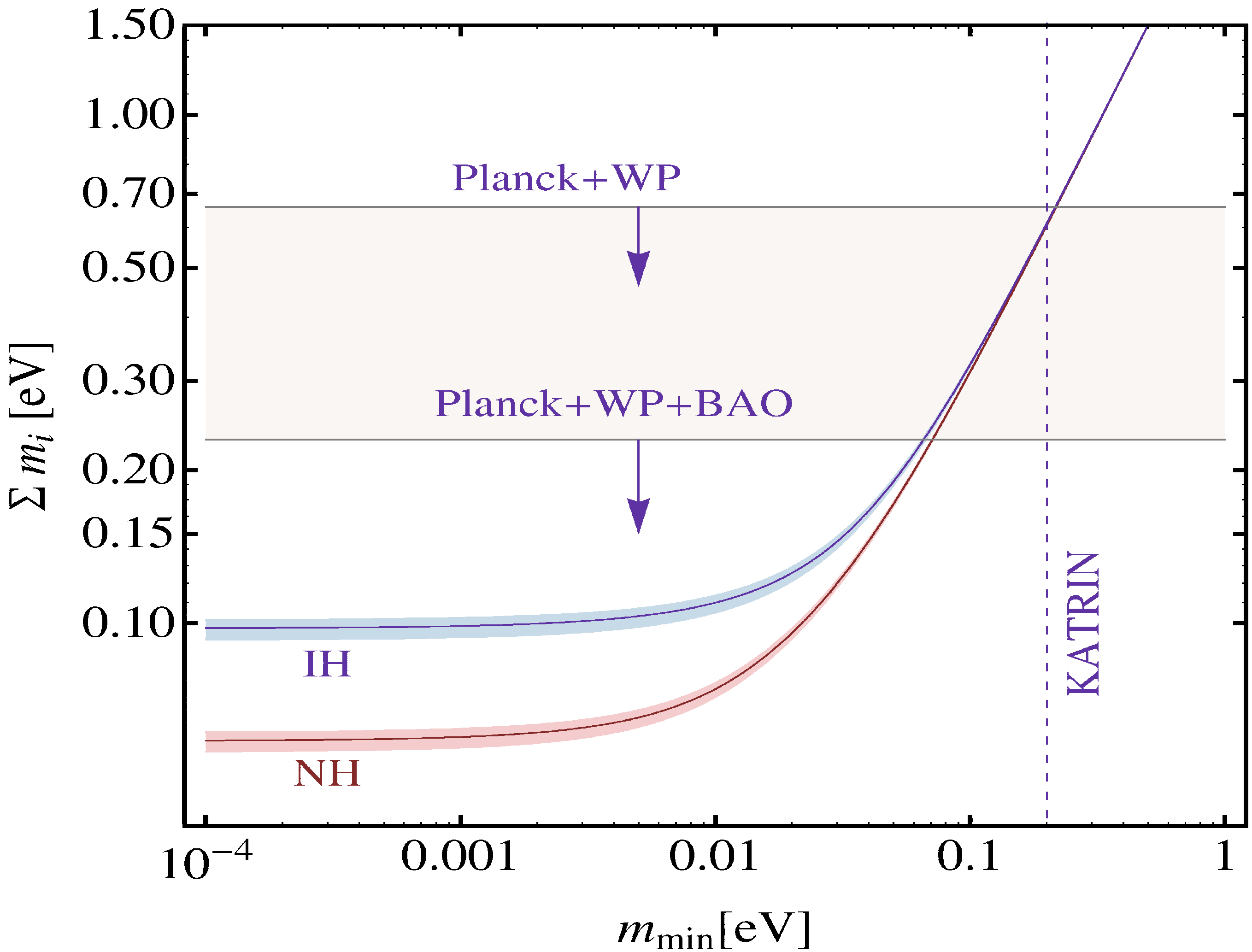
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP, Planck and currently taking data EUCLID experiments might allow to determine

$$\sum_j m_j : \qquad \delta \cong (0.01 - 0.04) \text{ eV}.$$

Similar sensitivity ($\delta \cong 0.03$ eV) is planned to be reached in CMB-S4 experiment, and/or combining data from DESI and CMB-S4 experiments ($\delta \cong 0.012 - 0.020$ eV).

Talks by M. Archidiacono and W. Elbers at Neutrino 2024, June 17-22, Milano

Mass and Hierarchy from Cosmology



NH ($m_1 \cong 0$): $\sum_j m_j \cong m_2 + m_3 = \sqrt{\Delta m^2_{21}} + \sqrt{\Delta m^2_{31}} \cong 0.061$ **eV** **(3σ max);**

IH ($m_3 \cong 0$): $\sum_j m_j \cong m_1 + m_2 \cong 2\sqrt{\Delta m^2_{23}} \cong 0.098$ **eV** **(3σ min);** $\sum_j m_j \gtrsim 0.098$ **eV.**

Reference 3-ν Mixing Scheme

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

Data: 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 0.5$ **eV;**
KATRIN: $m_{\bar{\nu}_e} < 0.45$ **eV;**
Cosmology: $\sum_j m_j$ < **0.064 - 0.616 eV** **(95% CL; 2503.07752).**
The value of min(m_j) and "mass ordering" unknown.

Δm^2_{21}, $|\Delta m^2_{31}|$ - **known (sgn(Δm^2_{31}) - unknown).**

ν_j, $m_j \neq 0$: **nature - Dirac or Majorana - unknown.**

The PMNS matrix U - 3×3 **unitary:** θ_{12}, θ_{13}, θ_{23} - **known; CPV phases** δ, α_{21}, α_{31} - **unknown.**

Thus, 5 known + 4 unknown parameters + MO.
"Known" = measured; "unknown" = not measured.

m_e, m_μ, m_τ **also known - used as input.**

Of fundamental importance are:

• determining the status of CP symmetry in the lepton sector and high precision measurement of δ (T2K, NOνA T2HK, DUNE); leptonic CPV might be at the origin of matter-antimatter (or baryon) asymmetry of the Universe; critical test of symmetry origin of the ν-mixing pattern.

• the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics) (LEGEND (GERDA, MJORANA), KamLAND-Zen II, CUORE (CUPID), nEXO (EXO), SNO+, PANDA-X, NEXT, ...);

• determination of the type of spectrum neutrino masses possess, or the "neutrino mass ordering" (T2K + NOνA; JUNO; ORCA; T2HK+HK(atm.data); DUNE; INO);

• determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology).

• High precision determination of $\sin^2\theta_{23}$ (T2HK+HK(atm.data); DUNE) - relevant, e.g., for ν-osc. tomography of the Earth (ORCA), and $\sin^2\theta_{12}$ (JUNO); both crucial for tests of ideas on origin of the ν-mixing pattern.

The program of research extends beyond 2040.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in $U_{\rm PMNS}$;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in $U_{\rm PMNS}$?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in $U_{\rm PMNS}$ be the leptogenesis CPV parameters at the origin of BAU?

BS3νRM: eV scale sterile ν's; NSIs; ChLFV processes ($\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- - e^-$ conversion on (A,Z)); $\nu-$related BSM physics at the TeV scale (N_{jR}, H^{--}, H^-, etc.).

Research in Neutrino Physics: we strive to understand at deepest level what are the origins of neutrino masses and mixing and what determines the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years. And we try to understand what are the implications of the remarkable discovery that neutrinos have mass, mix and oscillate for elementary particle physics, cosmology and for better understanding of the Earth, the Sun, the stars, formation of Galaxies, the Early Universe, i.e., for better deeper understanding of Nature in general.

The Framework: the Reference 3-ν Mixing Scheme

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary.

ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 0.5$ eV.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

The Problem

Understanding the origin of the peculiar pattern of neutrino mixing of two large and one small mixing angles is one of the major problems in neutrino physics.

It is a part of the more general highly challenging and still unresolved fundamental problem in particle physics of understanding the origins of the patterns of the charged lepton and neutrino masses and of neutrino mixing, of quark masses and mixing, and of CP violation in the quark and lepton sector, i.e., understanding the origins of the lepton and quark flavours.

"Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn't have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses."

From "Model Physicist", CERN Courier, 13 October 2017.

The renewed attempts to seek new better solutions of the flavour problem than those already proposed were stimulated primarily by the remarkable progress made in the studies of neutrino oscillations, which began 24 years ago with the discovery of oscillations of atmospheric ν_μ and $\bar{\nu}_\mu$ by SuperKamiokande experiment. This lead, in particular, to the determination of the pattern of the 3-neutrino mixing, which turn out to consist of two large and one small mixing angles.

The Lepton Flavour Problem

Consists of three basic elements (sub-problems), namely, understanding:

• **Why $m_{\nu_j} <<< m_{e,\mu,\tau}, m_q$, $q = u,c,t,d,s,b$ ($m_{\nu_j} \lesssim 0.5$ eV, $m_l \geq 0.511$ MeV, $m_q \gtrsim 2$ MeV);**

• **The origins of the patterns of**
i) neutrino mixing of 2 large and 1 small angles ($\theta^l_{12} = 33.4^\circ$, $\theta^l_{23} = 42.4^\circ$ (49.0°), $\theta^l_{13} = 8.59^\circ$),
and of ii) Δm^2_{ij}, i.e., of $\Delta m^2_{21} \ll |\Delta m^2_{31}|$, $\Delta m^2_{21}/|\Delta m^2_{31}| \cong 1/30$.

• **The origin of the hierarchical pattern of charged lepton masses:**
$m_e \ll m_\mu \ll m_\tau$, $m_e/m_\mu \cong 1/200$, $m_\mu/m_\tau \cong 1/17$.

The first two added new important aspects to the flavour problem.

$m_{\nu_j} <<< m_{e,\mu,\tau}, m_q$, $q = u,c,t,d,s,b$:

seesaw mechanism(s), Weinberg operator, radiative ν mass generation, extra dimensions.
However, additional input (symmetries) needed to explain the pattern of lepton mixing and to get specific testable predictions.

The Neutrino Mixing Problem

The most elegant, simple and testable solution of the neutrino mixing problem is arguably provided by the non-Abelian discrete symmetry approach.

In what concerns the lepton flavour problem, in the last 6 years a very successful and attractive approach based on Modular Invariance has been and continues to be developed.

The Non-Abelian Discrete Symmetry Approach

With the observed pattern of neutrino mixing Nature is sending us a Message. The Message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. In my opinion, Nature gave us also a hint what the content of Nature's Message is.

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}}(=\frac{\pi}{5}) - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$,
$\theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 \mp 0.10.$

- $U_{\rm PMNS}$ **due to new approximate symmetry?**

A Natural Possibility: $U_{\rm PMNS} = U_{\rm l}^{\dagger} U_{\nu}$

$$\mathbf{U}_{\rm PMNS} = \mathbf{U}_{\rm l}^{\dagger}(\theta^{\ell}_{\rm ij}, \delta^{\ell})\; \mathbf{Q}(\psi,\omega)\mathbf{U}_{\rm TBM,BM,LC,...}\; \mathbf{\bar{P}}(\xi_1,\xi_2)\,,$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1,\xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix} ; \quad \mu - \tau \ \text{symmetry}: \quad \theta^{\nu}_{23} = \mp\pi/4 ;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} ; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \quad \theta^{\nu}_{12} = \pi/6 .$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

GRB and HG mixing: W. Rodejohann et al., 2009.

$U_{\text{TBM(BM)}}$...: Groups A_4, T', S_4 (S_4),...(vast literature)

E. Ma, G. Rajasekaran, hap-ph/0106291; K. Babu, E. Ma, J.F.W. Valle, hep-ph/0206292; G. Altarelli, F. Feruglio, hep-ph/0512103; C.S. Lam, 0708.3665 and 0804.2622; W. Grimus, L.Lavoura, 0809.0226; Z.-Z. Xing, 1106.3244; S. Zhou, 1205.0761; F. Feruglio, C. Hagedorn, R. Ziegler, 1211.5560; M. Holthausen, M. Lindner, M.A. Schmidt, 1211.6953; A. Meroni, S.T.P., M. Spinrath, 1312:1966; S.T.P., 1405.6006; ...
(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King, Ch. Luhn, arXiv:1301.1340;...)

- U_{GRA}: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

- U_{LC}: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

- U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter;
$s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

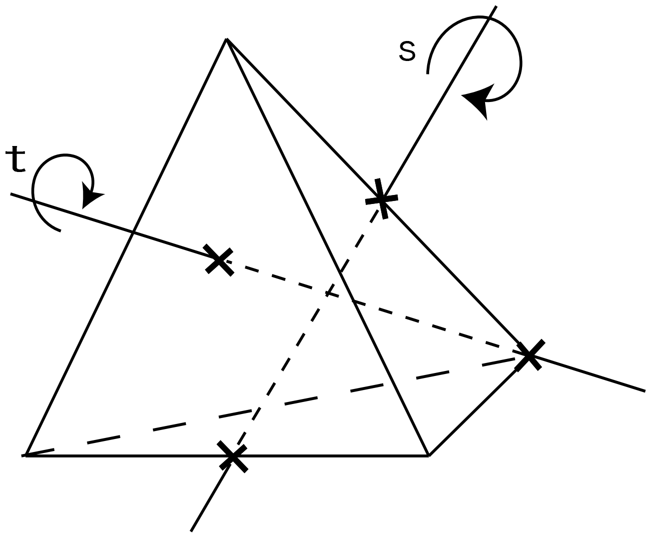
• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

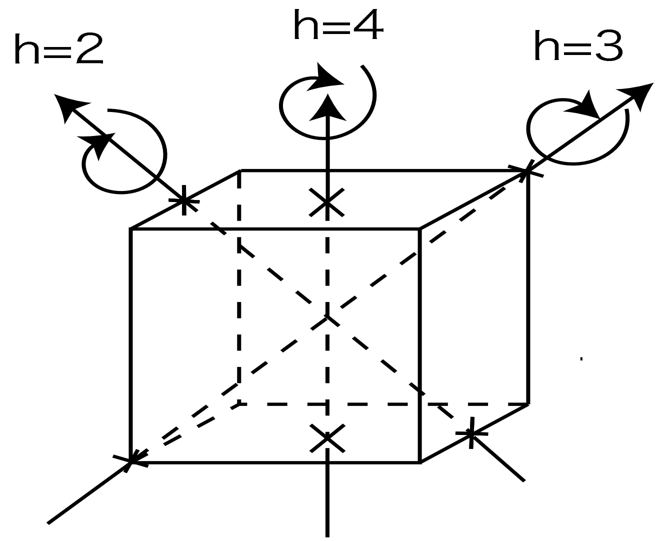
For all symmetry forms considered we have: $\theta^{\nu}_{13} = 0$, $\theta^{\nu}_{23} = \mp\pi/4$.
They differ by the value of θ^{ν}_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^{\nu}_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.

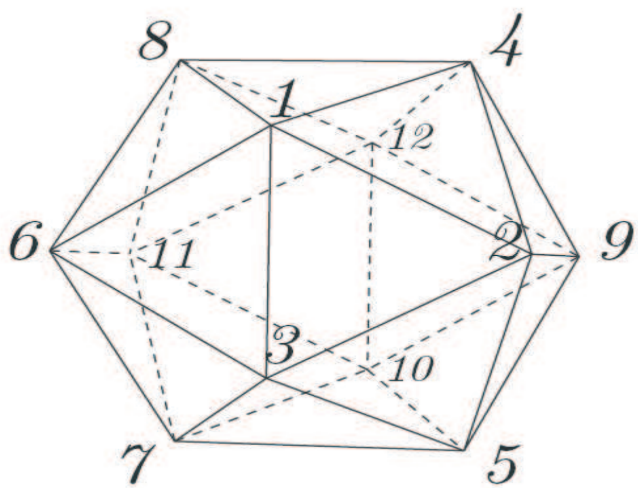
The observed pattern of 3-ν mixing, two large and one small mixing angles,
$\theta_{12} \cong 33^\circ$, $\theta_{23} \cong 45^\circ \pm 6^\circ$ **and** $\theta_{13} \cong 8.4^\circ$,
can most naturally be explained by extending the Standard Model (SM) with a flavour symmetry corresponding to a non-Abelian discrete (finite) group G_f.

$G_f = A_4$, T', S_4, A_5, D_{10}, D_{12},...

Vast literature; reviews: G. Altarelli, F. Feruglio, 1002.0211; H. Ishimori et al., 1003.3552; M. Tanimoto, AIP Conf.Proc. 1666 (2015) 120002; S. King , Ch. Luhn, 1301.1340; D. Meloni, 1709.02662; STP, 1711.10806; F. Feruglio, A. Romanino, 1912.06028







Examples of symmetries: A_4, S_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

Group	Number of elements	Generators	Irreducible representations
S_4	24	S, T (U)	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, $\mathbf{3}'$
S_4'	48	S, T (R)	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, $\mathbf{3}'$, $\hat{\mathbf{1}}$, $\hat{\mathbf{1}}'$, $\hat{\mathbf{2}}$, $\hat{\mathbf{3}}$, $\hat{\mathbf{3}}'$
A_4	12	S, T	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$
T'	24	S, T (R)	$\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$, $\mathbf{3}$
A_5	60	$\tilde{S}$, $\tilde{T}$	$\mathbf{1}$, $\mathbf{3}$, $\mathbf{3}'$, $\mathbf{4}$, $\mathbf{5}$
A_5'	120	$\tilde{S}$, $\tilde{T}$	$\mathbf{1}$, $\mathbf{3}$, $\mathbf{3}'$, $\mathbf{4}$, $\mathbf{5}$, $\hat{\mathbf{2}}$, $\hat{\mathbf{2}}'$, $\hat{\mathbf{4}}$, $\hat{\mathbf{6}}$.

Number of elements, generators and irreducible representations of S_4, S_4', A_4, $A_4' \equiv T'$, A_5 and A_5' discrete groups.

Predictions and Correlations

$U_{\nu} = U_{\rm TBM,BM,GRA,GRB,HG}\,\bar{P}(\xi_1,\xi_2)$; θ^{ν}_{12};

$U_{\ell}^{\dagger} = R_{12}(\theta^{\ell}_{12})\,Q$, $Q = {\rm diag}(e^{i\varphi},1,1)$: $\theta^{\ell}_{12},\varphi$

(the "minimal" = simplest case ($SU(5)\times T'$,...)

$U_{\ell}^{\dagger} \;=\; R_{12}(\theta^{\ell}_{12})\,R_{23}(\theta^{\ell}_{23})Q$, $Q \;=\; {\rm diag}(1,e^{-i\psi},e^{-i\omega})$,

(next-to-minimal case): $\theta^{\ell}_{12},\hat{\theta}^{\ell}_{23},\phi$

$\theta_{12},\theta_{23},\theta_{13},\delta$ **in terms of** $\theta^{\ell}_{12},\hat{\theta}^{\ell}_{23},\phi$ **+** θ^{ν}_{12}

$\cos\delta = \cos\delta(\theta_{12},\theta_{23},\theta_{13};\theta^{\nu}_{12},...)$,

$J_{CP} = J_{CP}(\theta_{12},\theta_{23},\theta_{13},\delta) = J_{CP}(\theta_{12},\theta_{23},\theta_{13};\theta^{\nu}_{12},...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

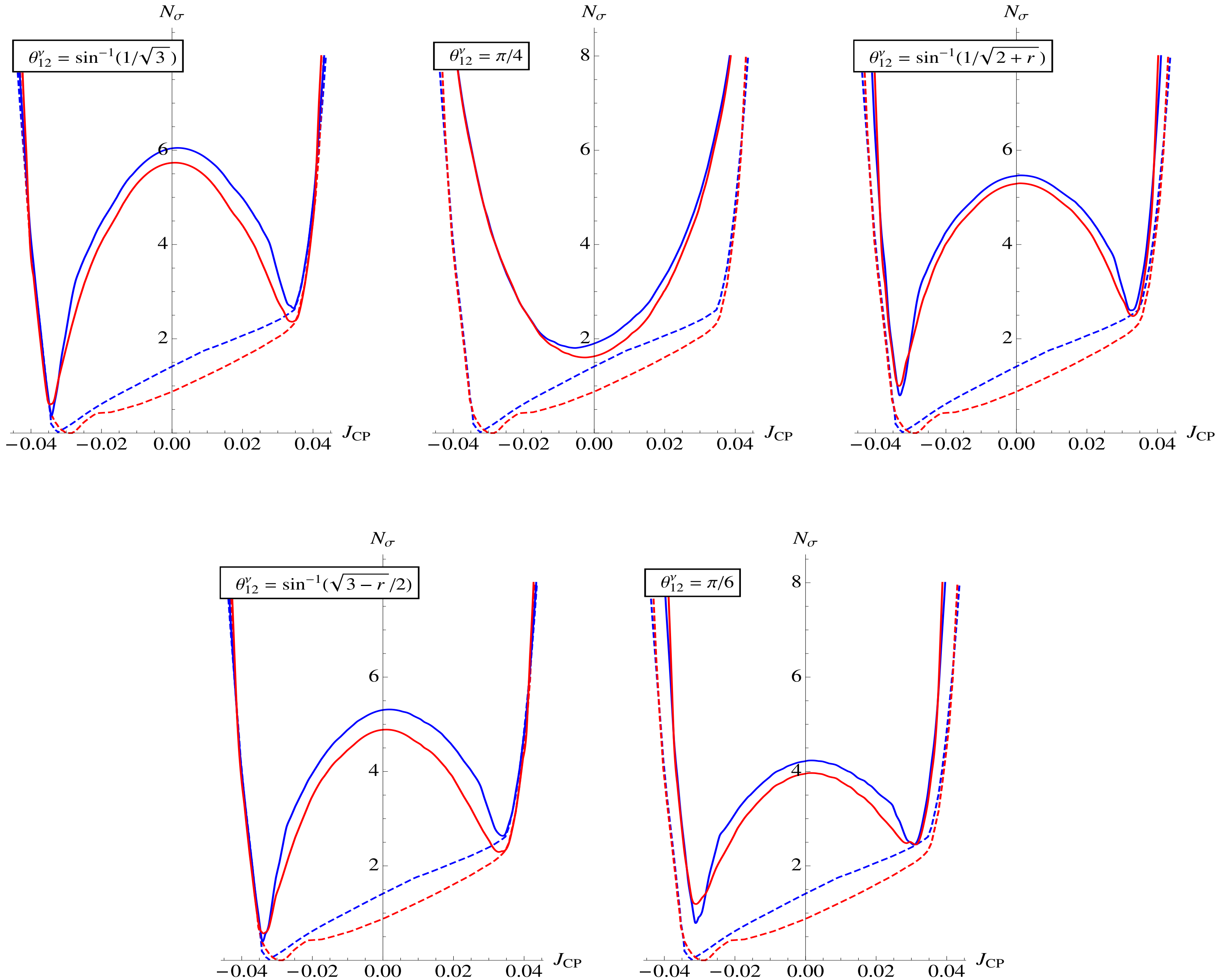
For arbitrary fixed θ^{ν}_{12} and any θ_{23} ("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right].$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.



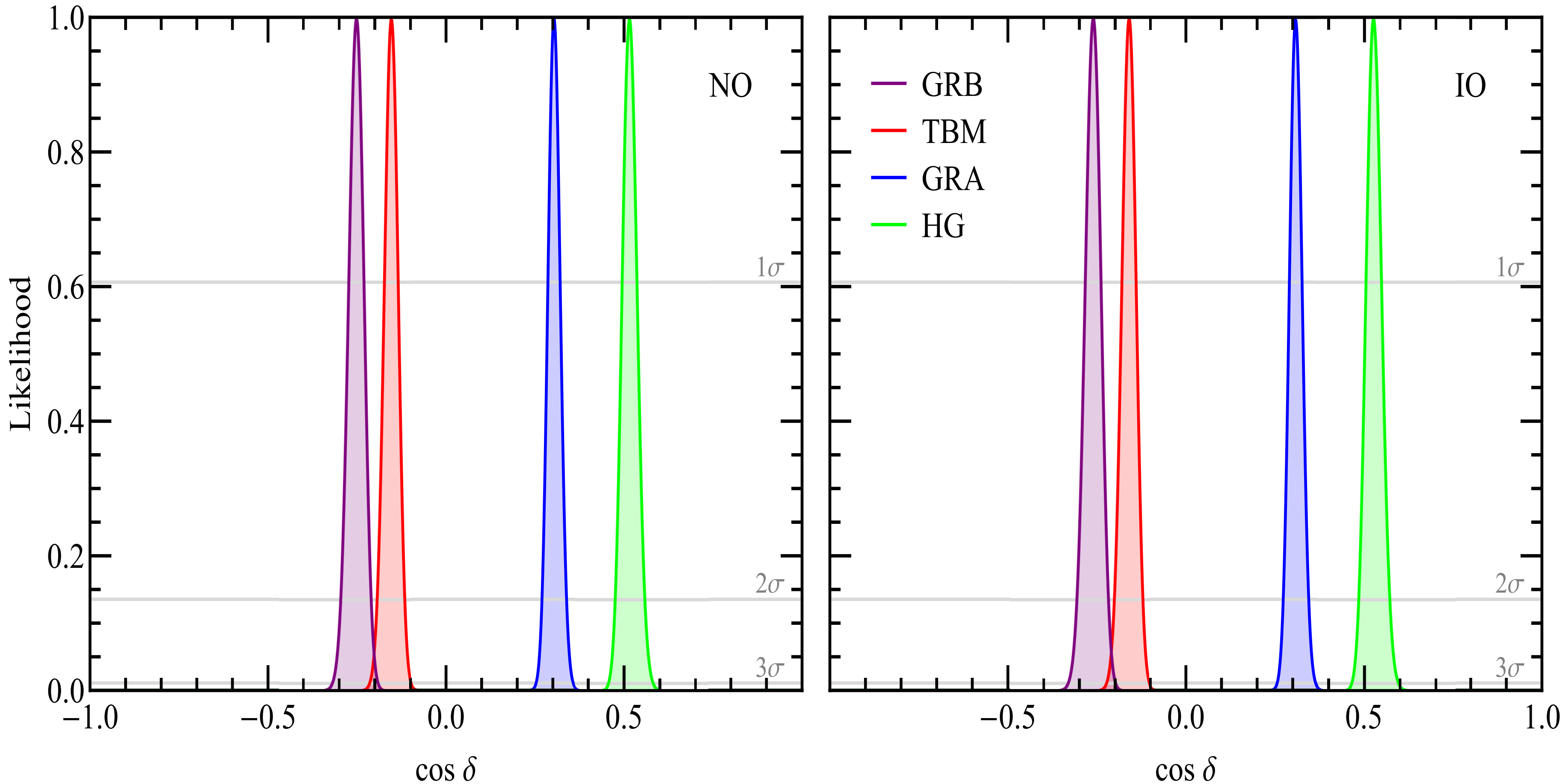
I. Girardi, S.T.P., A. Titov

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2K, NOνA combined).**

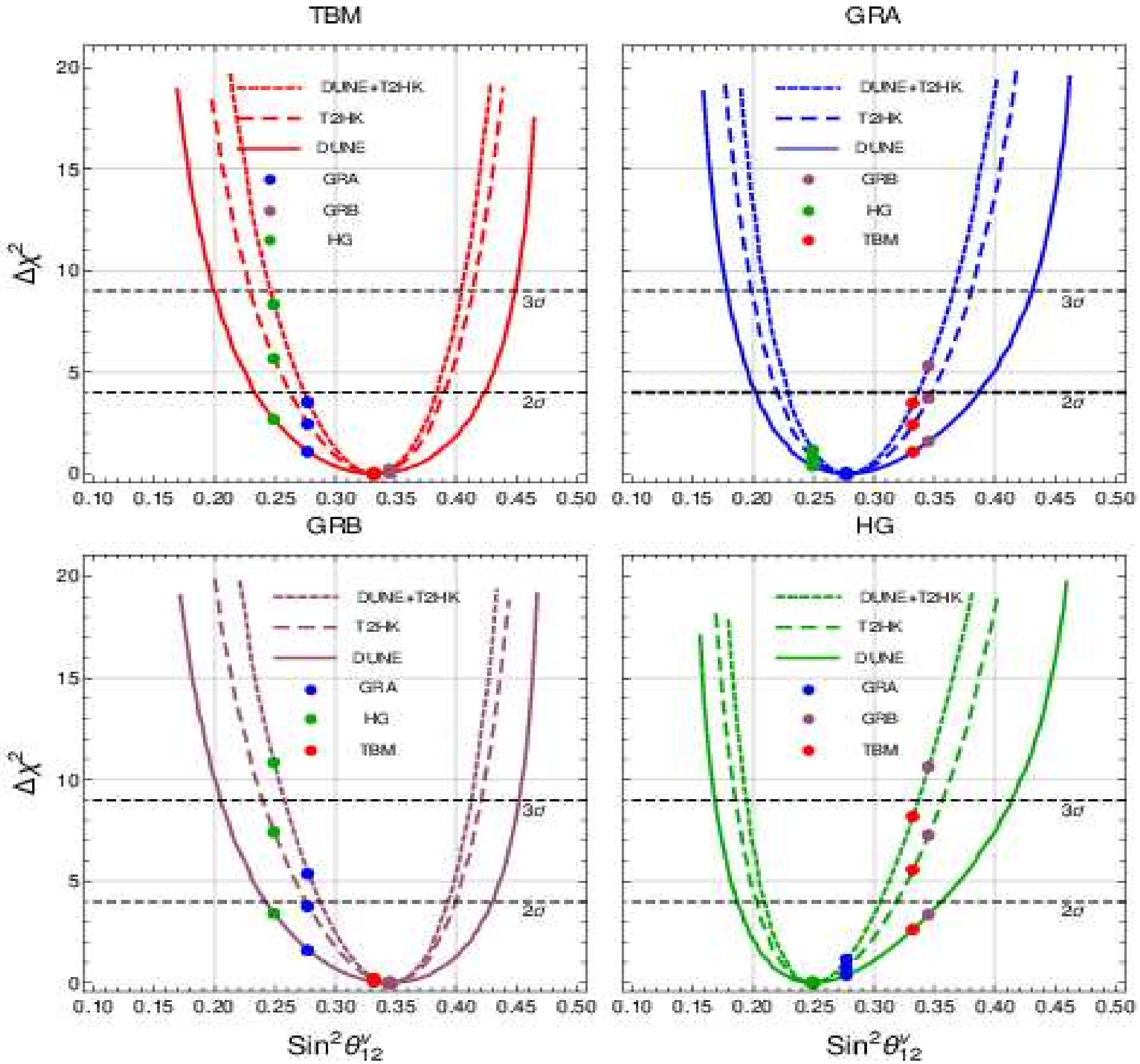


I. Girardi, S.T.P., A. Titov

b.f.v. of $\sin^2\theta_{ij}$ **(Esteban et al., Jan., 2018) + the prospective precision used.**

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right] .$$

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



Agarwalla, Chatterjee, STP, Titov, arXiv:1711.02107

GRB - HG $> 3\sigma$; GRA - GRB $\geq 2\sigma$; TMB - HG $\cong 3\sigma$; TMB - GRA $\cong 2\sigma$. DUNE + T2HK prospective data used.

How does it Work.

Choose G_f.

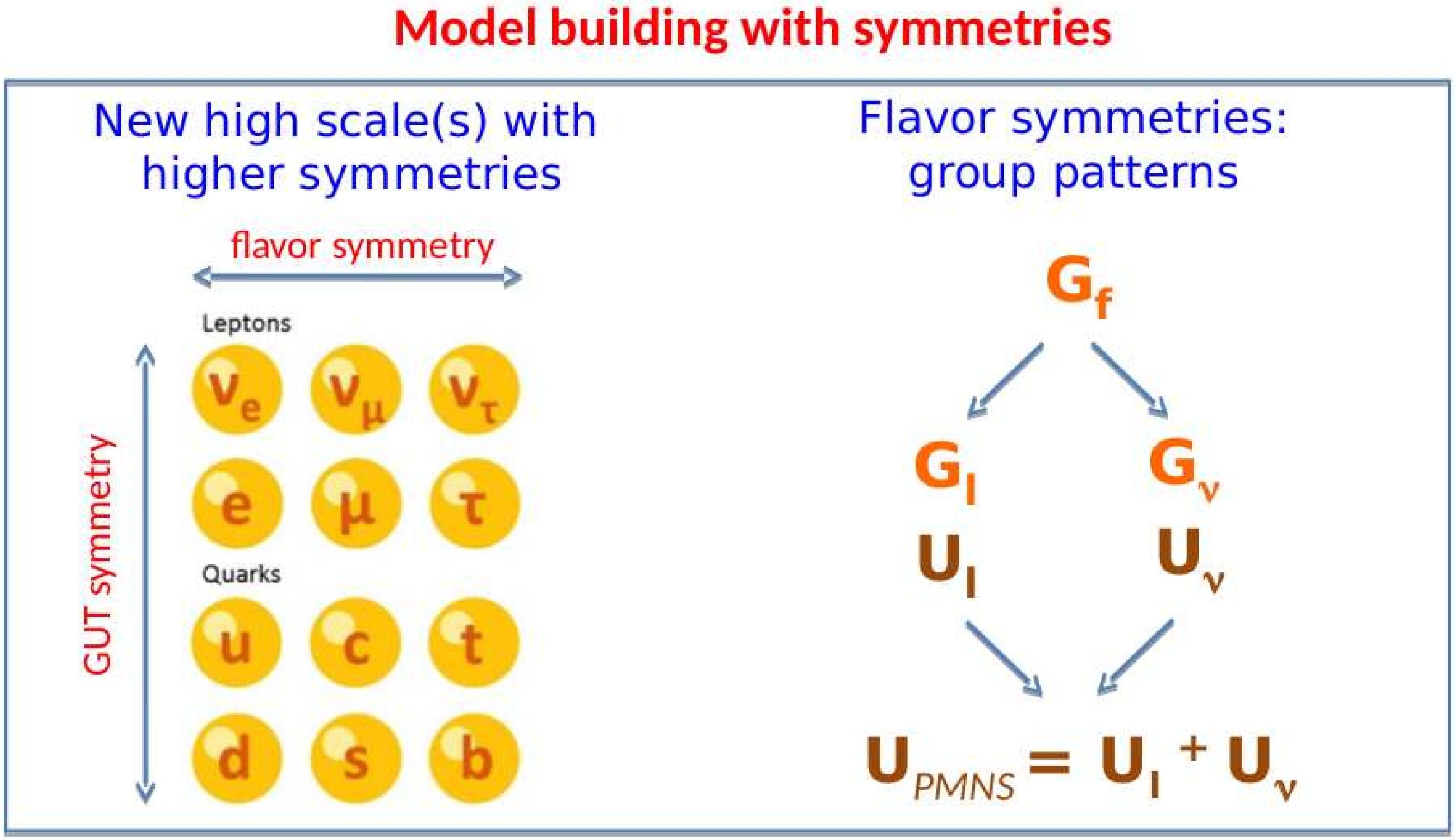
$\nu_{eL}(x), \nu_{\mu L}(x), \nu_{\tau L}(x)$: **assigned to $\rho^{(\nu)}(g_f)$ - irreducible representation of G_f, where g_f is an element of G_f.**

$e_L(x), \mu_L(x), \tau_L(x)$: **assigned to $\rho^{(e)}(g_f)$ - IRREP of G_f.**

$G_f = S_4$, A_4, T', A_5: $\rho^{(\nu)}(g_f)$, $\rho^{(e)}(g_f)$ - **triplet IRREP.**

$e_R(x), \mu_R(x), \tau_R(x)$: **singlets of G_f.**

How Does it Work



E. Lisi, TAUP 2019

ν_j, **Majorana mass term,** $m_j \neq m_k$, $j \neq k = 1,2,3$: $G_\nu = Z_2 \times Z_2$, Z_2

$G_e = Z_2$; Z_n, $n > 2$; $Z_n \times Z_m$, $n, m \geq 2$

M_e - charged lepton mass matrix (L-R convention).

U_e: $U_e^\dagger M_e M_e^\dagger U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)$.

G_e - residual symmetry group of $M_e M_e^\dagger$:

$\rho^{(e)}(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger$,

$\rho^{(e)}(g_e)$ generator(s) of G_e in the triplet rep.

$\rho^{(e)}(g_e)$ and $M_e M_e^\dagger$ commute: both are diagonalised by U_e.
$\rho^{(e)}(g_e)$ - known! Thus, U_e - fixed!

M_ν - neutrino Majorana mass matrix (R-L convention).

U_ν: $U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$.

G_ν - residual symmetry group of M_ν:

$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$,

g_ν: an element of G_ν,
$\rho(g_\nu)$ generator of G_ν in the triplet repr.

$\rho(g_\nu)$ and $M_\nu^\dagger M_\nu$ commute: both are diagonalised by U_ν.
$\rho(g_\nu)$ - known! Thus, U_ν-fixed.

$$U_{\text{PMNS}} = U_e^\dagger U_\nu$$

A_4: $G_e = Z_3^T = \{1, T, T^2\}$, $G_\nu = Z_2^S = \{1, S\}$

$(S^2 = T^3 = (ST)^3 = \mathbf{I})$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \omega = e^{i2\pi\tau/3} \ (\mathrm{A-F}).$$

$U_e = \mathbf{I}$, $U_{\mathrm{PMNS}} = U_e^\dagger U_\nu = U_{\mathrm{TBM}} U_{13}(\theta^\nu_{13}, \alpha)$, θ^ν_{13}, α - free.

W. Grimus, L. Lavoura, 2008

$$\sin^2\theta_{12} = \frac{1}{3(1-\sin^2\theta_{13})} \cong 0.34;$$

$$\cos\delta = \frac{\cos 2\theta_{23}\cos 2\theta_{13}}{\sin 2\theta_{23}\sin\theta_{13}(2-3\sin^2\theta_{13})^{\frac{1}{2}}};$$ **if** $\theta_{23} = \frac{\pi}{4}$, $\delta = \pm\frac{\pi}{2}$.

The Symmtry Breaking

The correct lepton mixing pattern in a model with non-Abelian discrete symmetry G_f is determined by the appropriate choice of residual symmetries G_e and G_ν and is not directly related to the charged lepton and neutrino mass generation.
The breaking of G_f has to ensure the correct generation of the fermion masses and keep G_e and G_ν intact.

Examples of Predictions and Correlations II.

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}(1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong \frac{1}{2}(1 \mp 0.022)$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545;\ 0.604$.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.340$.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 - 2\sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.319$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$\theta^{\nu}_{12}, ...$ - **known (fixed) parameters, depend on the underlying symmetry**.

The Approach is testable/falsifyable experimentally!

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Prospective (useful/requested) precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE; T2K+NOνA(?)).**

$\delta(\delta) \leq 14^\circ$ **at** $\delta = 3\pi/2$ **(**$\delta(\delta) = 10^\circ$**)**
(ESSνSB: $\sim$8%, A. Alekou et al., EPJ ST 231 (2022) 379; THKK?; DUNE: accounting for both the 1st and 2nd probability maxima, Jogesh Rout, Poonam Mehta et al., PRD 2021, S. Goswami et al., 2012.04958)

The Power of Data

Systematic analysis (I. Girardi *et al.*):
all possible combinations of residual symmetries G_e and G_ν of the lepton flavour symmetry groups $G_f = S_4$, A_4, T' and A_5, leading to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, were considered.

(A) $G_e = Z_2$ **and** $G_\nu = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$;

(B) $G_e = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$ **and** $G_\nu = Z_2$;

(C) $G_e = Z_2$ **and** $G_\nu = Z_2$.

In these cases $U_e^\dagger$ and/or U_ν of $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, are partially (or fully) determined by residual discrete symmetries of $G_f = S_4$, A_4, T', A_5.

$G_f = A_4$, S_4, T', A_5.

A_4: 3 Z_2, 4 Z_3, 1 $Z_2 \times Z_2$ subgroups (total 8).

T': similar to A_4.

S_4: 9 Z_2, 4 Z_3, 3 Z_4, 4 $Z_2 \times Z_2$ subgroups (total 20).

A_5: has 15 Z_2, 10 Z_3, 6 Z_5, 5 $Z_2 \times Z_2$ subgroups (36).

In the case of A_4 (T') symmetry only there are 64 models (up to permutation of rows and columns). Of these only 8 lead to distinct predictions for $U_{\rm PMNS}$, while only 5 cases a priori can be phenomenologically viable, i.e., they lead to $U_{\rm PMNS}$ without zero entries. Similar analyses can be performed for the S_4 and A_5 symmetries.

A_4:
$(G_e, G_\nu) = (Z_2, Z_3)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_3, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **C1 - C9.**

For A_4, S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2\theta_{ij}$ from the current data and the requirement $|\cos\delta| \leq 1$.

More specifically:

A. $G_e = Z_2$, $G_\nu = Z_n$, $n > 2$ **or** $Z_n \times Z_m$, $n,m \geq 2$;
U_ν **fixed;** $U_e = U_{ij}(\theta^e_{ij}, \delta_{ij})$, $ij = 12(A1); 13(A2), 23(A3)$;
θ_{23}, $\cos\delta$ **(**θ_{12}, θ_{13}**) predicted.**

B. $G_e = Z_n$, $n > 2$ **or** $G_e = Z_n \times Z_m$, $n,m \geq 2$, $G_\nu = Z_2$;
$U_\nu = U_{\rm sym} U_{ij}(\theta^\nu_{ij}, \delta_{ij}) Q_0$, $ij = 13, 12, 23$**(B1,B2,B3);**
U_e **fixed:** θ_{12}, $\cos\delta$ **(**θ_{23}, θ_{13}**) predicted.**

C. $G_e = Z_2$ **and** $G_\nu = Z_2$: U_e **- up to** $U_{ij}(\theta^e_{ij}, \delta^\varepsilon_{ij})$,
U_ν **- up to** $U_{ij}(\theta^\nu_{ij}, \delta^\nu_{ij})$, $ij = 12; 13, 23$ **(C1 - C9);**
θ_{12} **or** θ_{23} **or** $\cos\delta$ **predicted.**

For A_4, S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2\theta_{ij}$ from the current data and the requirement $|\cos\delta| \leq 1$.

Pattern A: $G_e = Z_2$ and $G_\nu = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$.

Cases A1-A2, θ_{23}, δ predicted.

Case A1 (U_e up to U in 1-2 plane)

$$\sin^2\theta_{23} = 1 - \frac{\cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,,$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{23} - \cos^2\theta_{12}) + \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\circ_{13}\cos\theta^\circ_{23}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23})^{\frac{1}{2}}}\,,$$

where θ°_{13} and θ°_{23} are fixed once the flavour symmetry group G_f and the residual symmetry subgroups G_e and G_ν are specified.

Case A2 (U_e up to U in 1-3 plane)

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,,$$

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{12}) + \sin^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\sin\theta^\circ_{23}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{23})^{\frac{1}{2}}}\,,$$

where θ°_{12} and θ°_{23} are fixed once G_f, G_e and G_ν are specified.

Case A3 (U_e up to U in 2-3 plane):

$\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$ **and** $\sin^2\theta_{12} = \sin^2\theta^\circ_{12}$ **are predicted, while** $\cos\delta$ **remains unconstrained.**

Pattern B: $G_e = Z_k$, $k > 2$ or $Z_m \times Z_n$, $m, n \geq 2$ and $G_\nu = Z_2$.

Cases B1-B2, θ_{12}, δ predicted.

Case B1 (U_ν up to U in 1-3 plane)

$$\sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}\,,$$

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23} - \cos^2\theta_{23}) + \sin^2\theta^\circ_{12}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\sin\theta^\circ_{12}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}\,,$$

where θ°_{12} **and** θ°_{23} **are fixed once the symmetries are specified.**

Case B2 (U_ν up to U in the 2-3 plane)

$$\sin^2\theta_{12} = 1 - \frac{\cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}}{1-\sin^2\theta_{13}}\,,$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{12} - \cos^2\theta_{23}) + \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\cos\theta^\circ_{12}\cos\theta^\circ_{13}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13})^{\frac{1}{2}}}\,.$$

Case B3 (U_ν up to U in the 1-2 plane)

$\sin^2\theta_{13} = \sin^2\theta^\circ_{13}$ **and** $\sin^2\theta_{23} = \sin^2\theta^\circ_{23}$ **are predicted; and no sum rule for** $\cos\delta$**.**

Pattern C: $G_e = Z_2$ and $G_\nu = Z_2$. In this case, both U_e and U_ν are determined up to $U(2)$ transformations in the i-j and k-l planes, respectively. Depending on the planes in which the free $U(2)$ transformations are performed, we have nine possibilities. We number them as cases C1–C9. Four of them lead to sum rules for $\cos\delta$, which we summarise bellow.

C1, $(ij,kl) = (12,13)$: $$\cos\delta = \frac{\sin^2\theta^\circ_{23} - \cos^2\theta_{12}\sin^2\theta_{23} - \cos^2\theta_{23}\sin^2\theta_{12}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,,$$

C3, $(ij,kl) = (12,23)$: $$\cos\delta = \frac{\sin^2\theta_{12}\sin^2\theta_{23} - \sin^2\theta^\circ_{13} + \cos^2\theta_{12}\cos^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,,$$

C4, $(ij,kl) = (13,23)$: $$\cos\delta = \frac{\sin^2\theta^\circ_{12} - \cos^2\theta_{23}\sin^2\theta_{12} - \cos^2\theta_{12}\sin^2\theta_{13}\sin^2\theta_{23}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,,$$

C8, $(ij,kl) = (13,13)$: $$\cos\delta = \frac{\cos^2\theta_{12}\cos^2\theta_{23} - \cos^2\theta^\circ_{23} + \sin^2\theta_{12}\sin^2\theta_{23}\sin^2\theta_{13}}{\sin\theta_{13}\sin 2\theta_{23}\sin\theta_{12}\cos\theta_{12}}\,.$$

The two cases, C5 and C9, yield correlations between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$:

$$\text{C5, } (ij,kl) = (23,13): \quad \sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}\,,$$

$$\text{C9, } (ij,kl) = (23,23): \quad \sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}-\sin^2\theta_{13}}{1-\sin^2\theta_{13}}\,.$$

In cases C2 and C7, instead, there are correlations between $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$:

$$\text{C2, } (ij,kl) = (13,12): \quad \sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}\,,$$

$$\text{C7, } (ij,kl) = (12,12): \quad \sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}-\sin^2\theta_{13}}{1-\sin^2\theta_{13}}\,.$$

Finally, in case C6, $(ij,kl) = (23,12)$, $\sin^2\theta_{13}$ is predicted to be equal to $\sin^2\theta^\circ_{13}$.
In cases C2, C5, C6, C7 and C9, $\cos\delta$ remains unconstrained.

$G_f = A_4$: **B1 is the only one phenomenologically viable case with** $(G_e, G_\nu) = (Z_3, Z_2)$**, which yields** $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$ **and corresponds to the TBM mixing matrix corrected from the right by a** $U(2)$ **transformation in the 1-3 plane;**
$\cos\delta = -0.353$**,** $\sin^2\theta_{12} = 0.341$**.**

In the case of $G_f = S_4$ there are 8 viable cases. We summarise them in following Table.

[t]

(G_e, G_ν)	Case	$\sin^2\theta^\circ_{ij}$	$\cos\delta$	$\sin^2\theta_{ij}$
(Z_3, Z_2)	**B1**	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	-0.353	$\sin^2\theta_{12} = 0.341$
	B2S$_4$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (1/6, 1/5)$	0.167	$\sin^2\theta_{12} = 0.318$
(Z_2, Z_2)	**C1**	$\sin^2\theta^\circ_{23} = 1/4$	-1^*	**not fixed**
	C2S$_4$	$\sin^2\theta^\circ_{23} = 1/2$	**not fixed**	$\sin^2\theta_{23} = 0.511$
	C3	$\sin^2\theta^\circ_{13} = 1/4$	-1^*	**not fixed**
	C4	$\sin^2\theta^\circ_{12} = 1/4$	1^*	**not fixed**
	C7S$_4$	$\sin^2\theta^\circ_{23} = 1/2$	**not fixed**	$\sin^2\theta_{23} = 0.489$
	C8	$\sin^2\theta^\circ_{23} = 3/4$	1^*	**not fixed**

The viable cases for $G_f = S_4$. The values of $\cos\delta$ and $\sin^2\theta_{12}/\sin^2\theta_{23}$ are obtained using the best fit values of the relevant (not fixed) mixing angles for NO. In the cases marked with an asterisk, physical values of $\cos\delta$ cannot be obtained employing the best fit values of the mixing angles, but they are achievable fixing two angles to their best fit values and varying the third one in its 3σ range.

For $G_f = A_5$, requiring the compatibility with the data in the way explained above, we find 13 viable cases. They are presented in the Table that follows.

(G_e, G_ν)	Case	$\sin^2\theta^\circ_{ij}$	$\cos\delta$	$\sin^2\theta_{ij}$
(Z_2, Z_3)	**A1A$_5$**	$(\sin^2\theta^\circ_{13}, \sin^2\theta^\circ_{23}) = (0.226, 0.436)$	0.727	$\sin^2\theta_{23} = 0.554$
	A2A$_5$	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.226, 0.436)$	-0.727	$\sin^2\theta_{23} = 0.446$
(Z_3, Z_2)	**B1**	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	-0.353	$\sin^2\theta_{12} = 0.341$
(Z_5, Z_2)	**B1A$_5$**	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.276, 1/2)$	-0.405	$\sin^2\theta_{12} = 0.283$
$(Z_2 \times Z_2, Z_2)$	**B2A$_5$**	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (0.095, 0.276)$	-0.936	$\sin^2\theta_{12} = 0.331$
	B2A$_5$II	$(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (1/4, 0.127)$	1^*	$\sin^2\theta_{12} = 0.331$
(Z_2, Z_2)	**C1**	$\sin^2\theta^\circ_{23} = 1/4$	-1^*	**not fixed**
	C3A$_5$	$\sin^2\theta^\circ_{13} = 0.095$	1^*	**not fixed**
	C3	$\sin^2\theta^\circ_{13} = 1/4$	-1^*	**not fixed**
	C4A$_5$	$\sin^2\theta^\circ_{12} = 0.095$	-0.799	**not fixed**
	C4	$\sin^2\theta^\circ_{12} = 1/4$	1^*	**not fixed**
	C8	$\sin^2\theta^\circ_{23} = 3/4$	1^*	**not fixed**
	C9A$_5$	$\sin^2\theta^\circ_{12} = 0.345$	**not fixed**	$\sin^2\theta_{12} = 0.331$

The exact algebraic forms of the irrational values of $\sin^2\theta^\circ_{ij}$ in the Table have been found in arXiv:1509.02502 They are related to the golden ratio $r = (1+\sqrt{5})/2$ as follows: $2/(4r^2 - r) \approx 0.226$, $r/(6r-6) \approx 0.436$, $1/(2+r) \approx 0.276$, $1/(4r^2) \approx 0.095$, $1/(3+3r) \approx 0.127$, and $(3-r)/4 \approx 0.345$.

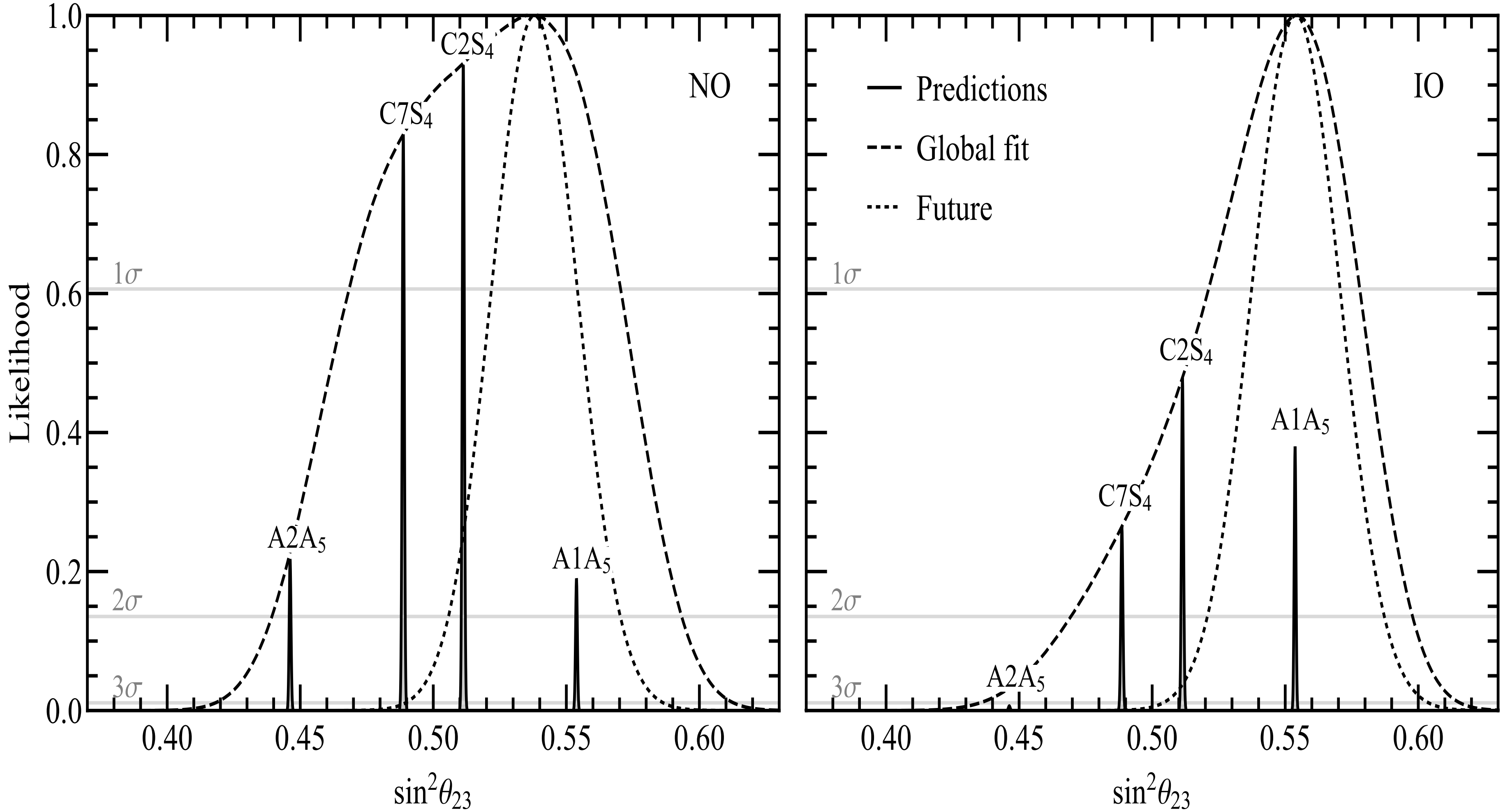
We note that case B1 is common to all the three flavour symmetry groups A_4, S_4 and A_5, while cases C1, C3, C4 and C8 are shared by S_4 and A_5. Thus, we have 16 cases in total, which lead to different predictions for $\sin^2\theta_{12}$ or $\sin^2\theta_{23}$ and/or $\cos\delta$. As we will see in the next subsection performing a statistical analysis of these predictions, two cases, namely, C4 and B2A$_5$II, are globally disfavoured at more than 3σ confidence level.
Thus, the total number of phenomenologically viable cases reduces to 14.

Phenomenologically Viable Predictions

A1 (A2), A_5 ($G_e = Z_2$, $G_\nu = Z_3$ (Dirac ν_j)):
$\sin^2\theta_{23} \cong 0.553$ (0.447); $\cos\delta \cong 0.716$ (-0.716).

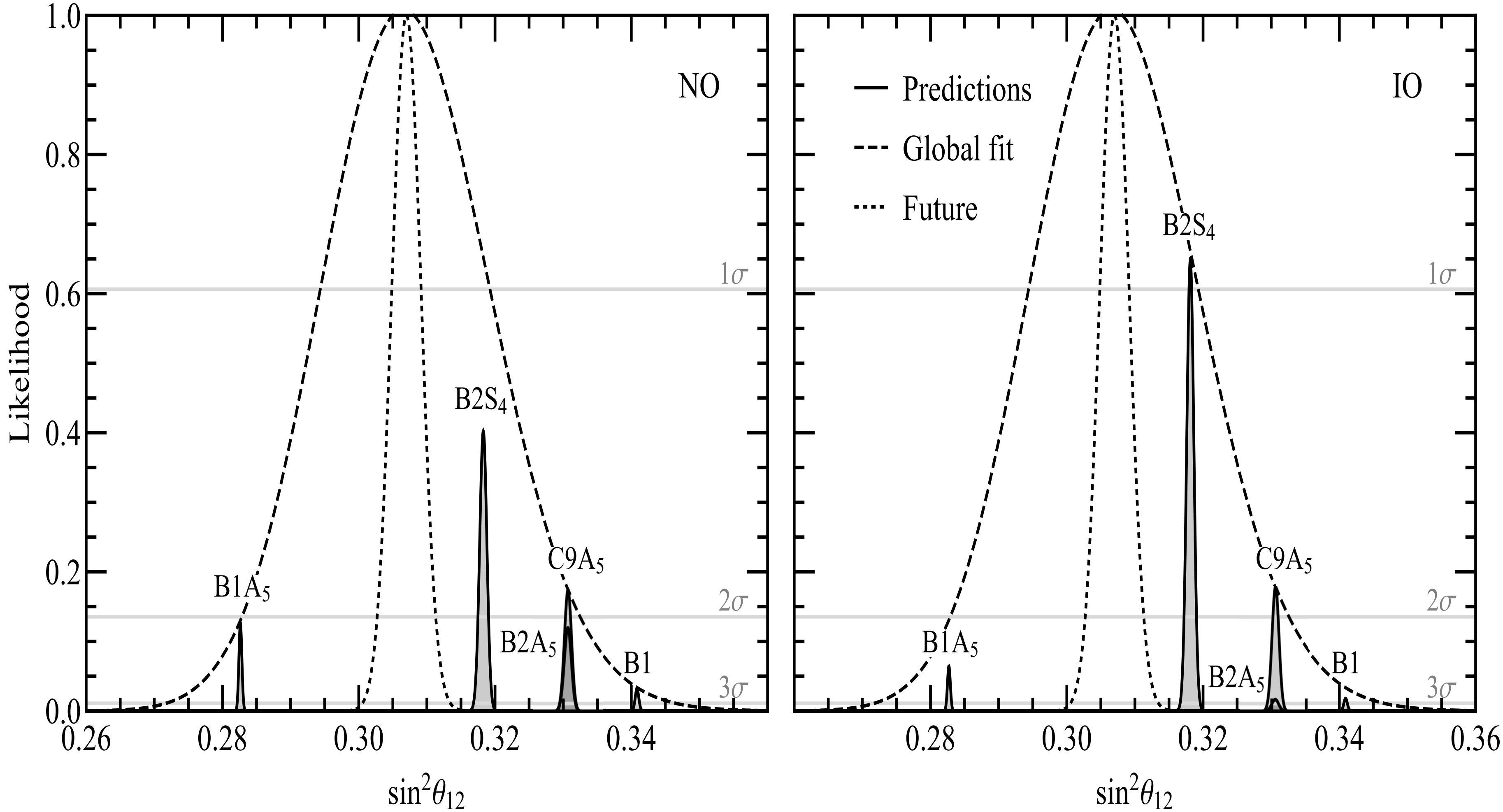
B1, A_4 (T', S_4, A_5) ($G_e = Z_3^T$, $G_\nu = Z_2^S$):
$U_{\rm PMNS} = U_{\rm TBM} U_{13}(\theta^\nu_{13}, \delta_{13}) Q_0$;
$\sin^2\theta_{12} = 1/(3\cos^2\theta_{13}) \cong 0.340$; $\cos\delta \cong 0.570$.

B2, S_4 ($G_e = Z_3^T$, $G_\nu = Z_2^{SU}$):
$\sin^2\theta_{12} \cong (1 - 2\sin^2\theta_{13})/3 = 0.319$; $\cos\delta \cong -0.269$.



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO).**

A total of 6 models would survive out of the currently viable 14 (of the extremely large number) considered if $\delta(\sin^2\theta_{23}) = 3\%$, $\delta(\sin^2\theta_{12}) = 0.7\%$ and the current b.f.v. would not change:

A1A$_5$, C3, C3A$_5$, C4A$_5$, C8, C2S$_4$.
A1A$_5$: $\cos\delta = 0.727$, $\sin^2\theta_{23} = 0.554$;
C2S$_4$: $\sin^2\theta_{23} = 0.511$.

C3, C3A$_5$, C8, C4A$_5$: predict $\cos\delta$.
C4A$_5$: $\cos\delta = -0.799$;
C3A$_5$, C8: $\cos\delta = 1^*$; **C3:** $\cos\delta = -1^*$.

In the cases marked with an asterisk, physical values of $\cos\delta$ cannot be obtained employing the best fit values of the mixing angles, but they are achievable fixing two angles to their best fit values and varying the third one in its 3σ range.

Will be constrained further by the data on δ.

JUNO

20 kt LS detector of reactor $\bar{\nu}_e$ via IBD
$\bar{\nu}_e + p \rightarrow n + e^+$; $E_{res} = 3\%/\sqrt{E}$; $L \cong 53$ km;
thermal power of the used reactors: 26.6 GW;
Sphere with a diameter of 38 m.
Cost: 300×10^6 US Dollars.
Built in China by international collaboration of more than 700 scientists from 74 Institutions in 17 countries/regions. Started data-taking on August 25, 2025.
After 6 years of operation: NMO at 3σ (using reactor ν data only). Adding ν_{atm} data can improve the sensitivity by $(0.8 - 1.4)\sigma$.

The idea put forward in S.T.P., M. Piai, PLB 553 (2002) 94 (hep-ph/0112074).

Based on: $P_{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \neq P_{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$

$$P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{atm}L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m^2_{\odot}L}{2E_\nu}\right)$$

$$+ \frac{1}{2}\sin^2 2\theta_{13}\sin^2\theta_{12}\left(\cos\left(\frac{\Delta m^2_{atm}L}{2E_\nu} - \frac{\Delta m^2_{\odot}L}{2E_\nu}\right) - \cos\frac{\Delta m^2_{atm}L}{2E_\nu}\right), \quad \Delta m^2_{\odot} \equiv \Delta m^2_{21},$$

$$P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{atm}L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}\left(1 - \cos\frac{\Delta m^2_{\odot}L}{2E_\nu}\right)$$

$$+ \frac{1}{2}\sin^2 2\theta_{13}\cos^2\theta_{12}\left(\cos\left(\frac{\Delta m^2_{atm}L}{2E_\nu} - \frac{\Delta m^2_{\odot}L}{2E_\nu}\right) - \cos\frac{\Delta m^2_{atm}L}{2E_\nu}\right).$$

$\Delta m^2_{\text{atm}} = \Delta m^2_{31(32)}(NO)$, $\Delta m^2_{\text{atm}} = \Delta m^2_{32(31)}(IO)$,

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

Spectrum of e^+ - sensitive to the difference between $P^{NO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ and $P^{IO}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ - can be used to determine neutrino mass ordering. Optimal L exists; $E_{res} \sim 3\%/\sqrt{E}$ required.

S.T.P., M. Piai, 2001

JUNO (China, International collaboration)

S. Choubey, S.T.P., M. Piai, PRD 68 (2003) 113006 ((hep-ph/0306017): **can measure $\sin^2\theta_{12}$, Δm^2_{21} and Δm^2_{31} with exceptionally high precision.**

After 6 years of datataking:

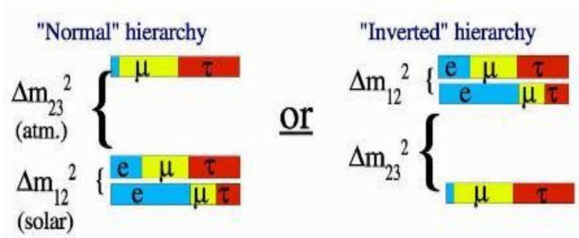
$\sin^2\theta_{12}$: 0.5%; Δm^2_{21}: 0.3%; Δm^2_{31}: 0.2% (1σ)

(Y. Wang, talk given at CERN on March 20, 2024).

Wide program of research: atmospheric ν oscillations, solar neutrinos, SN neutrinos, geo-neutrinos, nucleon decay; distant future: $(\beta\beta)_{0\nu}$ decay.

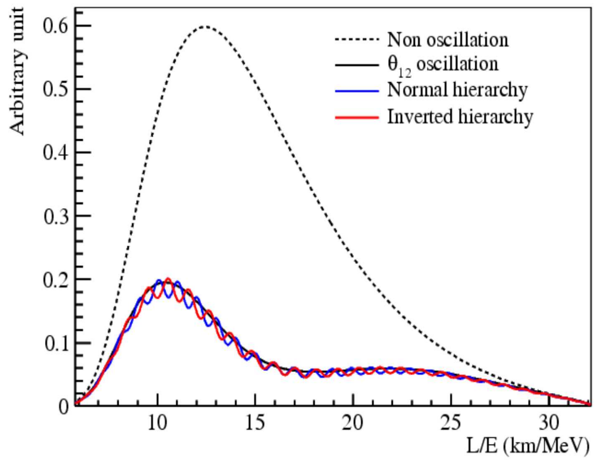

Guangzhou
Guang Zhou
Dongguan
Huizhou
Zhaoqing
CNS
2.5 h drive
Shen Zhen
Huizhou NPP
Lufeng NPP
Daya Bay
NPP
Zhongshan
Zhu Hai
Zhujiang River Estuary
Hong Kong
Hong Kong
Macau
Macau
JUNO site
53 km
53 km
Taishan NPP
Yangjiang NPP

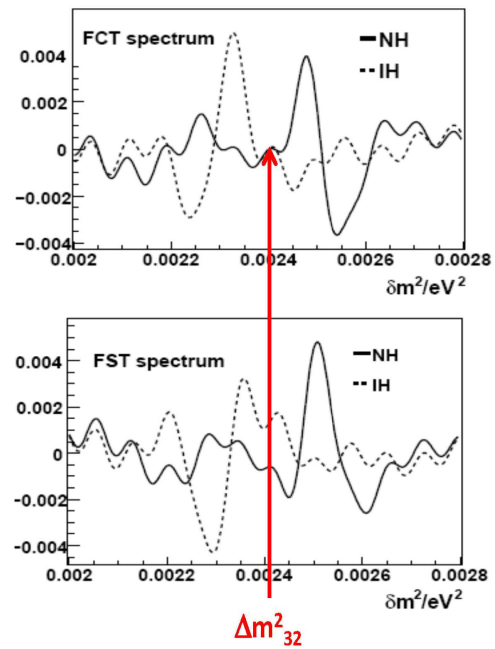
Mass Ordering by Reactor Neutrinos



$$\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$$
$$\mathrm{NH:}\ |\Delta m^2_{31}| = |\Delta m^2_{32}| + |\Delta m^2_{21}|$$
$$\mathrm{IH:}\ |\Delta m^2_{31}| = |\Delta m^2_{32}| - |\Delta m^2_{21}|$$

$$\frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} \sim 3\%$$





$$P_{ee}(L/E) = 1 - P_{21} - P_{31} - P_{32}$$
$$P_{21} = \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21})$$
$$P_{31} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31})$$
$$P_{32} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32})$$

S. Petcov and Piai, Phys. Lett. B 553, 94-106(2002)
J. Learned et al., PRD 78(2008)071302
L. Zhan, YFW et al., PRD 78(2008)111103

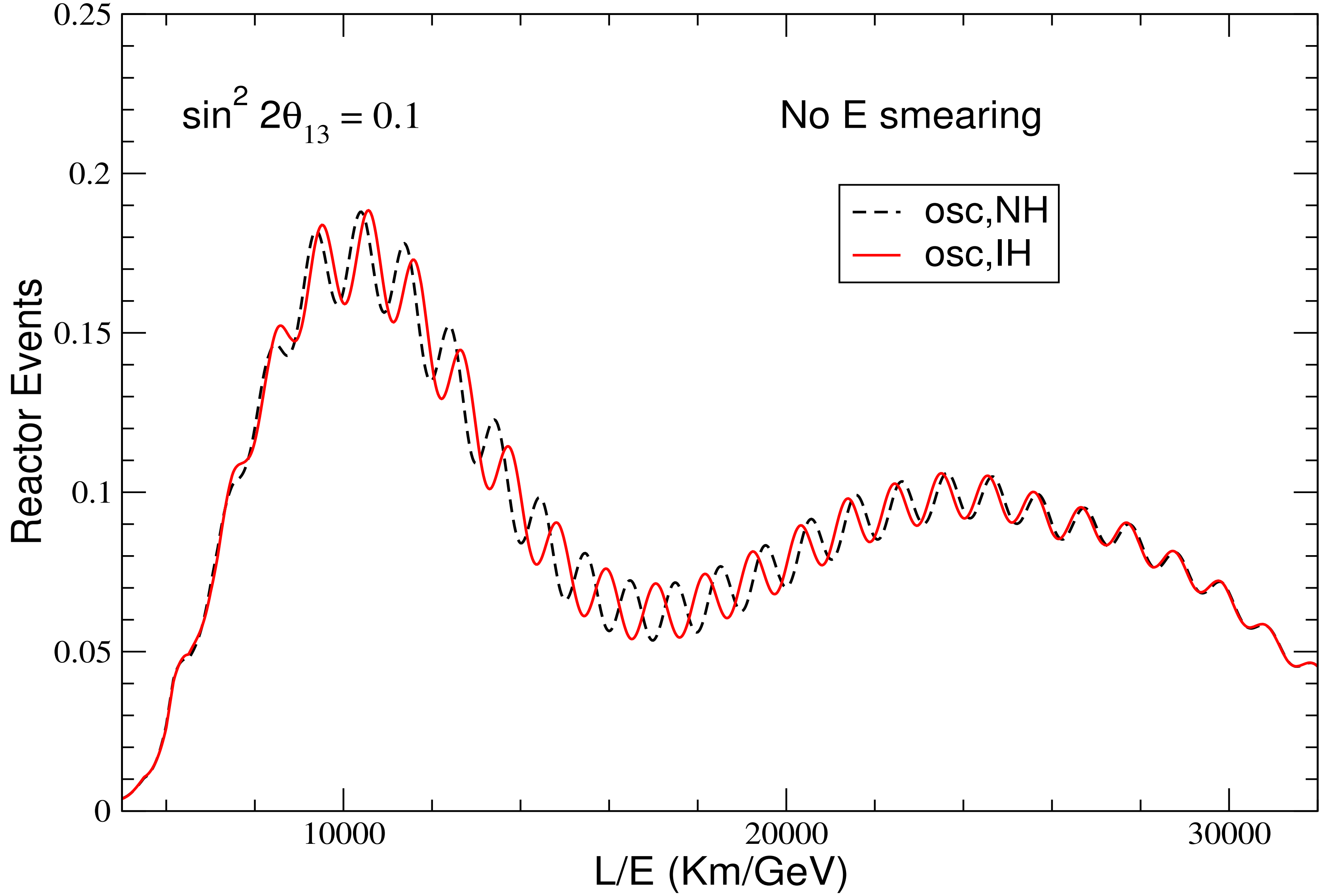
5

Y. Wang, talk given at CERN on March 20, 2024

" Very large detector, $\sin\theta_{13} \gtrsim 0.10$, $E_{res} \sim 3\%/\sqrt{E}$**,... required. Probably, the experiment will never be realised... "**

"However, as it is well known, "Only those who wager can win." (W. Pauli, Letter to Participants of a Physics Meeting in Tubingen, (H. Geiger et al.), Germany, December 4, 1930."

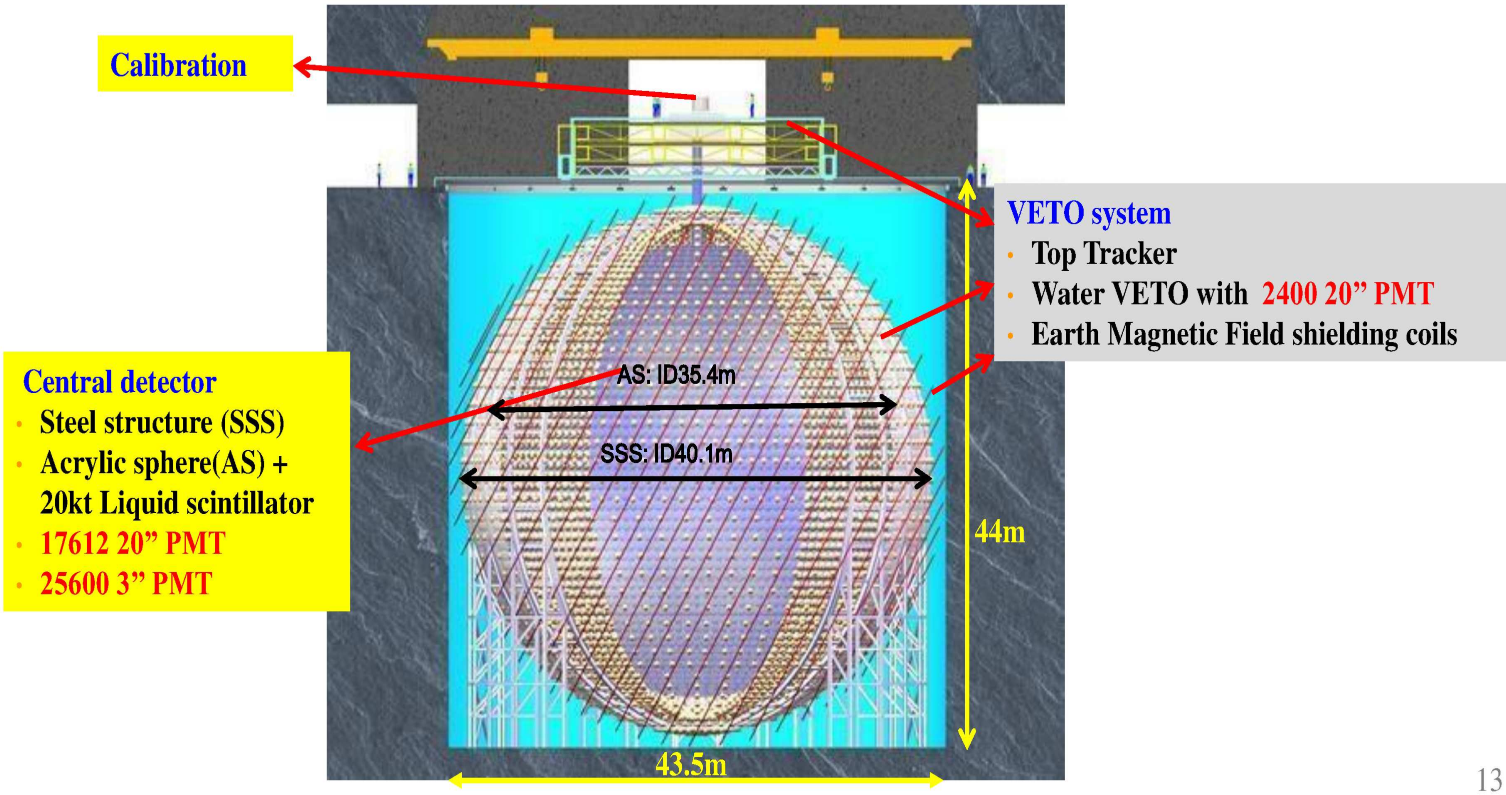
S.T.P., M. Piai, PLB 553 (2002) 94 (hep-ph/0112074)

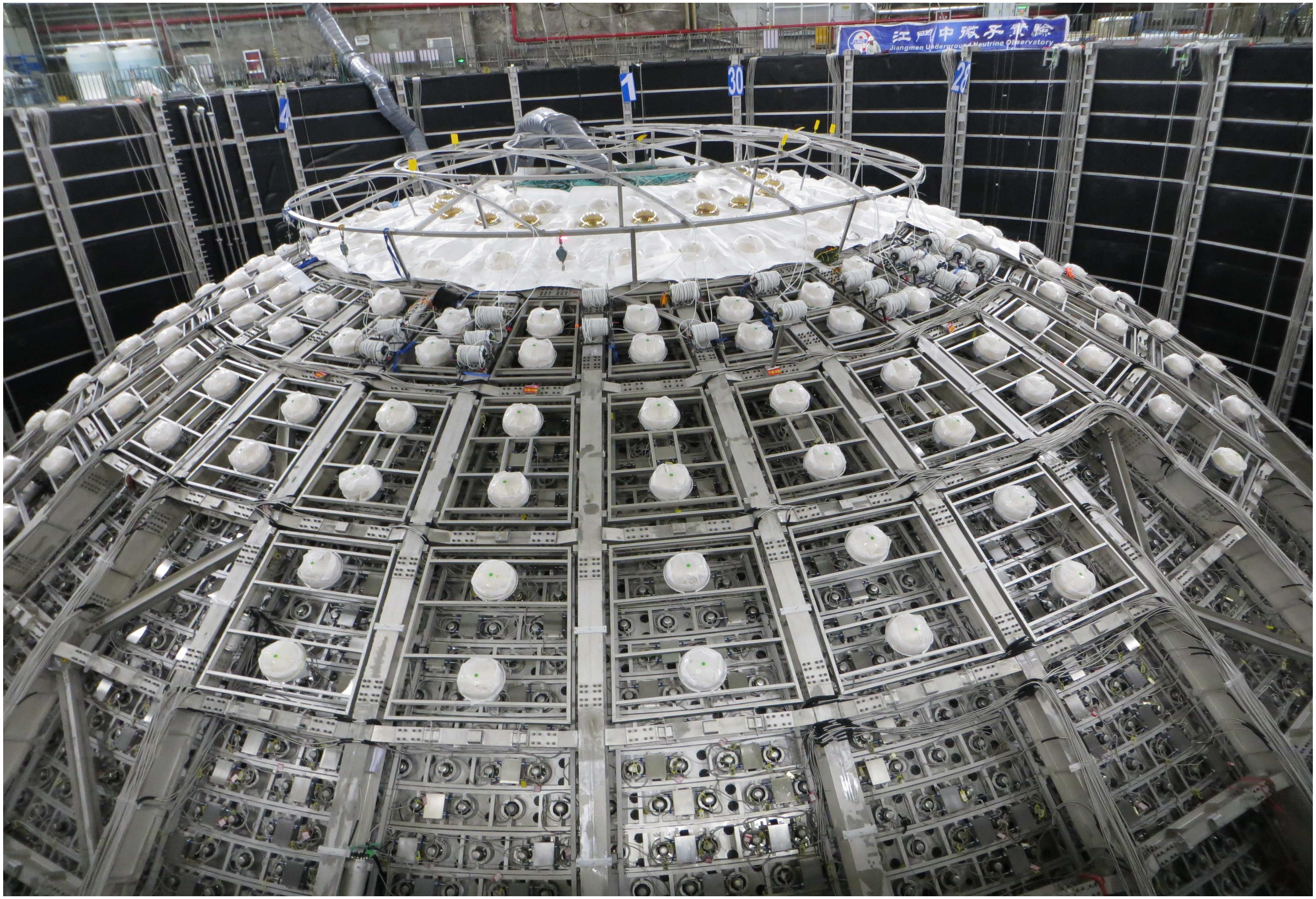


P. Ghoshal, S.T.P., JHEP 03 (2011) 058 (arXiv:1011.1646)

Concept of JUNO for Mass Ordering

- Two-layer structure for simplicity and cost: stainless steel frame + Acrylic tank
- Water as VETO and Buffer ➔ radiopurity control of water




JUNO
江門中微子實驗
Jiangmen Underground Neutrino Observatory

江門中微子實驗
Jiangmen Underground Neutrino Observatory

JUNO started taking physics data on August 26, 2025. The first results were released on November 11, 2025 (after 59.1 days):

$\sin^2\theta_{12} = 0.3092 \pm 0.0087$,
$\Delta m^2_{12} = (7.50 \pm 0.12) \times 10^{-5}$ eV^2

Improvemed by a factor of approximately 1.4 (1.6) the precision on $\sin^2\theta_{12}$ (Δm^2_{12}).

There are more than 25 papers on the implications of the first JUNO results.

$\sin^2\theta_{12}$ from Non-Abelian discrete symmetries

$U_{PMNS} = U_e^{\dagger} U_{\nu}$

Case 1: $A_4, S_4, A_5 \rightarrow (G_e, G_\nu) = (Z_3^{ST}, Z_2^{S})$,
$U_e = \mathbf{I}$, $U_\nu = U_{TB} U_{13}(\tilde{\theta}_{13}, \tilde{\phi}_{13})$

$$\sin^2\theta_{12} = \frac{1}{3\cos^2\theta_{13}} \cong 0.341\,, \quad \text{TB2}$$

Ruled out at 3.65σ

Case 2: $A_5 \rightarrow (G_e, G_\nu) = (Z_5, Z_2^{S})$,
$U_e = \mathbf{I}$, $U_\nu = U_{GR} U_{13}(\tilde{\theta}_{13}, \tilde{\phi}_{13})$

$$\sin^2\theta_{12} = \frac{1}{(2+r)\cos^2\theta_{13}} \cong 0.283\,, \; \text{r} = (1+\sqrt{5})/2 \cong 1.618\,.$$

Disfavoured at 3σ

Case 3: $A_5 \to (G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$

$$\sin^2\theta_{12} = 1 - \frac{1+r}{4\cos^2\theta_{13}} \cong 0.331\,.$$

Disfavoured at 3σ

Case 4: $\mathbf{S}_4 \to (G_e, G_\nu) = (Z_3^{ST}, Z_2^U)$**,**
$U_e = \mathbf{I}$**,** $U_\nu = U_{TB} U_{13}(\tilde{\theta}_{23}, \tilde{\phi}_{23})$

$$\sin^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{3\cos^2\theta_{13}} \cong 0.318\,, \quad \text{TB1}\,.$$

Compatible at 1.1σ

In all these cases $\cos\delta$ **is also predicted in terms of** $\sin^2\theta_{13}$ **and** $\sin^2\theta_{23}$**.**

Case 4: $\sin^2\theta_{12} = 0.318$; **for NO (IO) one has:**

$$\cos\delta = -\frac{(1-5\sin^2\theta_{13})\cot 2\theta_{23}}{2\sqrt{2}\sin\theta_{13}(1-3\sin^2\theta_{13})^{\frac{1}{2}}} = -0.131\ (0.219)\,,$$

where the best fit values of $\sin^2\theta_{13}$ **and** $\sin^2\theta_{23}$ **from I. Esteban et al., arXiv:2410.05380 (NuFit-6.0 results) where used to get the numerical values.**

A Note of Caution

Since any of the non-Abelian symmetries considered above are assumed to be realised at some high scale, the question arises how our results might be modified by renormalisation group (RG) running effects. The evaluation of the RG effects requires additional specific assumptions about the scale at which the symmetry is realised, the underlying model (Standard Model (SM), minimal supersymmetric SM, etc.), the nature — Dirac or Majorana — of massive neutrinos and the mechanism of neutrino mass generation (SM minimally extended with right-handed singlet neutrinos and conserved total lepton charge for Dirac neutrinos, or seesaw mechanism/Weinberg dimension-5 operator for Majorana neutrinos, etc.). They further depend on the type of spectrum of neutrino masses (NO or IO), on the value of the lightest neutrino mass and, in the case of massive Majorana neutrinos, on the Majorana phases. However, the RG effects are known to be negligible for the NO mass spectrum if the lightest neutrino mass is smaller than approximately 0.01 eV; for the IO spectrum and massive Majorana neutrinos, the RG effects on θ_{12} can be large, but are known to be particularly strongly suppressed for, e.g., specific values of the Majorana phases (see S. Antusch et al., arxiv:0305273). Thus, we can conclude conservatively that the results presented above are valid at least in the indicated cases.

These are only the first of many remarkable results expected from the JUNO experiment, which has unique physics capabilities. These include determining the neutrino mass spectrum, measuring three neutrino oscillation parameters with exceptional precision, studying oscillations of reactor, solar, and atmospheric neutrinos, detecting neutrinos from a supernova explosion and the diffuse supernova neutrino background, measuring the flux of geoneutrinos, searching for proton decay, and probing physics beyond the Standard Model. The JUNO experiment has reported impressive new results after only 59.1 days of data taking, achieving a factors of 1.4 and 1.6 improvement in the precision of solar neutrino oscillation parameters compared with previous experiments. With its rich physics programme and unique detector capabilities, JUNO is set to be a leading experiment in neutrino and particle physics for the next 20 years and possibly beyond 2045.

The future of Neutrino Physics is bright!