李政道研究所
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Cosmological consequences of axion models

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Based on *JCAP* 03 (2024) 061, *JCAP* 03 (2025) 014, *Phys.Rev.D* 113 (2026) 2, and *arXiv:2606.14098*
With J. Osinski, L. Roszkowski, U. Min, A. Ghoshal, D. Paul, A. Fowlie, and G. Herrera

The strong charge-parity problem

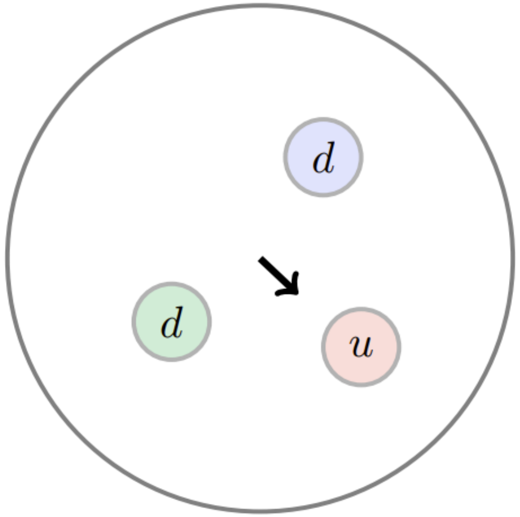
The strong CP problem asks why there is no observed charge-parity violation in strong interactions despite the SM gauge-group allowing it

$$\mathcal{L}_{\not{CP}} = \theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

Predicts an electric dipole moment of the neutron (and others)

$$d_n = (2.4 \pm 1.0)\, \theta \, \times 10^{-3}\, \mathrm{e\, fm}$$

Observations put this down to $|\theta| < 0.8 \times 10^{-10}$



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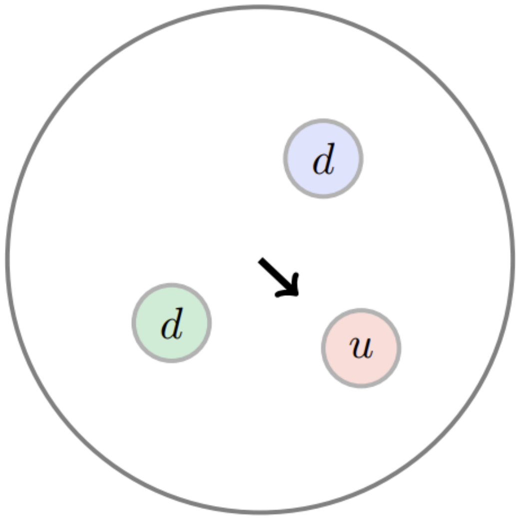
In fact, the observable depends on

$$\bar{\theta} = \theta + \arg\left(\det M_q\right)$$

Two independent phases cancel out precisely, $\bar{\theta} \lesssim 10^{-10}$

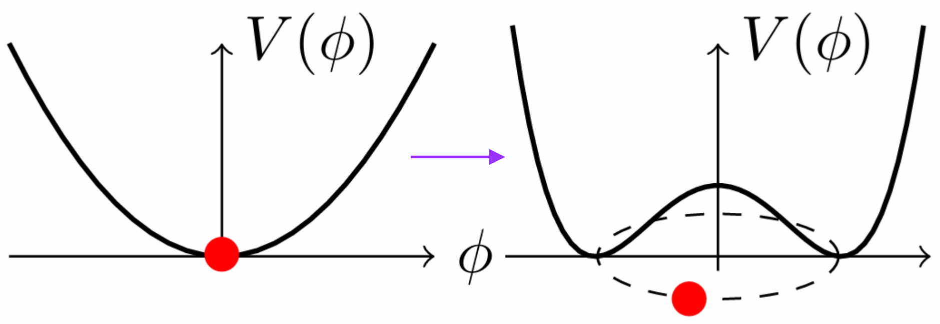


The Peccei-Quinn Mechanism

A new, spontaneously broken, global $U(1)$ symmetry proposed by R. D. Peccei and H. R. Quinn (1977). Referred to as $U(1)_{\text{PQ}}$, breaks at some scale $\sim f_a$.

Complex scalar field

$$\mathcal{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) + m^2 \Phi^* \Phi - \frac{\lambda}{4} \Phi^2 \Phi^{*2}$$



Symmetry breaking leads to a non-zero vacuum expectation value

$$\langle \Omega_\theta | \Phi | \Omega_\theta \rangle = \sqrt{\frac{2m^2}{\lambda}} e^{i\theta}$$

Expand around new minimum

$$\Phi \to \frac{1}{\sqrt{2}} (f_a + \phi) \; e^{ia/f_a}$$

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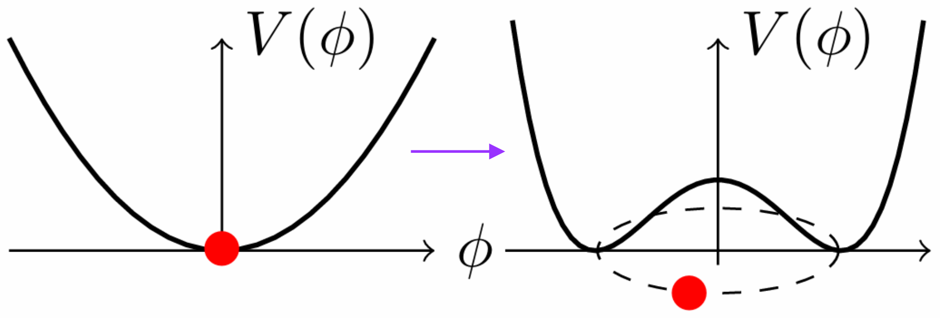

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Weinberg and Wilczek independently realized that this implied the existence of a new boson, the axion (1978).

The Peccei-Quinn Mechanism

The CP violating term now has a dynamical θ parameter

$$\mathcal{L}_{\text{eff}}^{a} = \frac{a\left(t,x\right)}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}^{a}\tilde{G}_{a}^{\mu\nu} = \theta\left(t,x\right)\frac{\alpha_s}{8\pi}G_{\mu\nu}^{a}\tilde{G}_{a}^{\mu\nu}$$

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QCD generates a potential for the axion field.

$$V_{\text{QCD}}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\frac{N\theta}{2}}$$

This potential dynamically drives the field to cancel the CP violation term!

$$\bar{\theta} \to 0$$

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$$\bar{\theta}\to 0$$

Also generate the mass of axion

$$m_a = \frac{m_\pi f_\pi}{f/N}\sqrt{\frac{m_u m_d}{2(m_u+m_d)^2}} \approx 6\,\mu\text{eV}\,\frac{10^{12}\,\text{GeV}}{f/N}.$$

Misalignment production of axion

Equation of motion for a scalar in expanding Universe

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 3H(t)\frac{\mathrm{d}}{\mathrm{d}t}\right)\theta(t) + \tilde{m}_a^2(t)\sin(\theta(t)) = 0$$

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$$\theta(t) = a(t)/f_a$$

$$\tilde{m}_a(T) \simeq m_a \times \begin{cases} 1 & \text{for } T \leq T_{\mathrm{QCD}}\,, \\ \left(\frac{T}{T_{\mathrm{QCD}}}\right)^{-4} & \text{for } T \geq T_{\mathrm{QCD}}\,, \end{cases}$$

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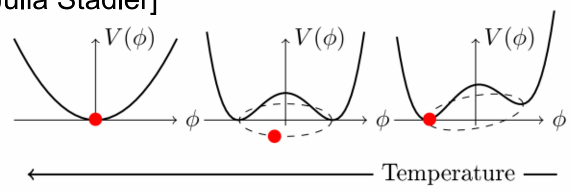
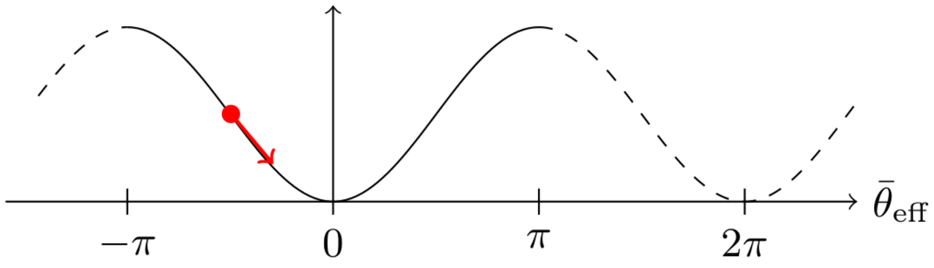
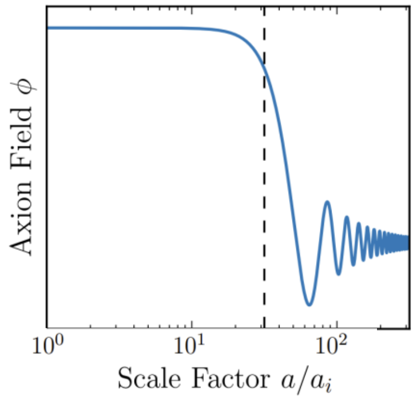
[Julia Stadler]



The initial field value is random



[D. Marsh, axion cosmology]



Axion field redshifts like a matter field at late times

$\rho_{\rm axion} \propto a^{-3}$

QCD axion as dark matter

Misalignment production in standard cosmology

$$\Omega_a h^2 \approx 0.12 \left(\frac{\theta_{\rm i}}{2.15}\right)^2 \left(\frac{28\ \mu\text{eV}}{m_a}\right)^{7/6}$$

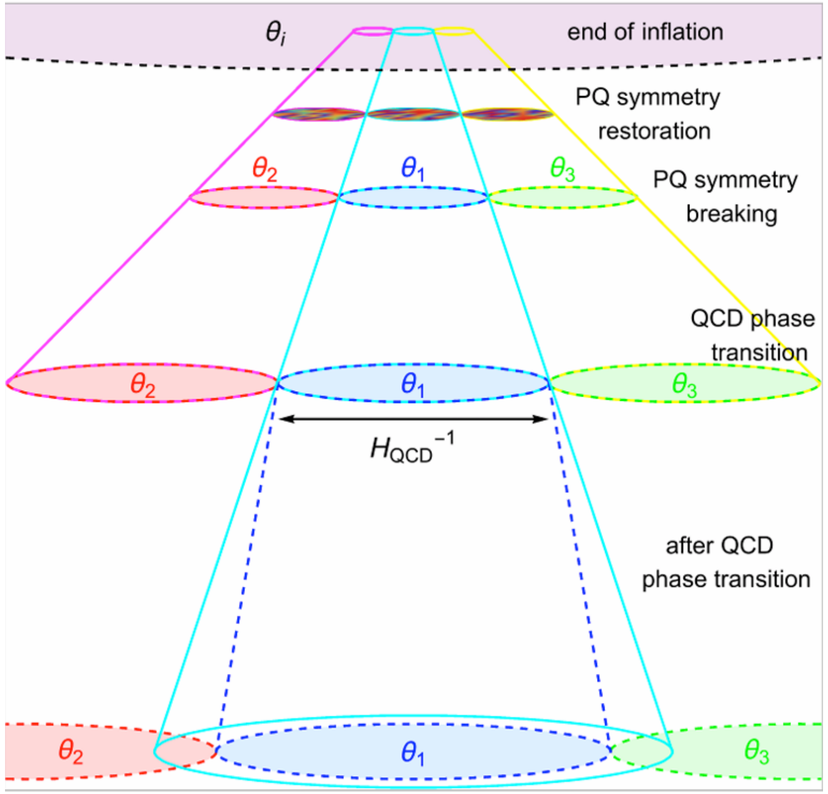
In the post-inflationary breaking you expect random θ_i in range $[-\pi, \pi)$.

Take random values in each Hubble patch

$$\theta_i \equiv \sqrt{\langle \theta_i^2 \rangle} = \frac{\pi}{\sqrt{3}} \simeq 1.81 \longrightarrow \approx 2.15$$

Anharmonic corrections

[K. Saikawa]



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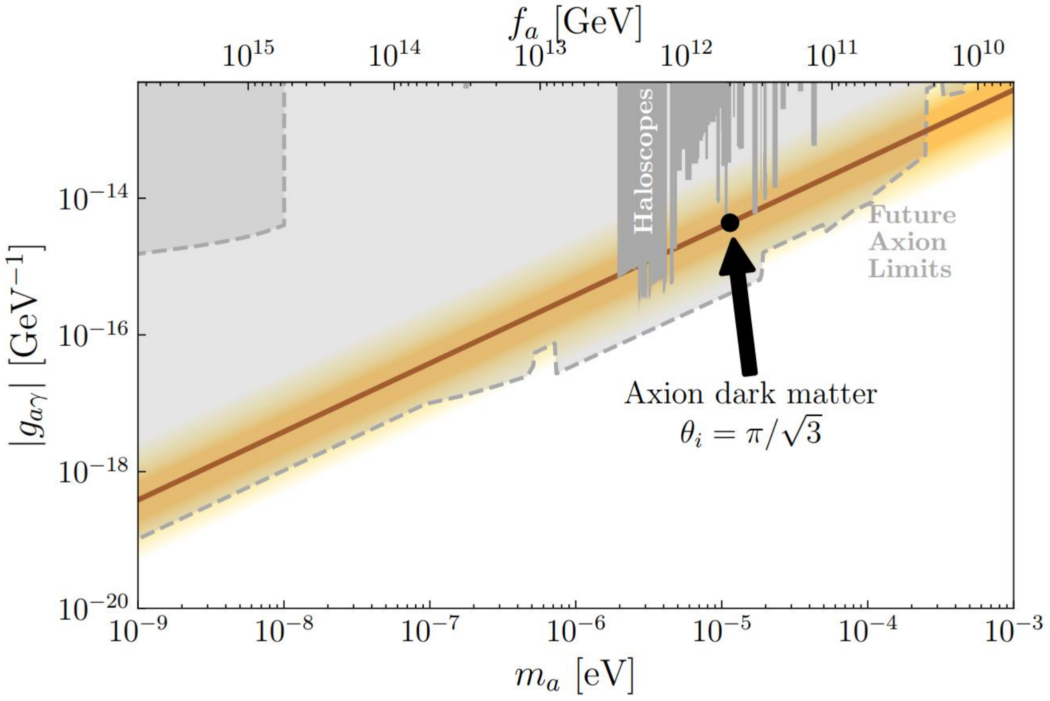

$$m_a = 5.7 \times 10^{-5}\,\text{eV} \left(\frac{10^{11}\,\text{GeV}}{f_a}\right)$$

PQ breaking and inflation

- If PQ symmetry is broken before inflation, the whole observable Universe has the same initial angle θ_i, QCD axion could be much lighter.

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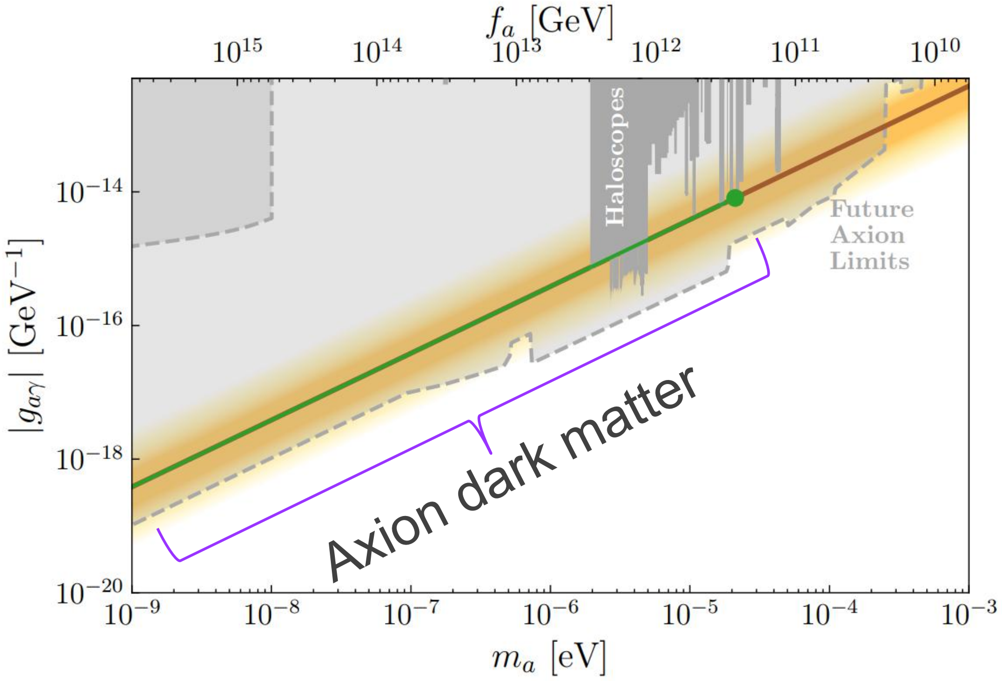

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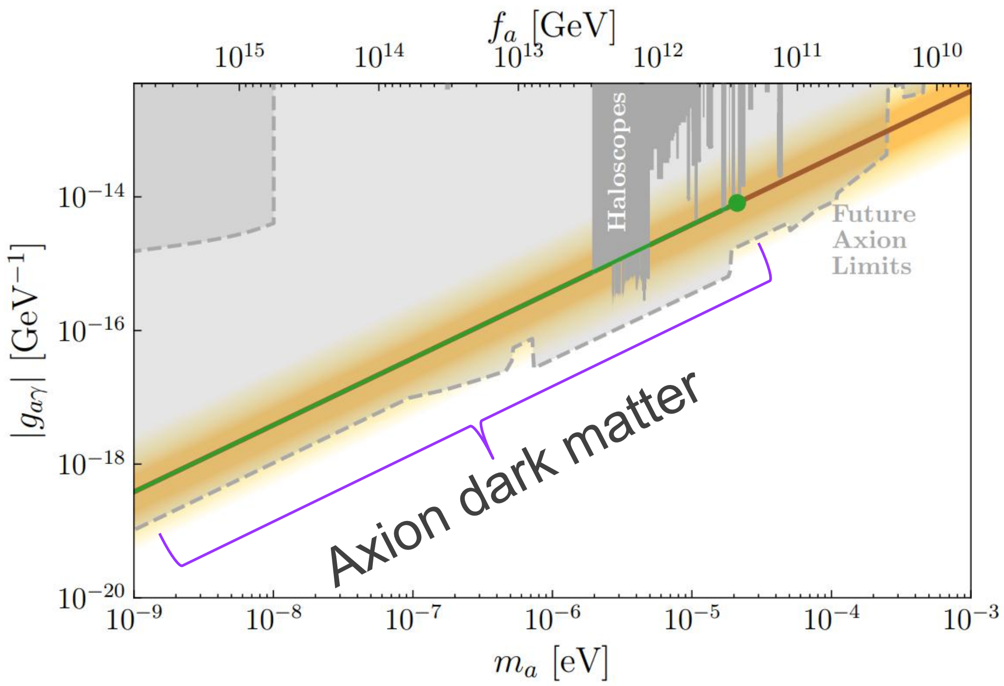
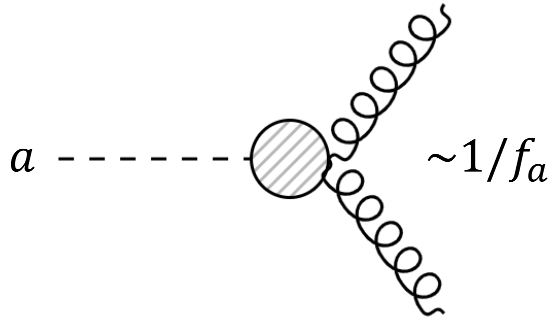


- The discovery of a light axion would be an indication of pre-inflationary PQ breaking.
- Other phenomenological consequences,
  - Thermal axion contributions to dark radiation.
  - Isocurvature bounds on scale of inflation.

Axion dark matter: a simple solution

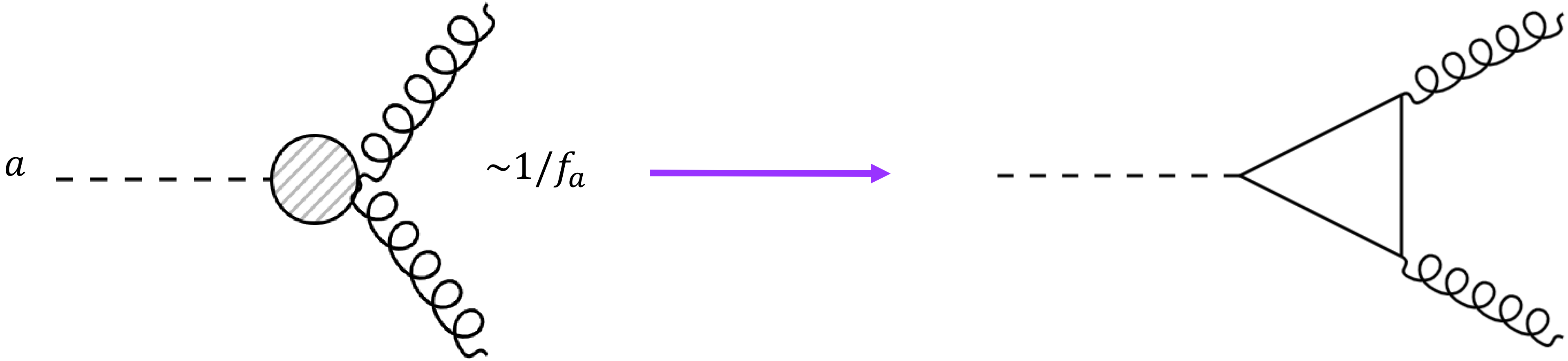
- The **QCD** axion provides an elegant solution to two big problems in fundamental physics.
- It explains why there is no observed CP violation in the strong sector.
- It has two potential mechanisms for producing a dark matter relic.
- All comes from the anomaly term

$$\mathcal{L}^a_{\text{eff}} = \frac{a\left(t,x\right)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$



- QCD and cosmology do the rest.

Axion dark matter: not so simple



Complicated by completions

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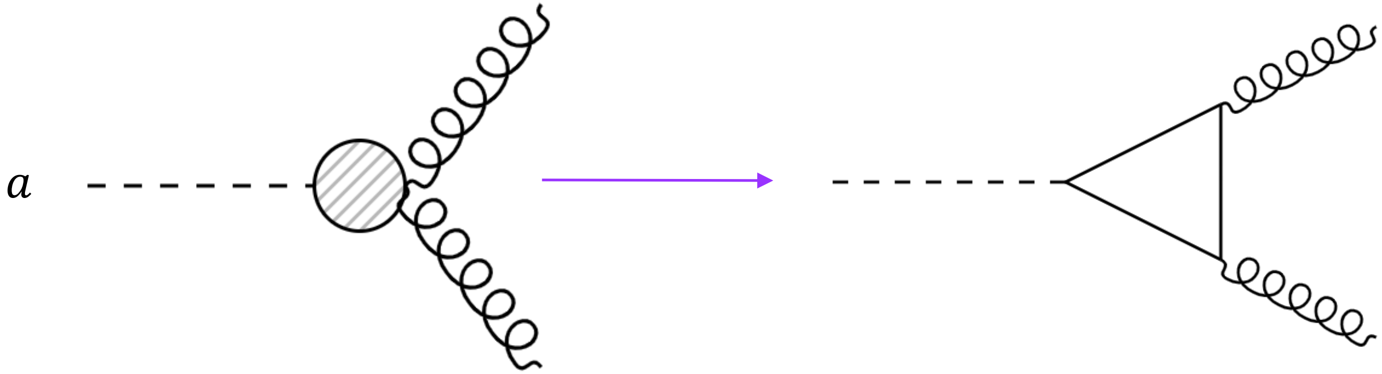

UV completions must involve strongly coupled particles.

KSVZ: SM fields are $U(1)_{\mathrm{PQ}}$ neutral

DFSZ: SM fields are charged under $U(1)_{\mathrm{PQ}}$

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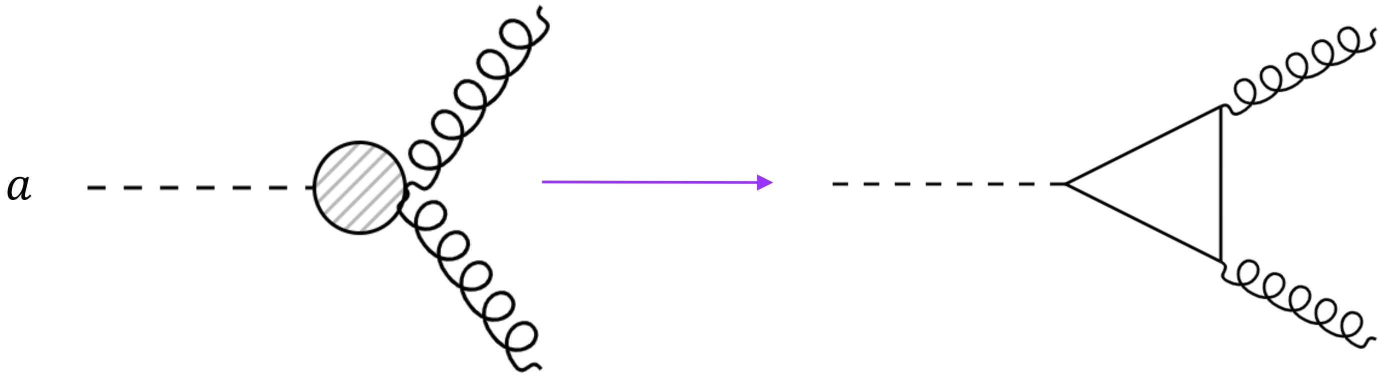

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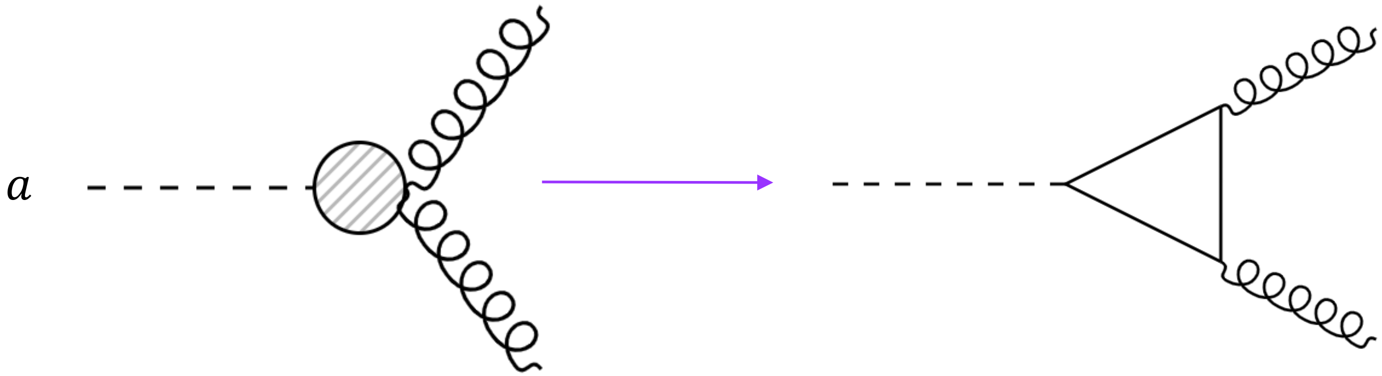

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KSVZ: SM fields are $U(1)_{\text{PQ}}$ neutral → Introduce heavy quarks which need to decay ✦

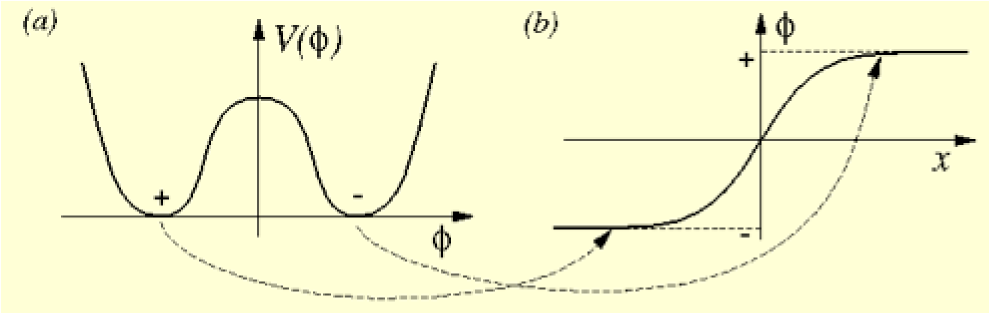
DFSZ: SM fields are charged under $U(1)_{\text{PQ}}$ → suffer from a domain wall problem ✦

✦ These problems can be avoided with pre-inflationary PQ breaking

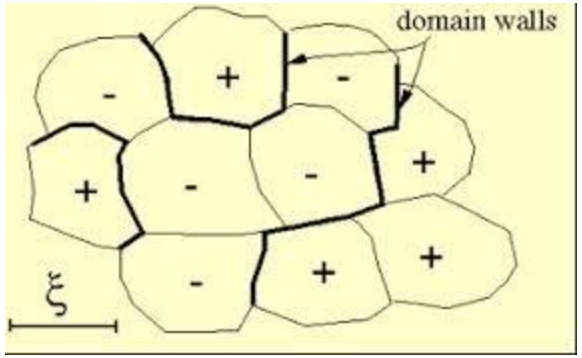
See family dependent DFSZ models by P. Cox et. al [arXiv:2310.16348]

Domain walls

- Topological defect where two (or more) distinct vacua are separated by a potential



[CTC outreach]



- Stable domain walls scale like $(\text{scale factor})^{-2}$ so can quickly dominate the Universe.
- Much effort has gone into getting domain walls to **decay** or be **destroyed**.
- I think its interesting to first explore models where this is **not a problem**.

Post-inflationary KSVZ

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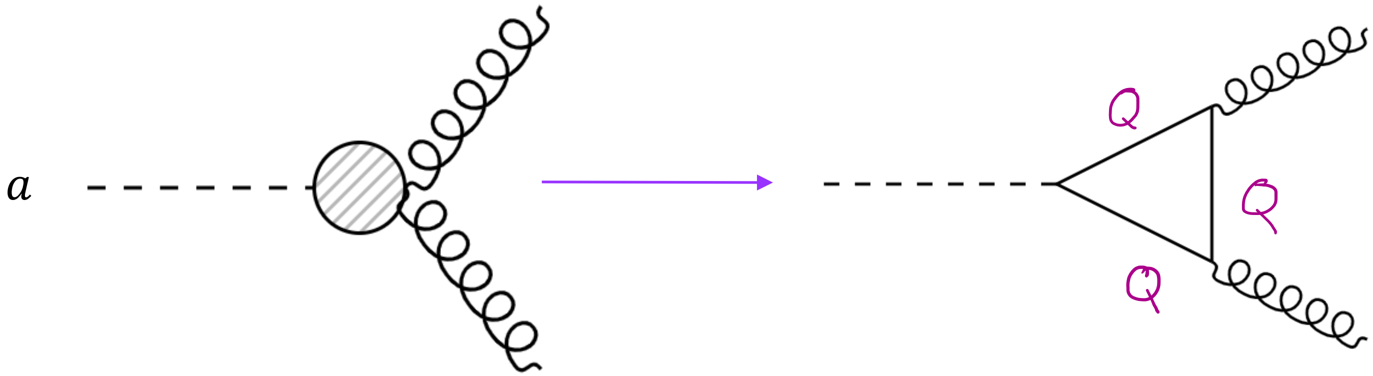

KSVZ is my focus today because there are simple models without a domain wall problem.

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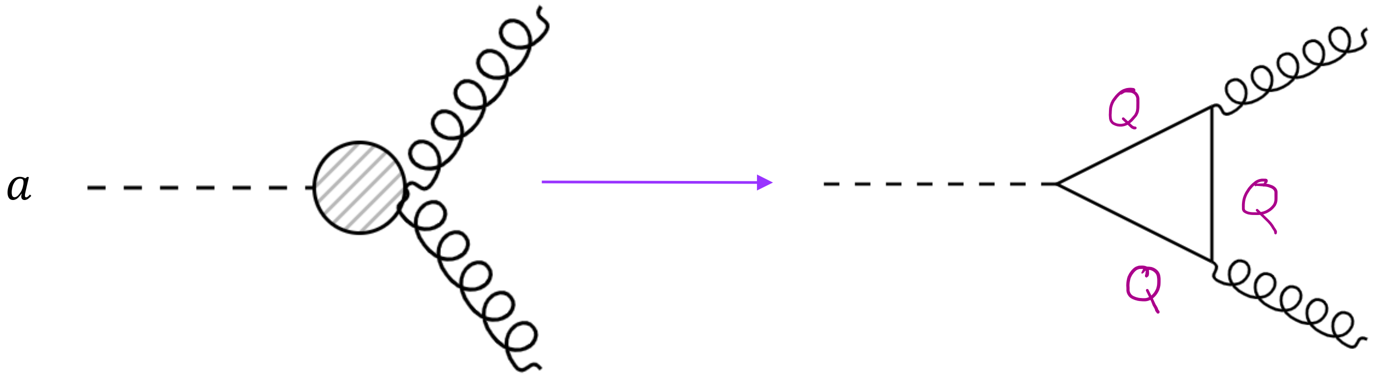

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NB: Q is the new “heavy quark”, not SM left-handed quark doublet (q_L).

Heavy quark disaster

- These new **Heavy Quarks (HQs)** thermally freeze-out, like dark matter (if stable).
- Strongly coupled dark matter is ruled out, and m_Q is typically too high!

$$\mathcal{L}_{\rm PQ} = |\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q \; - (y_Q \overline{Q}_L Q_R \Phi + \text{H.c})$$

So $m_Q \sim f_a$

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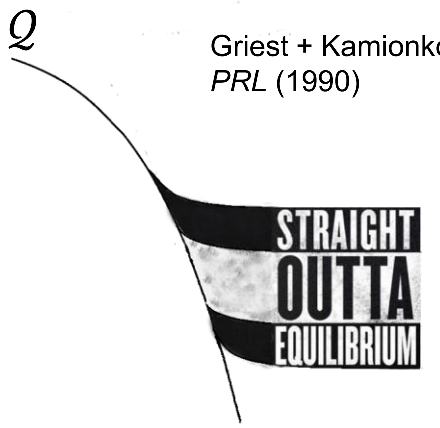
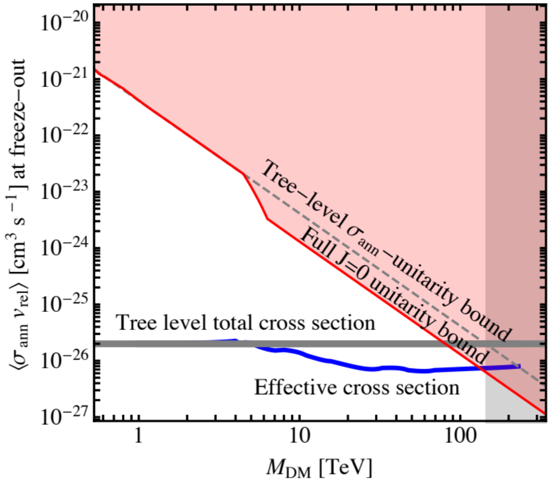
So $m_Q \sim f_a$

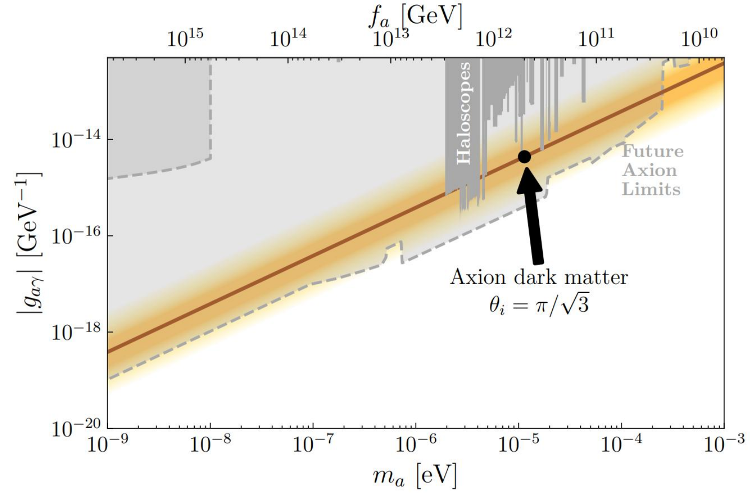
- Unitarity bound says anything above $m_Q \gtrsim 100\ \mathrm{TeV}$ will overclose the Universe.



Griest + Kamionkowski
PRL (1990)



Smirnov + Beacom
[arXiv:1904.11503]



Heavy quarks must decay

- Let's look at the symmetries,

$$\mathcal{L}_{\text{PQ}} = \underbrace{|\partial_\mu \Phi|^2 + \overline{Q} i \not{D} Q}\; - (y_Q \underbrace{\overline{Q}_L Q_R} \Phi + \text{H.c})$$

$$U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\phi \rightarrow U(1)_{\text{PQ}} \times U(1)_Q$$

- Must introduce Q-breaking term

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$$U(1)^3 \equiv U(1)_{Q_L} \times U(1)_{Q_R} \times U(1)_\phi \to U(1)_{\rm PQ} \times U(1)_Q$$

- Must introduce Q-breaking term
- This is only possible for some charge assignments
- For example,

$$R_Q: \left(3, 1, -\frac{1}{3}\right) \quad \text{and} \quad \overbrace{(\chi_L, \chi_R)}^{\text{PQ charge}} = \left(\frac{1}{2}, -\frac{1}{2}\right) \longrightarrow$$

In this case, decay impossible!

$SU(3)_c$ $SU(2)_W$ $U(1)_Y$

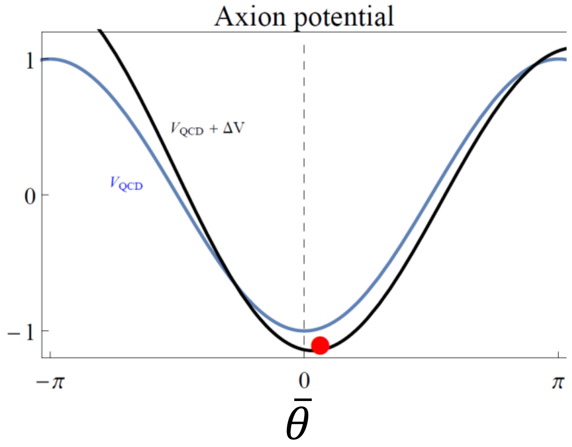
When to break a global U(1)?

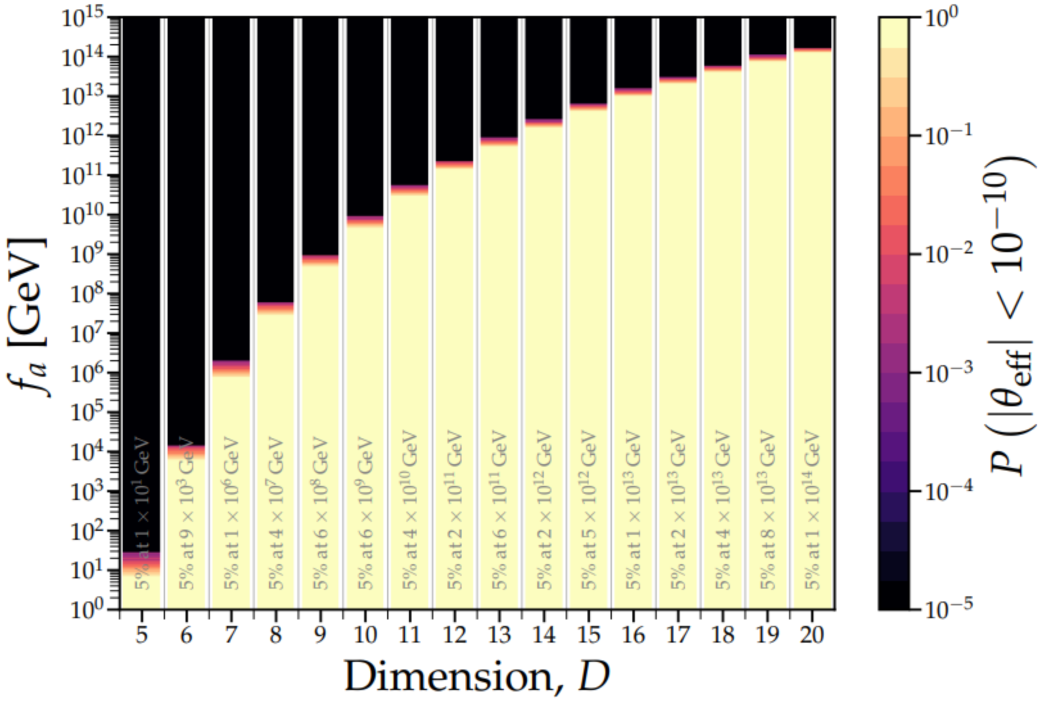
- Quantum gravity conjectures tell us to expect global symmetries break before the Planck scale!

PQ breaking

$$\mathcal{L}_{\rm UV} = \sum_{d=5}^{\infty} \sum_{k=1}^{d} c_{kd} \frac{|\Phi|^{d-k}\,\Phi^k}{M_{\rm Pl}^{d-4}} + \text{h.c.}$$

At dimension 5, this is enough to ruin the solution to CP problem.





[arXiv:2606.14098]

AC, A. Fowlie, G. Herrera

When can we break $U(1)_Q$?

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\rm DW} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_1: $(3, 1, -\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38}(g_1)$	$2/3$	1
R_2: $(3, 1, +\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34}(g_1)$	$8/3$	1
R_3: $(3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39}(g_1)$	$5/3$	2
R_4: $(3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27}(g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_5: $(3, 2, +\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22}(g_1)$	$29/3$	2
R_6: $(3, 3, -\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30}(g_2)$	$14/3$	3
R_7: $(3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27}(g_2)$	$20/3$	3

L. Di Luzio et. al. (PRL)
[arXiv:1610.07593]

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L. Di Luzio et. al. (PRL) [arXiv:1610.07593]

Can also go to the non-renormalizable level to determine the limit?

$$\mathcal{L}_{Qq} = \mathcal{L}_{Qq}^{d\leq 4} + \mathcal{L}_{Qq}^{d>4} = \mathcal{L}_{Qq}^{d\leq 4} + \frac{1}{\Lambda^{(d-4)}}\mathcal{O}^{d>4} + \text{h.c.}$$

Decays are suppressed by powers of Λ, and $\Lambda \neq f_a$.

$$\Gamma_{d,n_f} = \frac{m_Q}{4\,(4\pi)^{2n_f-3}\,(n_f-1)!\,(n_f-2)!}\left(\frac{m_Q^2}{\Lambda^2}\right)^{d-4}$$

Use Standard Cosmology

Decay terms are limited by

1) Ensuring misalignment doesn't overproduce axions
2) Ensuring Q decay occurs before BBN

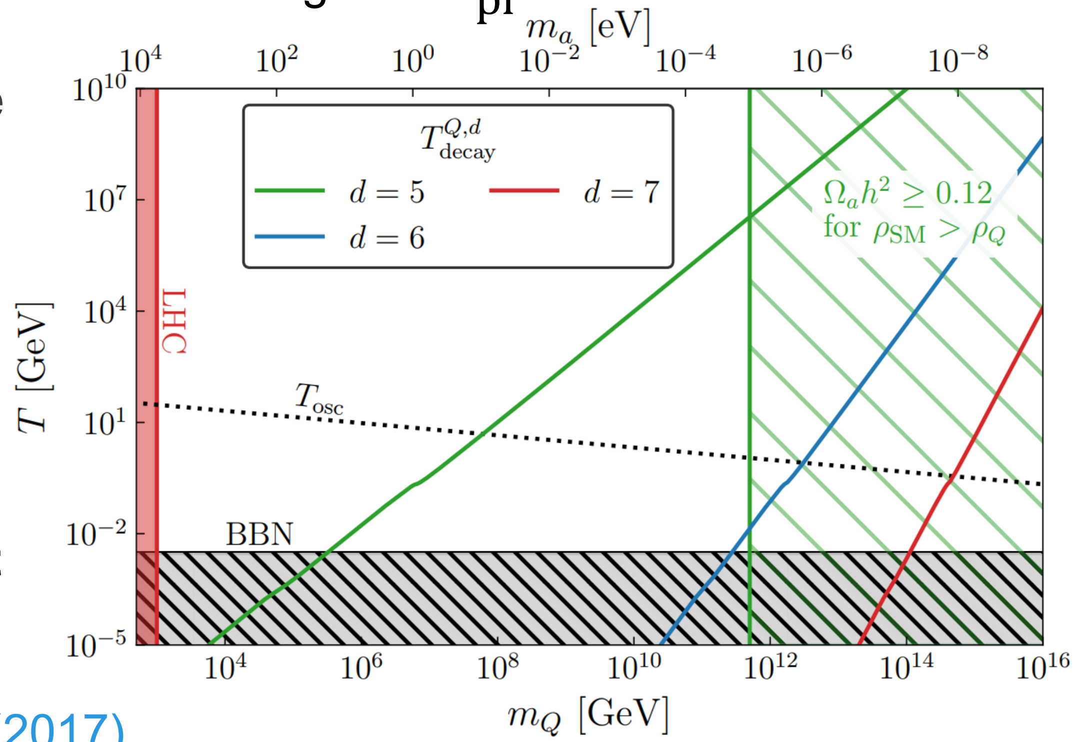
approximately

$$\tau \lesssim 0.01\,\mathrm{s}$$

***Preferred axion models* decay via dimension 5 at most!**

Put forward by Luzio, Mescia and Nardi in [*PRL* 118 (2017) 3, 031801] and [*PRD* 96 (2017) 7, 075003].

Assuming $\Lambda = M_{\mathrm{pl}}$



Not many choices for SM charges

Sticking with only renormalizable terms is already quite restrictive, especially if $N_{\text{DW}} = 1$

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_1: $(3, 1, -\frac{1}{3})$	$\overline{Q}_L d_R$	$9.3 \cdot 10^{38} (g_1)$	$2/3$	1
R_2: $(3, 1, +\frac{2}{3})$	$\overline{Q}_L u_R$	$5.4 \cdot 10^{34} (g_1)$	$8/3$	1
R_3: $(3, 2, +\frac{1}{6})$	$\overline{Q}_R q_L$	$6.5 \cdot 10^{39} (g_1)$	$5/3$	2
R_4: $(3, 2, -\frac{5}{6})$	$\overline{Q}_L d_R H^\dagger$	$4.3 \cdot 10^{27} (g_1)$	$17/3$	2

R_Q	$\mathcal{O}_{Qq}$	$\Lambda^{R_Q}_{LP}$[GeV]	E/N	N_{DW}
R_5: $(3, 2, +\frac{7}{6})$	$\overline{Q}_L u_R H$	$5.6 \cdot 10^{22} (g_1)$	$29/3$	2
R_6: $(3, 3, -\frac{1}{3})$	$\overline{Q}_R q_L H^\dagger$	$5.1 \cdot 10^{30} (g_2)$	$14/3$	3
R_7: $(3, 3, +\frac{2}{3})$	$\overline{Q}_R q_L H$	$6.6 \cdot 10^{27} (g_2)$	$20/3$	3

L. Di Luzio et. al.
[arXiv:1610.07593]

From here, can determine distinct models from PQ charges

Renormalizable

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1), \quad \text{Model A},$$
$$\mathcal{O}_4^H = y_{1,q} H \overline{q}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0), \quad \text{Model B},$$
$$\mathcal{O}_4^\Phi = y_{2,d} \Phi \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0), \quad \text{Model B},$$
$$\mathcal{O}_4^{\Phi^\dagger} = y_{3,d} \Phi^\dagger \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-1, -2), \quad \text{Model C}.$$

Dimension 5

$$\mathcal{O}_5^\Phi = \frac{\lambda_{2,d}}{\Lambda} \Phi^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (2, 1), \quad \text{Model D},$$
$$\mathcal{O}_5^{\Phi H} = \frac{\lambda_{2,q}}{\Lambda} \overline{Q}_R q_L H^\dagger \Phi, \quad \text{for } (\chi_L, \chi_R) = (2, 1), \quad \text{Model D},$$
$$\mathcal{O}_5^{\Phi^\dagger} = \frac{\lambda_{3,d}}{\Lambda} (\Phi^\dagger)^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-2, -3), \quad \text{Model E}.$$

Probing *preferred* models

Preferred axion models **decay via dimension 5 at most!**

$N_{\mathrm{DW}} = 1$

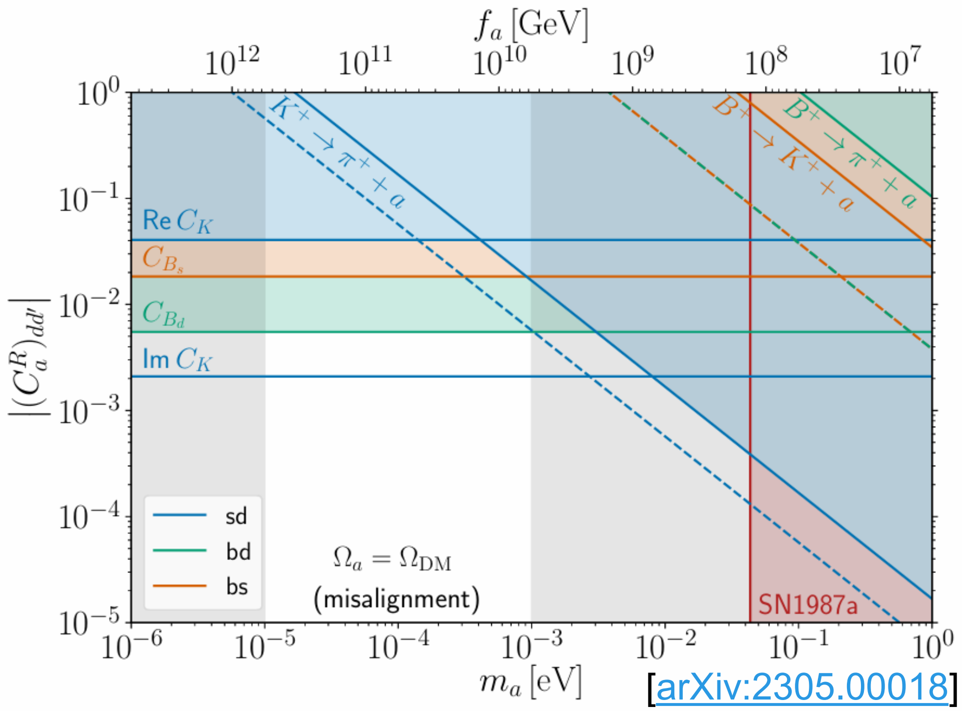
$$\mathrm{KSVZ\text{–}I} : (3,\ 1,\ -1/3),\ \text{or}$$
$$\mathrm{KSVZ\text{–}II} : (3,\ 1,\ +2/3).$$

$\mathrm{KSVZ-I}$

Mass mixing of Q and SM quarks

$$(\bar{d}_L\ \ \bar{s}_L\ \ \bar{b}_L\ \ \bar{Q}_L)\begin{pmatrix} m_d & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 \\ 0 & 0 & m_b & 0 \\ y_{2,d}f_a & y_{2,s}f_a & y_{2,b}f_a & m_Q \end{pmatrix}\begin{pmatrix} d_R \\ s_R \\ b_R \\ Q_R \end{pmatrix},$$

Leads to flavor changing decays, strongest constraint being from $K^+ \to \pi^+ a$



[arXiv:2305.00018]
Alonso-Alvarez et. al.

Probing *preferred* models

With cosmology?

$N_{\mathrm{DW}} = 1$

$$\text{KSVZ–I} : (3,\ 1,\ -1/3),\ \text{or}$$
$$\text{KSVZ–II} : (3,\ 1,\ +2/3).$$

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$

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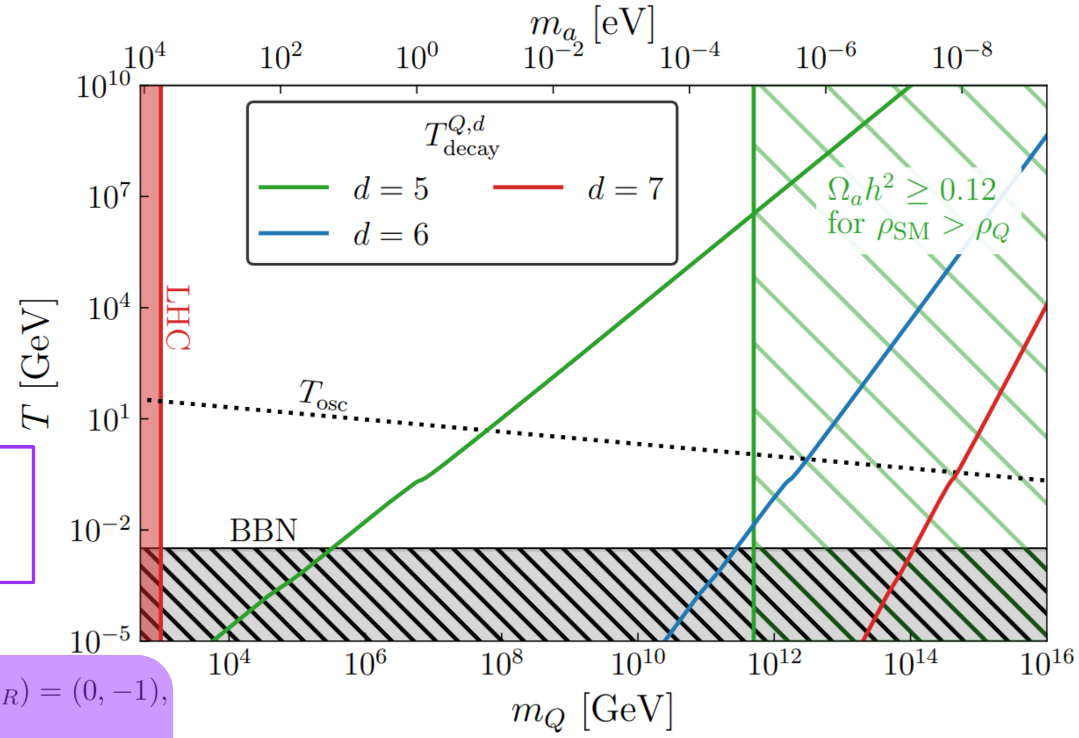
$$\mathcal{O}_5^\Phi = \frac{\lambda_{2,d}}{\Lambda} \Phi^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
$$\mathcal{O}_5^{\Phi H} = \frac{\lambda'_{2,d}}{\Lambda} \overline{Q}_R q_L H^\dagger \Phi, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$

$$\mathcal{O}_5^{\Phi^\dagger} = \frac{\lambda_{3,d}}{\Lambda} (\Phi^\dagger)^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-2, -3),$$

5 models for each KSVZ model type

$$\mathcal{O}_5^{|H|^2} = \frac{\lambda_d}{\Lambda} |H|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^{|\Phi|^2} = \frac{\lambda'_d}{\Lambda} |\Phi|^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_5^{H} = \frac{\lambda_{1,d}}{\Lambda} \Phi H \overline{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$



[arXiv:2411.17320]
AC + Ui Min

Probing *preferred* models

***Preferred axion models* decay via dimension 5 at most!**

$N_{\mathrm{DW}} = 1$

$$\mathrm{KSVZ{-}I} : (3,\ 1,\ -1/3),\ \text{or}$$
$$\mathrm{KSVZ{-}II} : (3,\ 1,\ +2/3).$$

$$\mathcal{O}_4^M = M_d \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$
$$\mathcal{O}_4^H = y_{1,d} H \bar{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^\Phi = y_{2,d} \Phi \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (1, 0),$$
$$\mathcal{O}_4^{\Phi^\dagger} = y_{3,d} \Phi^\dagger \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (-1, -2),$$
$$\mathcal{O}_5^\Phi = \frac{\lambda_{2,d}}{\Lambda} \Phi^2 \overline{Q}_L d_R, \quad \text{for } (\chi_L, \chi_R) = (2, 1),$$
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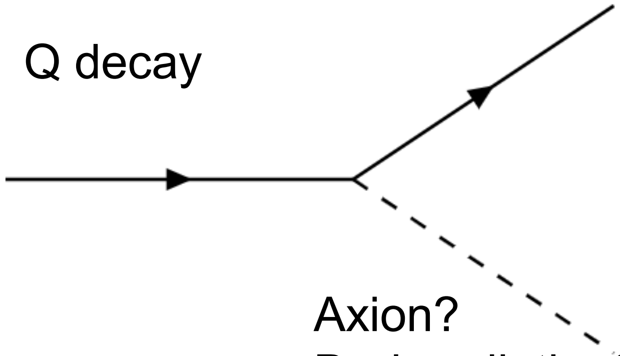
$$\mathcal{O}_5^H = \frac{\lambda_{1,d}}{\Lambda} \Phi H \bar{d}_L Q_R, \quad \text{for } (\chi_L, \chi_R) = (0, -1),$$

Q decay



Axion?
Dark radiation?
SM boson?

[arXiv:2411.17320]
AC + Ui Min

Light remnants of heavy quarks

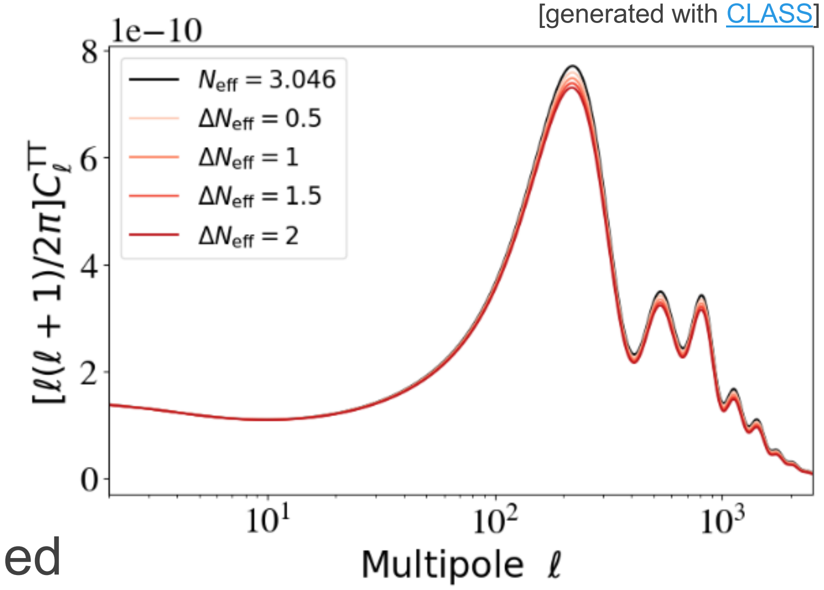
In the standard picture of the Big Bang, we have two particles species that remain relativistic until recombination.

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu$$

$$\rho_{\rm rad} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]$$

$$(N_{\rm eff})_{\rm P18} = 2.88^{+0.44}_{-0.42}$$

[generated with CLASS]



Consistent with 3 light neutrinos, additions constrained

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_{\rm DR} \longrightarrow \Delta N_{\rm eff} \equiv \left\{\frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM}\right\}\frac{\rho_a}{\rho_{\rm R}^{\rm SM}}$$

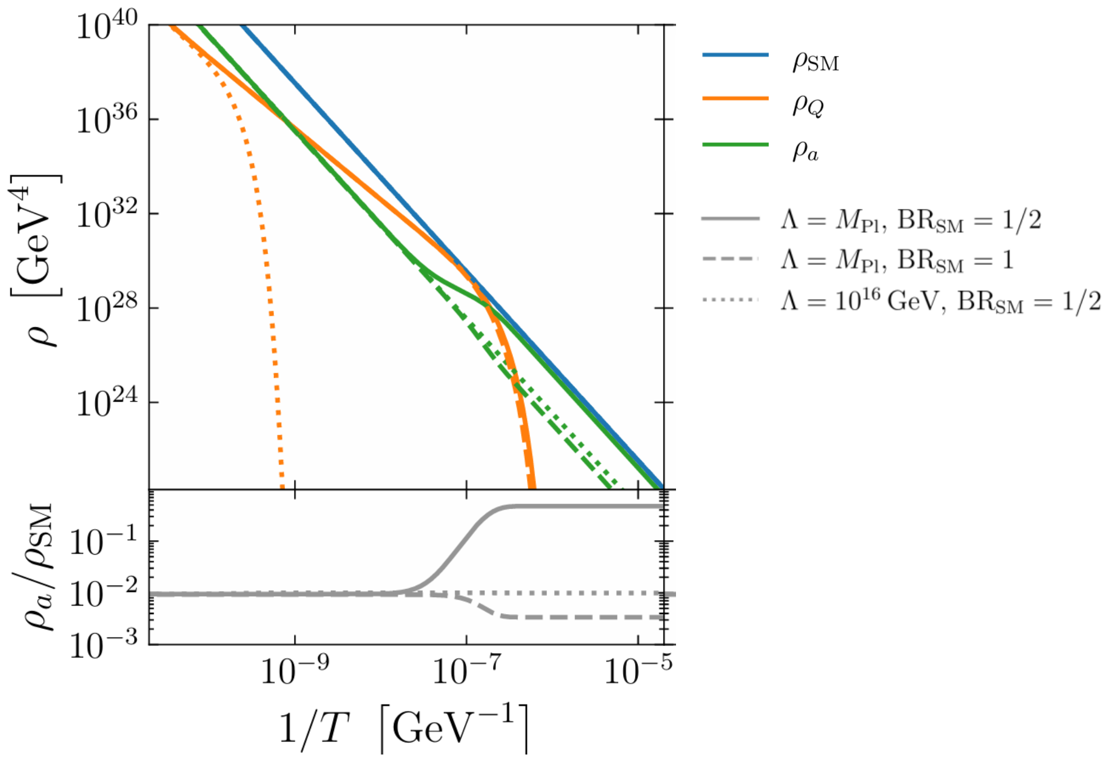
$$\Delta N_{\rm eff} \leq 0.276$$

CMB disfavors some models

Decay	Model A	Model B	Model C	Model D	Model E
$\Gamma(Q \to a\, d) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	—	$\frac{y_{2,d}}{\sqrt{2}}$	$\frac{y_{3,d}}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a}{\Lambda}$	$\frac{\lambda_{3,d} f_a}{\Lambda}$
$\Gamma(Q \to H\, q_L) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	$\frac{\lambda_{1,q} f_a}{\Lambda}$	$y_{1,q}$	—	$\frac{\lambda_{2,q} f_a}{\Lambda}$	—
$\Gamma(Q \to a\, a\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	—	$\frac{y_{2,d}}{2 f_a}$	$\frac{y_{3,d}}{2 f_a}$	—	$\frac{\lambda_{3,d}}{\Lambda}$
$\Gamma(Q \to H\, H\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_d}{\Lambda}$	—	—	—	—
$\Gamma(Q \to H\, q_L\, a) = \frac{1}{512\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_{1,q}}{\Lambda}$	—	—	$\frac{\lambda_{2,q}}{\Lambda}$	—
M_d	M_d	$\frac{y_{2,d} f_a}{\sqrt{2}}$	$\frac{y_{3,d} f_a}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a^2}{2\Lambda}$	$\frac{\lambda_{3,d} f_a^2}{2\Lambda}$

[arXiv:2411.17320]

AC + Ui Min (JCAP)



$$\Delta N_{\rm eff} \equiv \left\{ \frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM} \right\} \frac{\rho_a}{\rho_{\rm R}^{\rm SM}}$$

CMB disfavors some models

Decay	Model A	Model B	Model C	Model D	Model E
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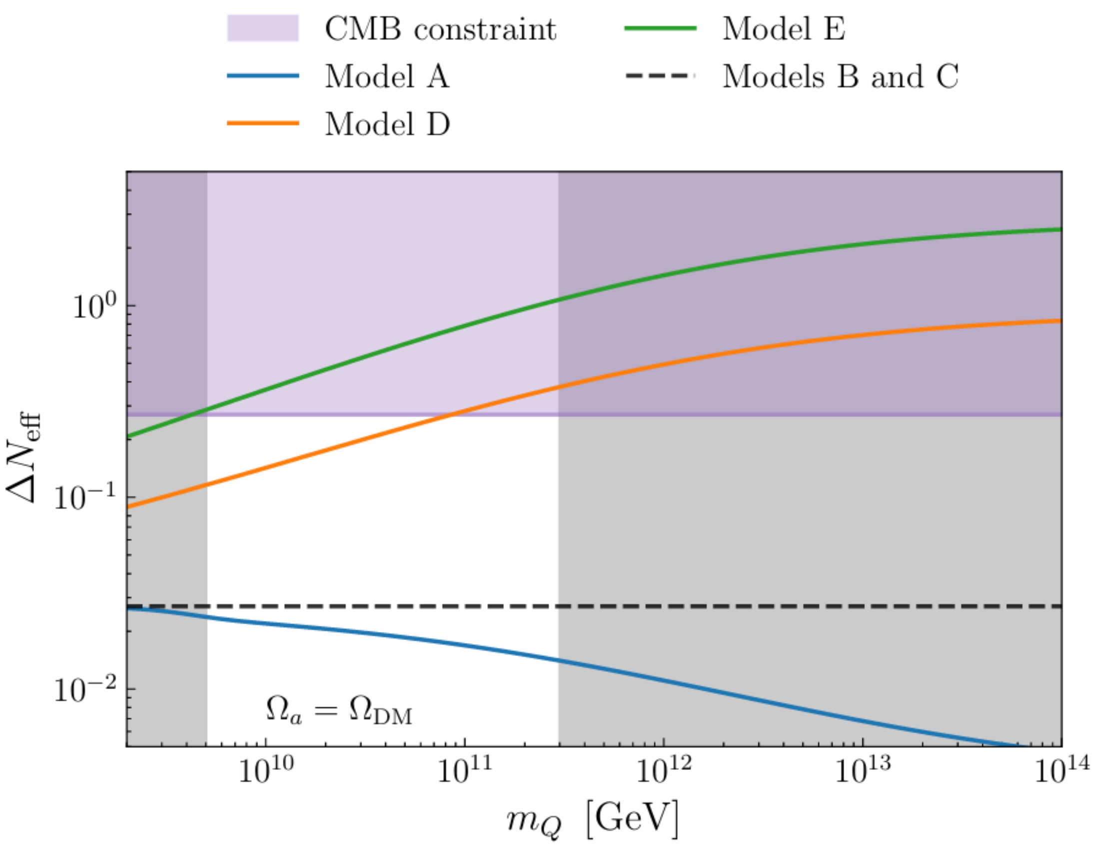
$\Gamma(Q \to H\, q_L) = \frac{3}{32\pi}\mathcal{C}^2 m_Q$	$\frac{\lambda_{1,q} f_a}{\Lambda}$	$y_{1,q}$	—	$\frac{\lambda_{2,q} f_a}{\Lambda}$	—
$\Gamma(Q \to a\, a\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	—	$\frac{y_{2,d}}{2 f_a}$	$\frac{y_{3,d}}{2 f_a}$	—	$\frac{\lambda_{3,d}}{\Lambda}$
$\Gamma(Q \to H\, H\, d) = \frac{1}{256\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_d}{\Lambda}$	—	—	—	—
$\Gamma(Q \to H\, q_L\, a) = \frac{1}{512\pi^3}\mathcal{C}^2 m_Q^3$	$\frac{\lambda_{1,q}}{\Lambda}$	—	—	$\frac{\lambda_{2,q}}{\Lambda}$	—
M_d	M_d	$\frac{y_{2,d} f_a}{\sqrt{2}}$	$\frac{y_{3,d} f_a}{\sqrt{2}}$	$\frac{\lambda_{2,d} f_a^2}{2\Lambda}$	$\frac{\lambda_{3,d} f_a^2}{2\Lambda}$

[arXiv:2411.17320]

AC + Ui Min (JCAP)

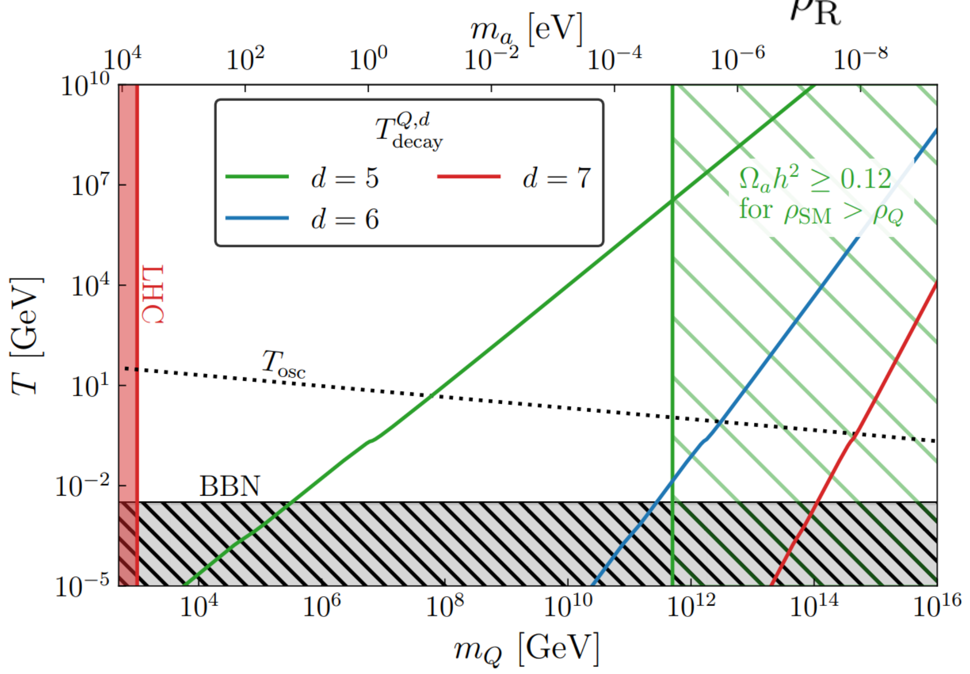


$$\Delta N_{\rm eff} \equiv \left\{ \frac{8}{7}\left(\frac{4}{11}\right)^{-\frac{4}{3}} + N_{\rm eff}^{\rm SM} \right\} \frac{\rho_a}{\rho_{\rm R}^{\rm SM}}$$

Preferred axion models too restrictive

- Constraints on heavy quark decay terms assumed standard cosmology
- Ignored heavy quark's impact on cosmology

$$\frac{\rho_Q}{\rho_R^{SM}} \sim 10^{10} \left(\frac{m_Q}{10^{12}\,\mathrm{GeV}}\right)^2 \left(\frac{1\,\mathrm{MeV}}{T}\right)$$



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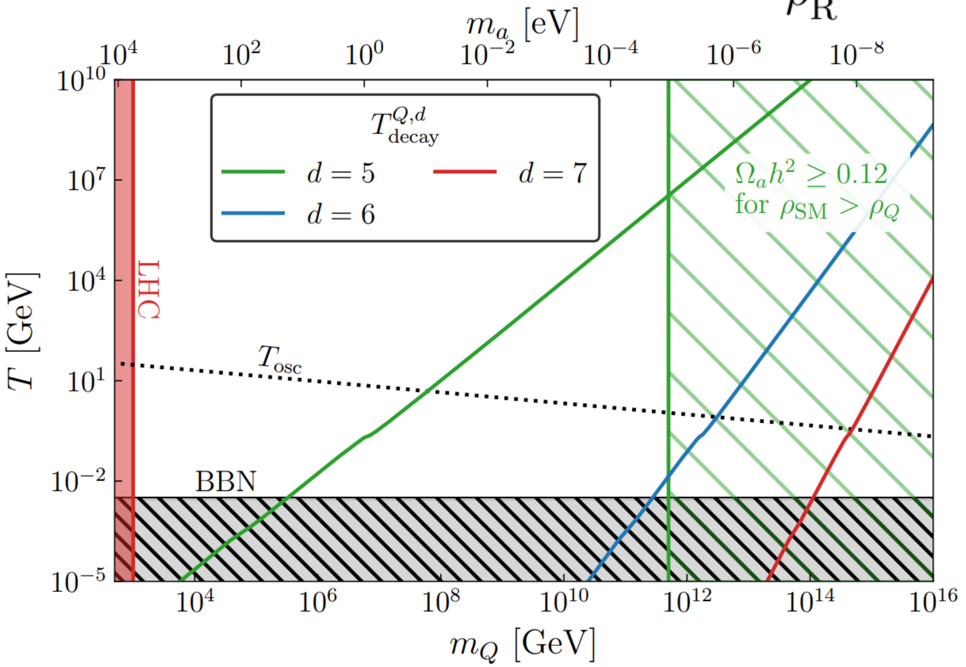
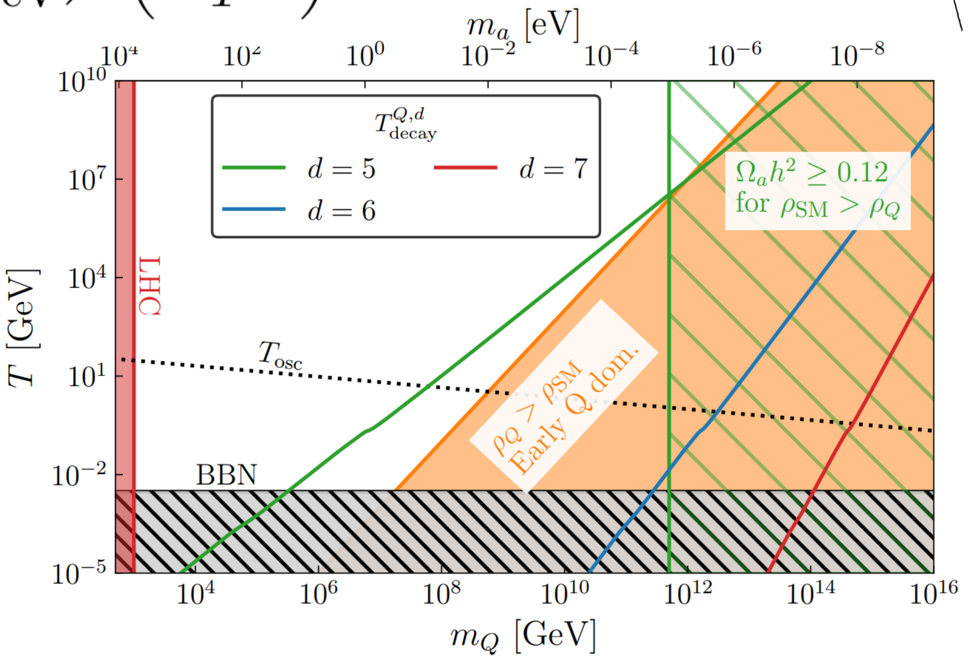
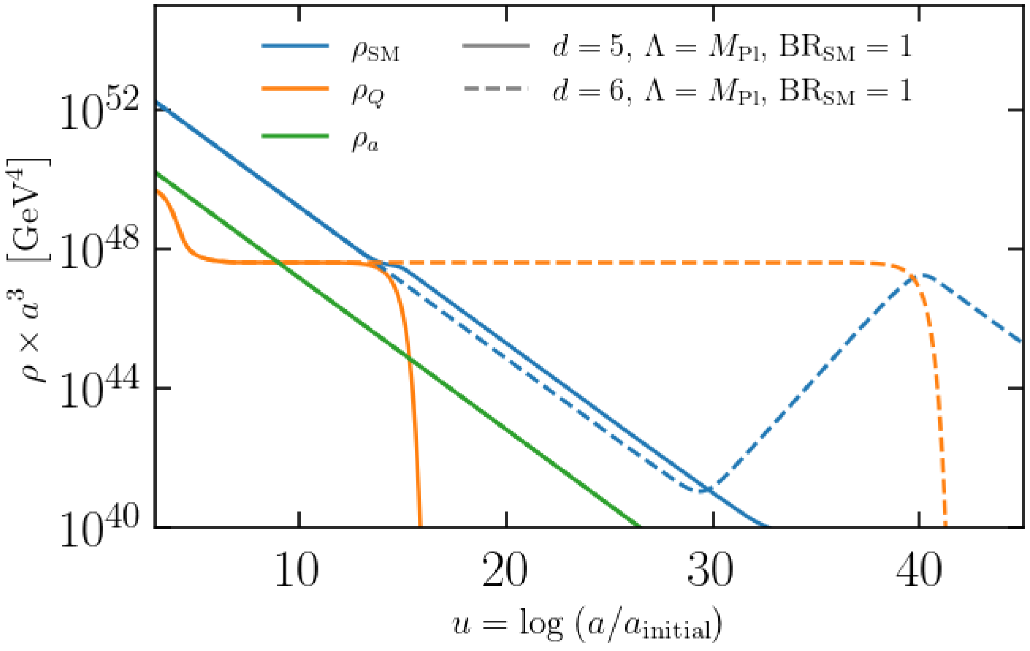
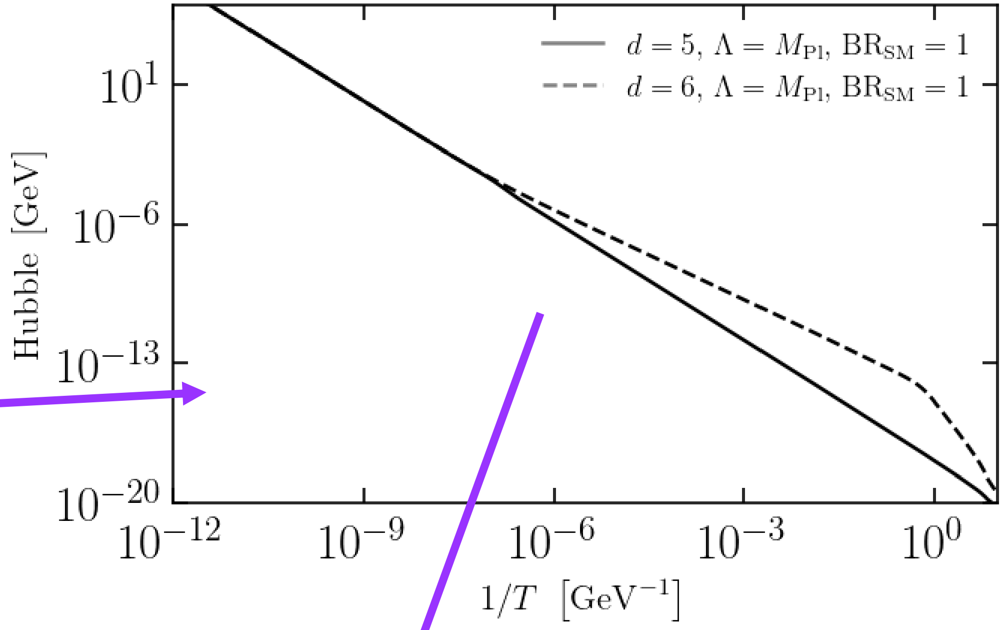




First you approximate, then you solve



Scale factor

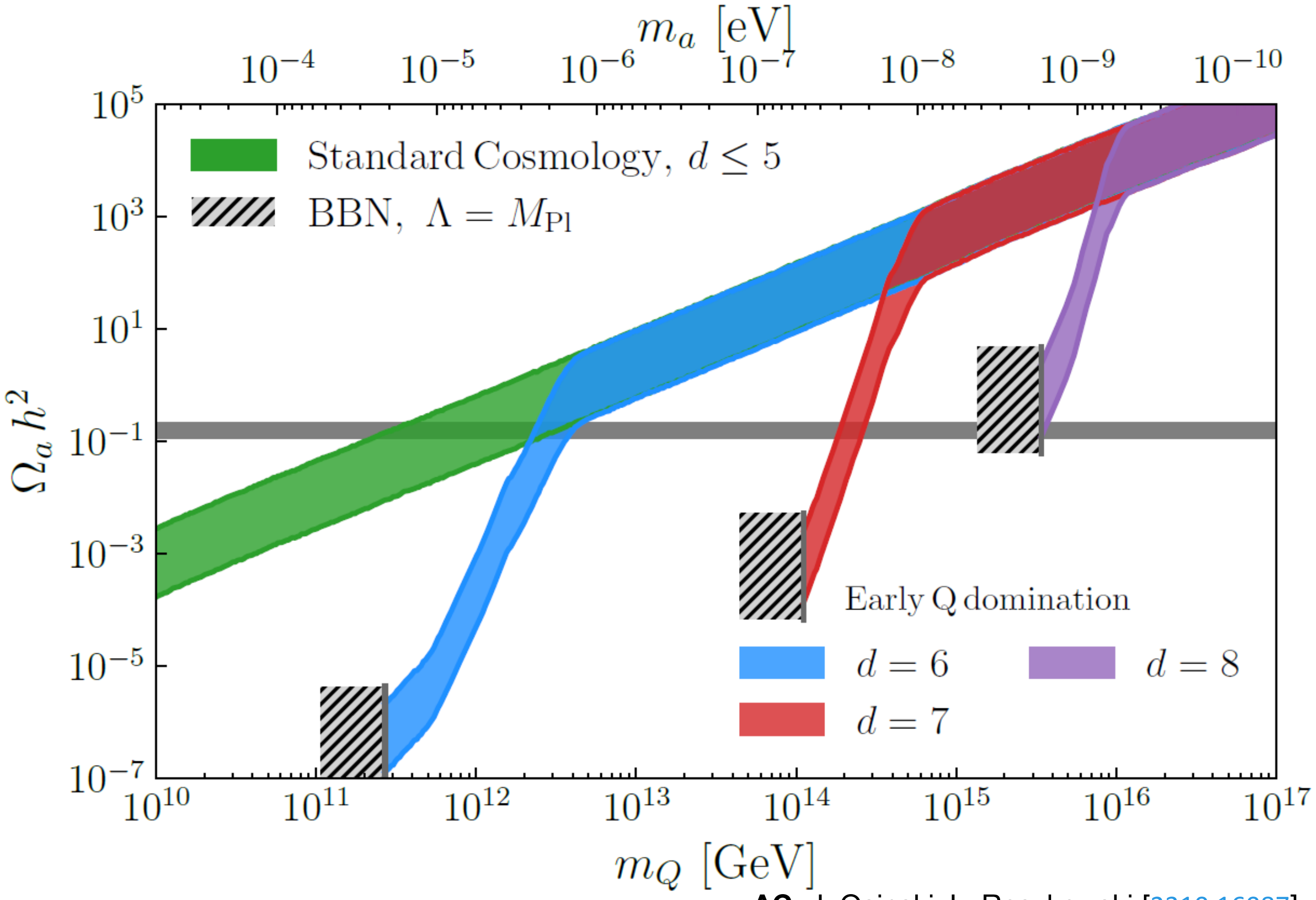


$$\left(\frac{d^2}{dt^2} + 3H(t)\frac{d}{dt}\right)\theta(t) + \tilde{m}_a^2(t)\sin(\theta(t)) = 0$$

[MiMes] misalignment solver

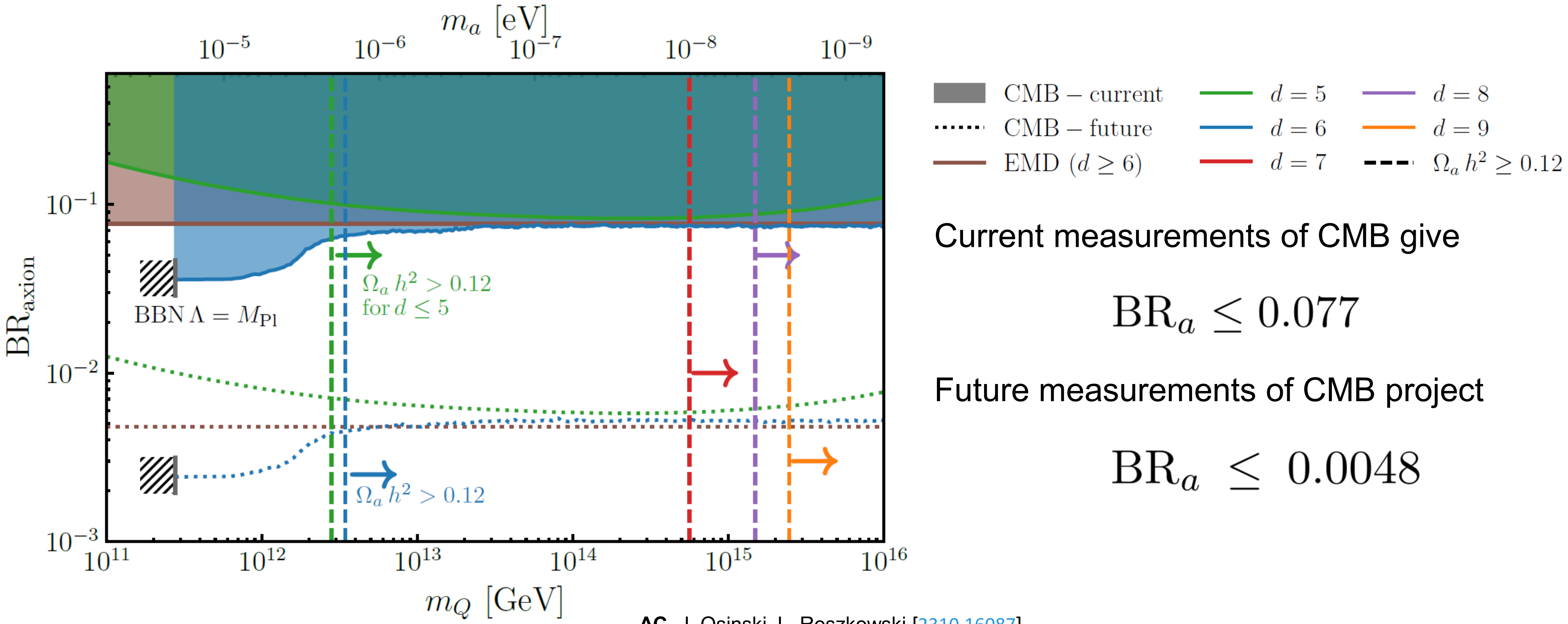
Heavy quark domination dilutes Ω_a

- We were the first to point out the axion models themselves could provide early matter domination.
- Plotting band of different initial angles $\theta_i \in \left[\frac{1}{2}, \frac{\pi}{\sqrt{3}}\right]$
- Dimension 6-7-8 now are viable.
- More axion models available and parameter space.



AC, J. Osinski, L. Roszkowski [2310.16087]

Constraints from dark radiation



Current measurements of CMB give

$$\mathrm{BR}_a \leq 0.077$$

Future measurements of CMB project

$$\mathrm{BR}_a \leq 0.0048$$

AC, J. Osinski, L. Roszkowski [2310.16087]

GUT-scale PQ breaking

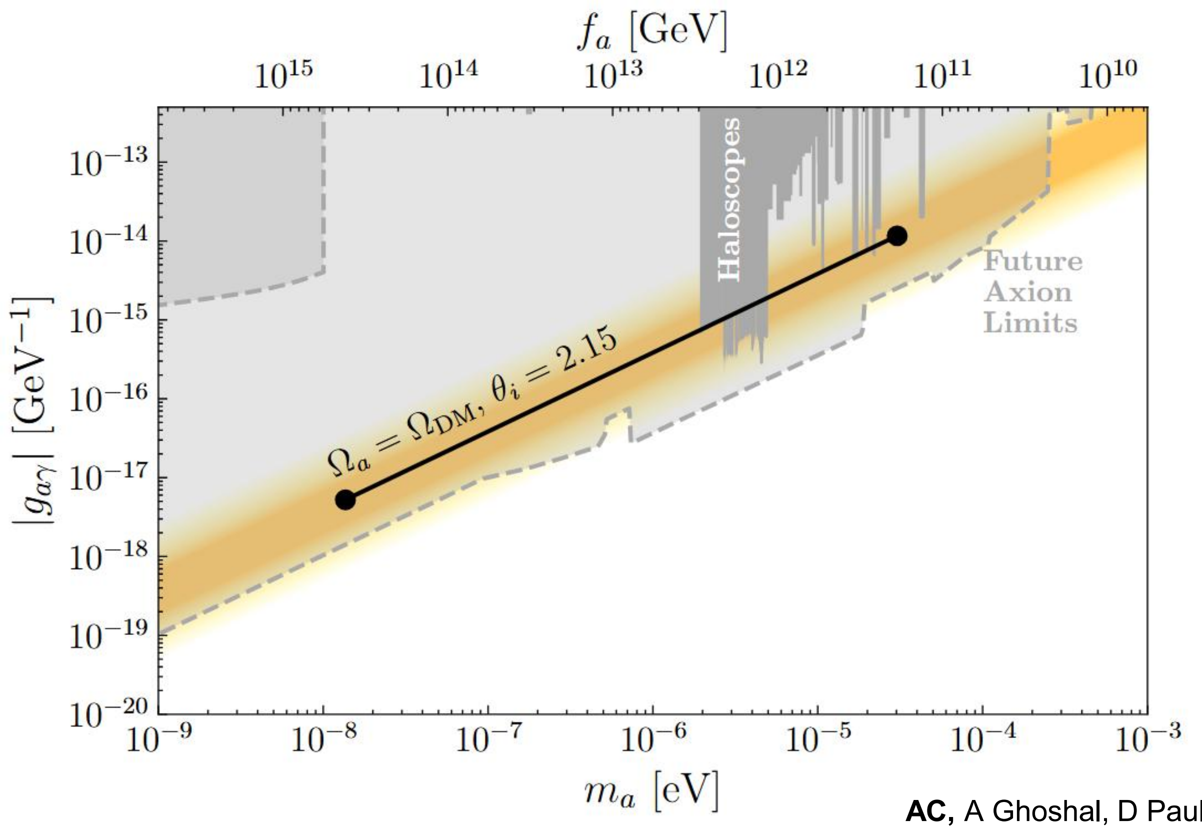
Previously taken $m_Q = f_a$ and $\Lambda = M_{\text{pl}}$

Can relax this and obtain order of magnitude lower mass.

The plot shows dimension 6 models

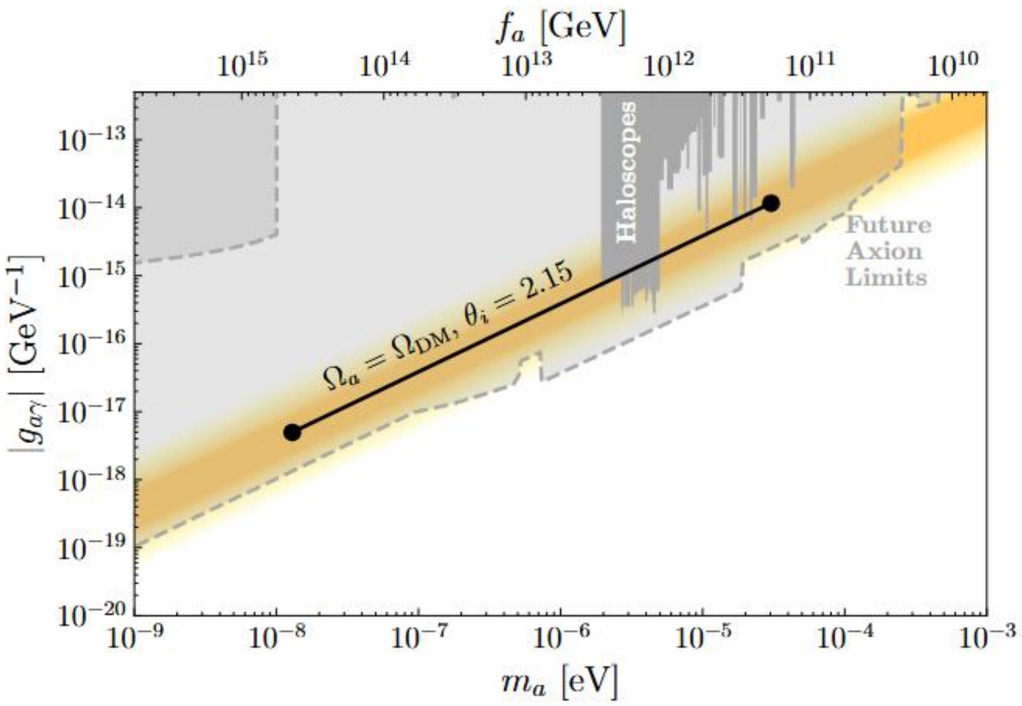
The point with smallest m_a corresponds to:

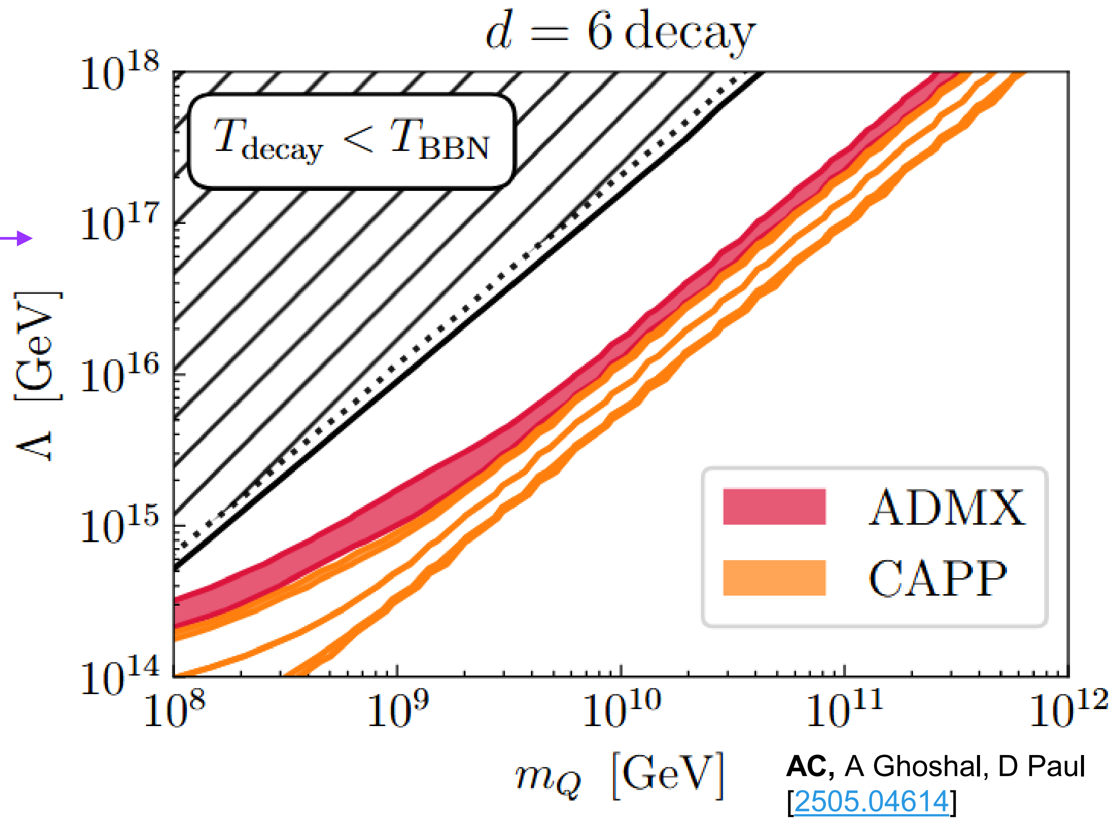
$$f_a = 4 \times 10^{14}\ \text{GeV}$$
$$m_Q = 4 \times 10^{11}\ \text{GeV}$$
$$\Lambda = 4 \times 10^{18}\ \text{GeV}$$



AC, A Ghoshal, D Paul [2505.04614] (PRD)

GUT-scale PQ breaking





AC, A Ghoshal, D Paul [2505.04614]

More models without domain walls

Recently, Di Luzio et. al. confirmed my findings and catalogued higher dimensional models

L. Di Luzio et. al. [arXiv:2412.17896]

Rep. $(\mathcal{C},\mathcal{I},6\mathcal{Y})$			E/N	$N_{\rm DW}$	Min. d	Example operator	LP [GeV]
3	1	-2	$2/3$	1	3	$\bar{\mathcal{Q}}_L\, d_R$	2.0×10^{39}
3	1	4	$8/3$	1	3	$\bar{\mathcal{Q}}_L\, u_R$	6.8×10^{35}
3	1	-14	$98/3$	1	6	$\bar{\mathcal{Q}}_L\, d_R\,(\bar{e}^c_R\, e_R)$	2.2×10^{22}
$\overline{3}$	1	8	$32/3$	1	6	$\bar{u}_R\,\gamma_\mu\, e_R\, \bar{d}_R\,\gamma^\mu\, \mathcal{Q}_R$	3.0×10^{28}
$\overline{3}$	1	-10	$50/3$	1	6	$(\bar{d}_R\, d^c_R)\,\bar{e}_R\,\mathcal{Q}_L$	6.4×10^{25}
3	1	16	$128/3$	1	6	$\bar{\mathcal{Q}}_L\, u_R\,(\bar{e}_R\, e^c_R)$	1.8×10^{21}
$\overline{3}$	1	20	$200/3$	1	9	$(\bar{d}^c_R\, d_R)\,(\bar{e}^c_R\, e_R)\,\bar{u}_R\,\mathcal{Q}_L$	6.2×10^{19}
3	1	22	$242/3$	1	9	$\bar{\mathcal{Q}}_L\, u_R\,(\bar{\ell}_L\,\ell^c_L)\,(\bar{e}_R\, e^c_R)$	2.0×10^{19}

$\mathrm{Br}_{\rm SM}\approx 1$

$\therefore\ \Delta N_{\rm eff}\ll 0.027$

$$g_{a\gamma}\equiv\frac{\alpha}{2\pi}\frac{1}{f_a}\left(\frac{E}{N}-1.92(4)\right)$$

GUT-scale PQ breaking & $N_{\mathrm{DW}} = 1$

Previously taken $m_Q = f_a$ and $\Lambda = M_{\mathrm{pl}}$

Can relax this and obtain order of magnitude lower mass.

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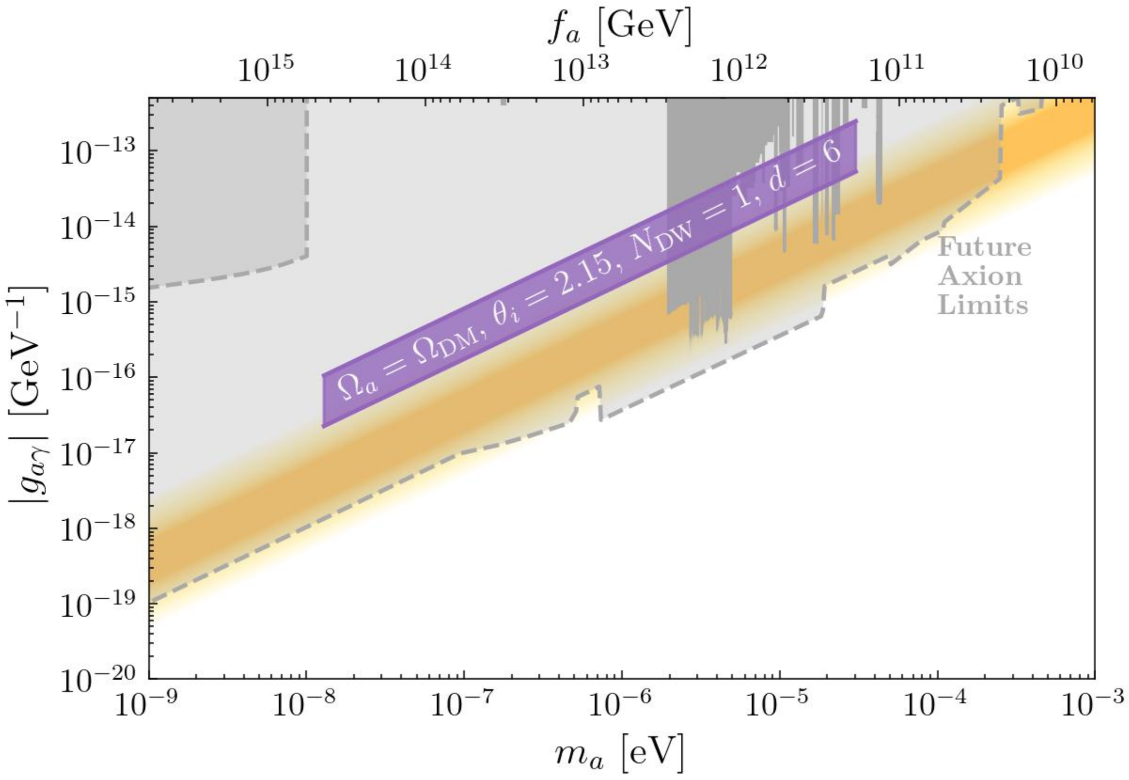


When does PQ break?

BEFORE INFLATION

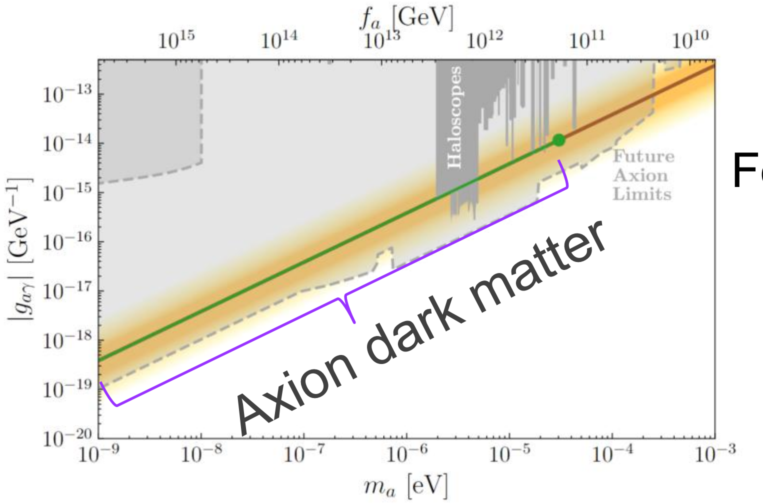
Can have $m_a \leq 10\ \mu\text{eV}$

No detectable dark radiation.

AFTER INFLATION

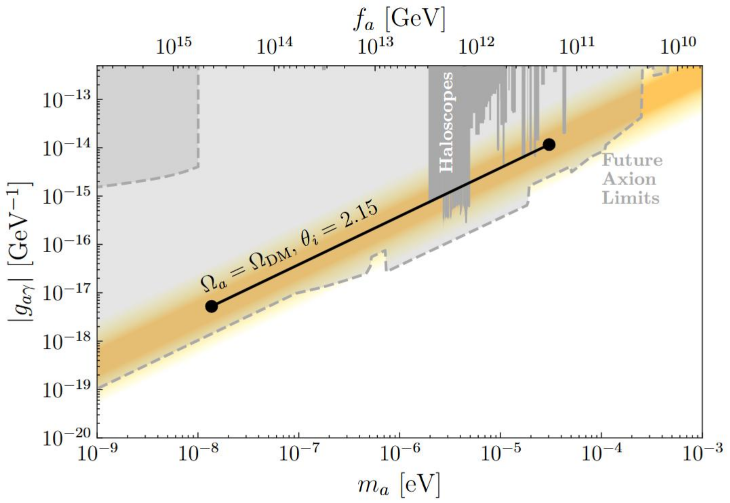
Now can have $m_a \leq 10\ \mu\text{eV}$ with HQD

The only models that survive have no detectable dark radiation.



For $m_a = 10^{-8}$ eV

$\theta_i = 10^{-2}$



Both scenarios have the same phenomenological output.

Primordial GWs as a tracer of HQD

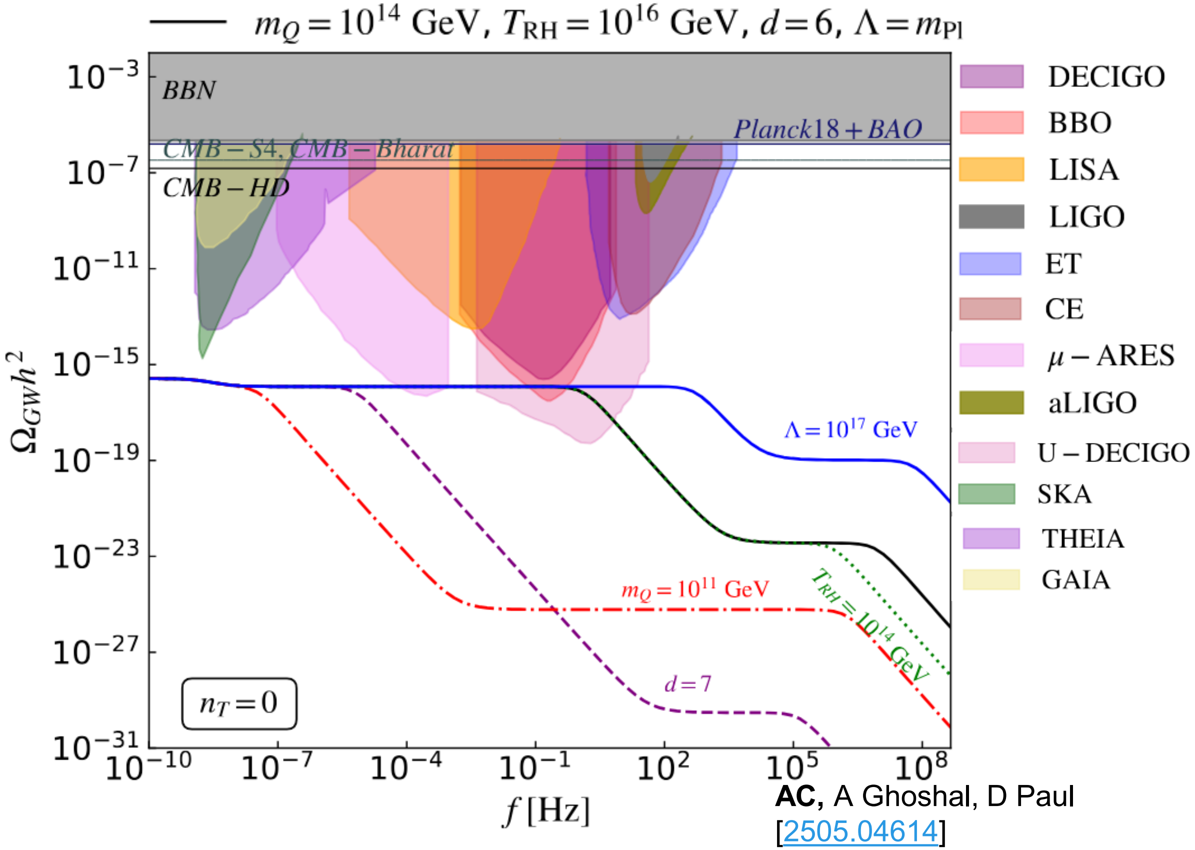
- These are tensor perturbations from inflation, its power spectrum parameterized by

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

- Current constraints on tensor to scalar ratio

$$r = \frac{A_T}{A_S} < 0.036$$

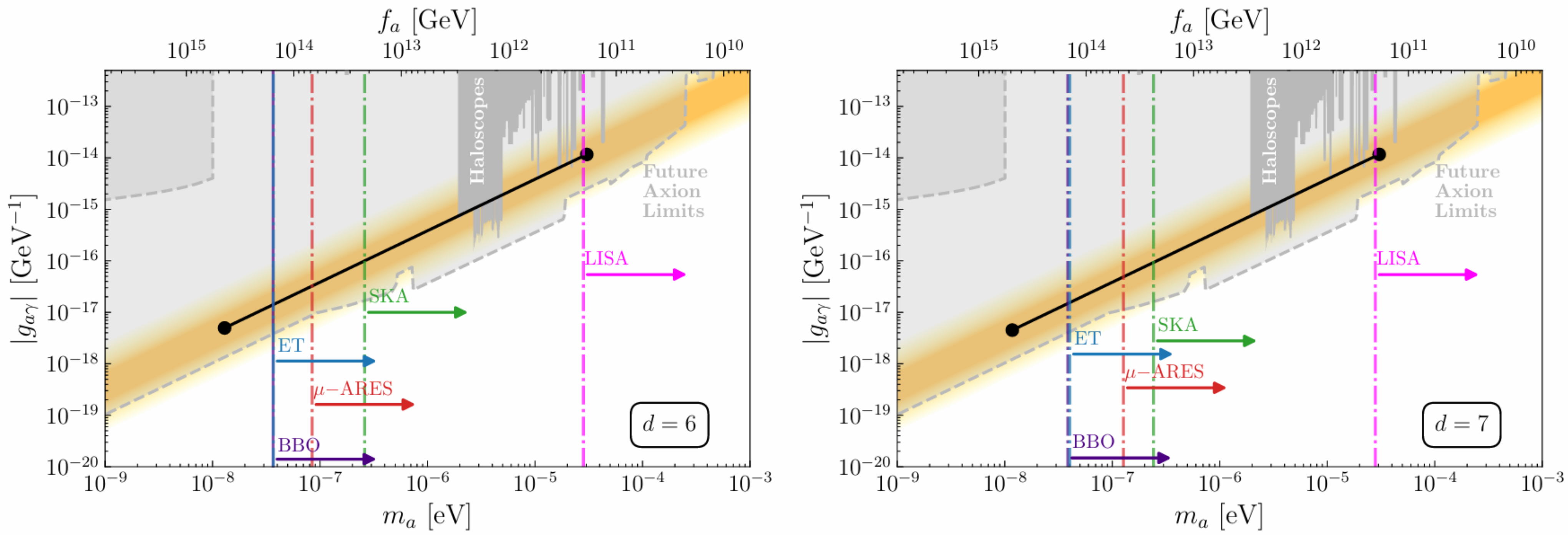
- The tilt n_T is determined in vanilla slow-roll inflation to be $n_T = -\frac{r}{8}$.



AC, A Ghoshal, D Paul [2505.04614]

Where GWs will be able to probe

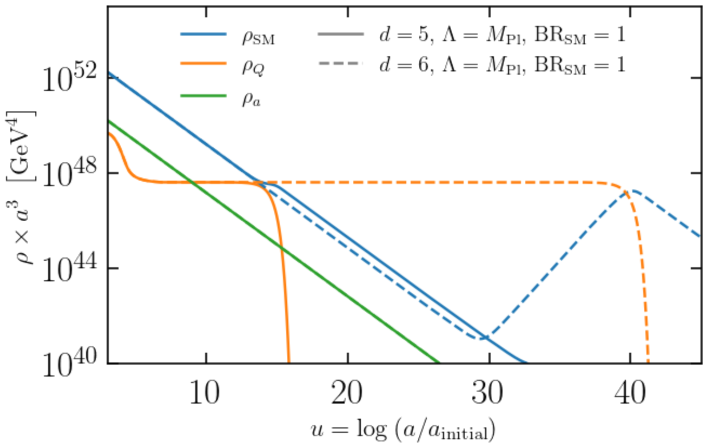
- We take an optimistically blue-tilted power spectrum to assess the maximum sensitivity of GW experiments.

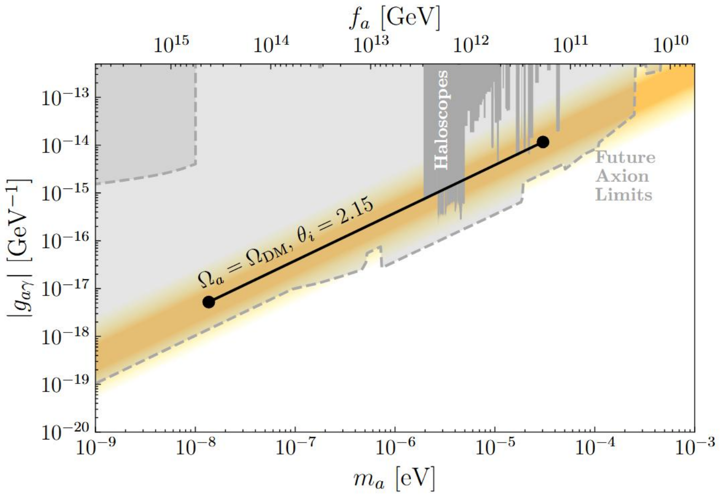


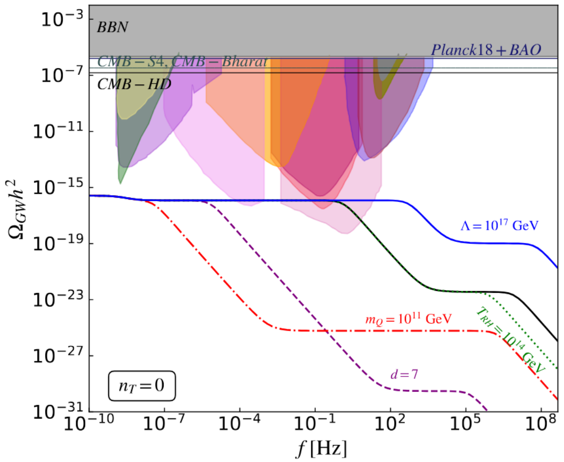
AC, A Ghoshal, D Paul [2505.04614]

Conclusions

- High energy axion models have phenomenological consequences.
- Heavy quark domination makes more models viable. Including some without a domain wall problem.
- Axion dark matter as light as $m_a = 10^{-8}$ eV can be achieved in post-inflationary breaking without adding additional fields.







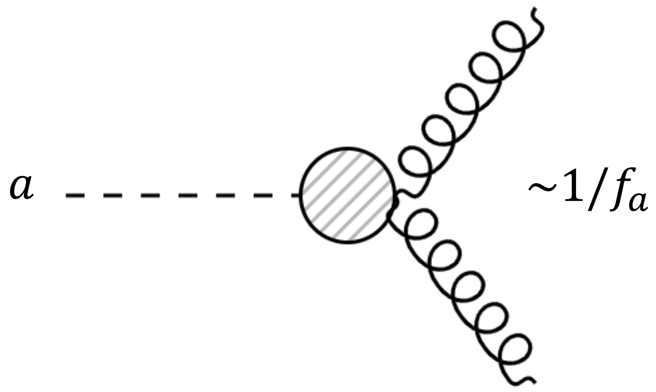
Thank you!

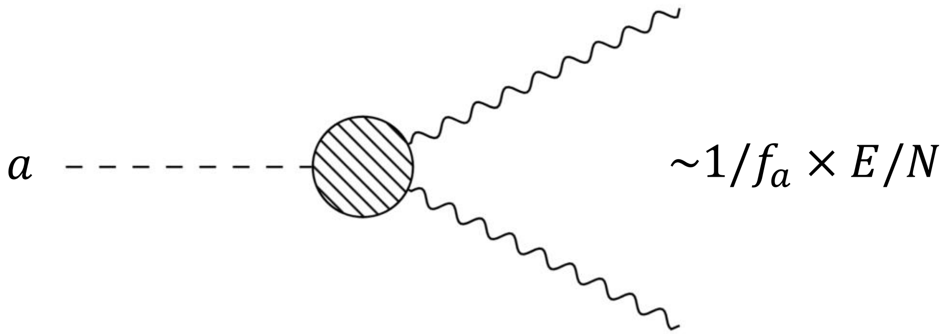
Back-up slides

谁让你非要问的！

Axion dark matter: Testable?

$$\mathcal{L}^a_{\text{eff}} = \frac{a(t,x)}{f_a}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a + \frac{a(t,x)}{f_a}\frac{\alpha_{\text{em}}}{8\pi}\frac{E}{N}F_{\mu\nu}\tilde{F}^{\mu\nu}$$



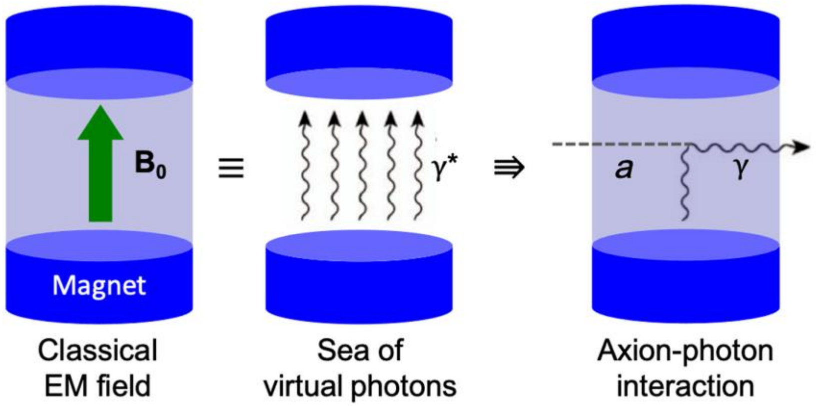


- Axion-photon coupling

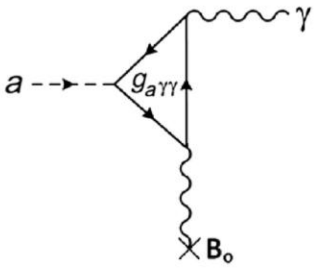
$$g_{a\gamma} \equiv \frac{\alpha}{2\pi}\frac{1}{f_a}\left(\frac{E}{N} - 1.92(4)\right)$$

Axion haloscope experiments

- These experiments try to convert the axion dark matter in the local environment (the halo) into photons.

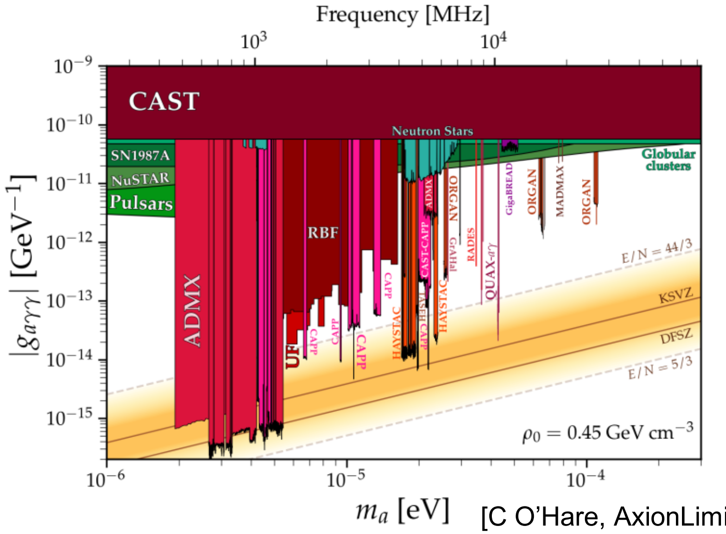


[Sci.Adv. 8 (2022) 8, amb9928]



- The use microwave cavity inside a strong magnetic field.
- Conversion is resonantly enhanced when $\omega = m_a$.
- Even when $E = 0$, pion contribution gives an effective $g_{a\gamma}$

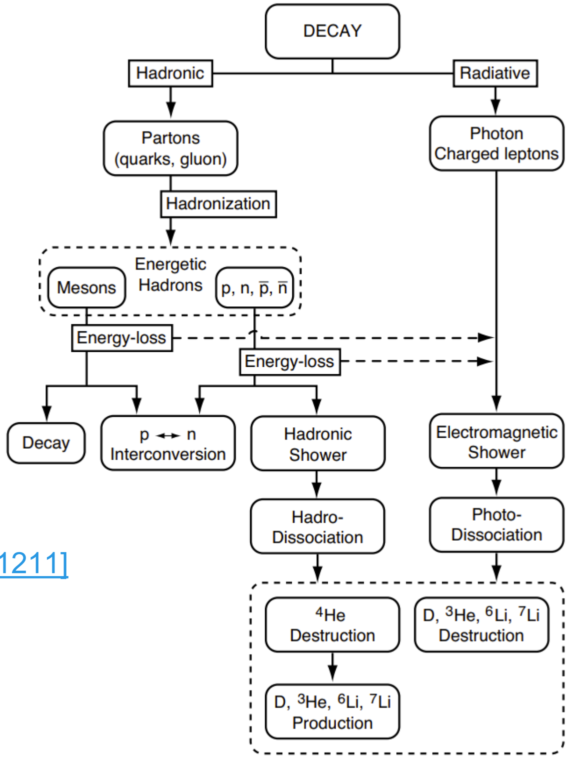
$$g_{a\gamma} \equiv \frac{\alpha}{2\pi}\frac{1}{f_a}\left(\frac{E}{N} - 1.92(4)\right)$$



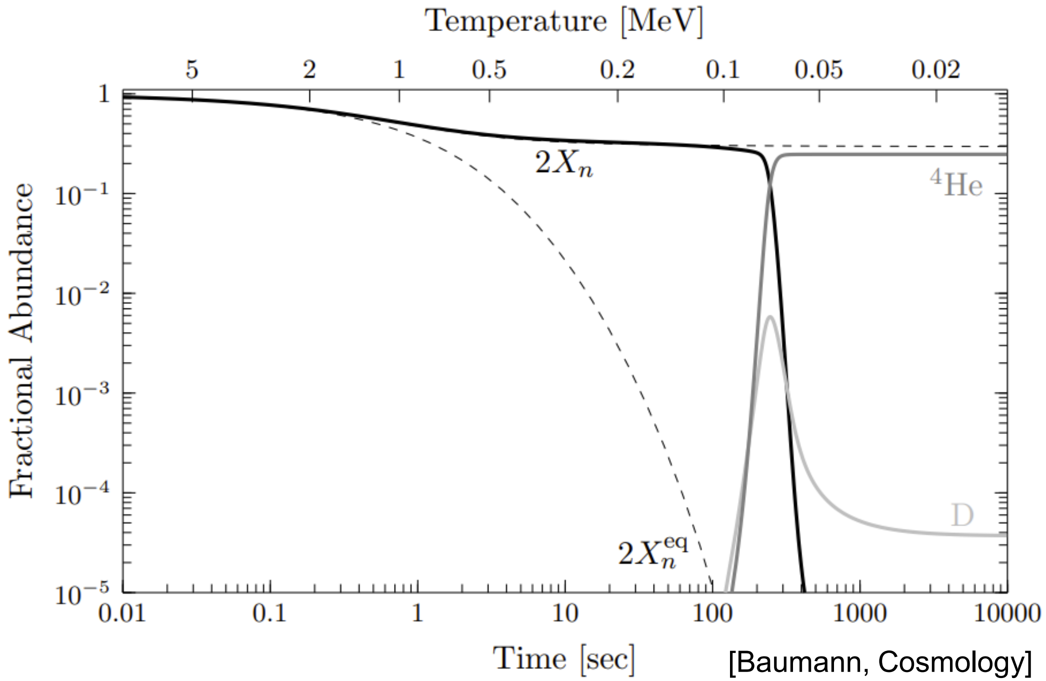
[C O'Hare, AxionLimits]

Big Bang nucleosynthesis (BBN)

Decays of heavy particles can disrupt the delicate balance of BBN.



[arXiv:1709.01211]



[Baumann, Cosmology]

Limits typically

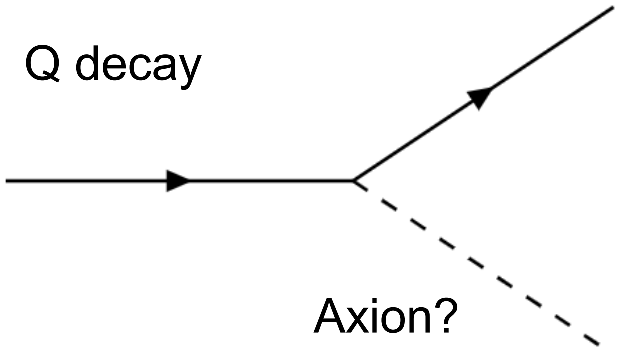
$\tau \lesssim 0.01\,\mathrm{s}$ or $T^Q_{\mathrm{decay}} \gtrsim T_{\mathrm{BBN}} \approx 3\,\mathrm{MeV}$

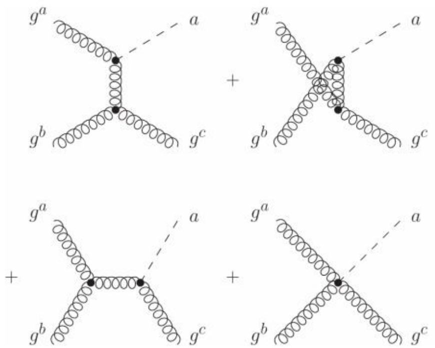
Decay of heavy quarks

- Heavy quark decay products may leave a trace in the early universe.

$$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = -4H\rho_a + \mathrm{BR}_{\mathrm{axion}}\Gamma_Q\rho_Q + \langle E^{\mathrm{axion}}_{\mathrm{scat}}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\mathrm{eq}}}\right),$$

$$\frac{\mathrm{d}n_Q}{\mathrm{d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\mathrm{eq}})^2\right].$$



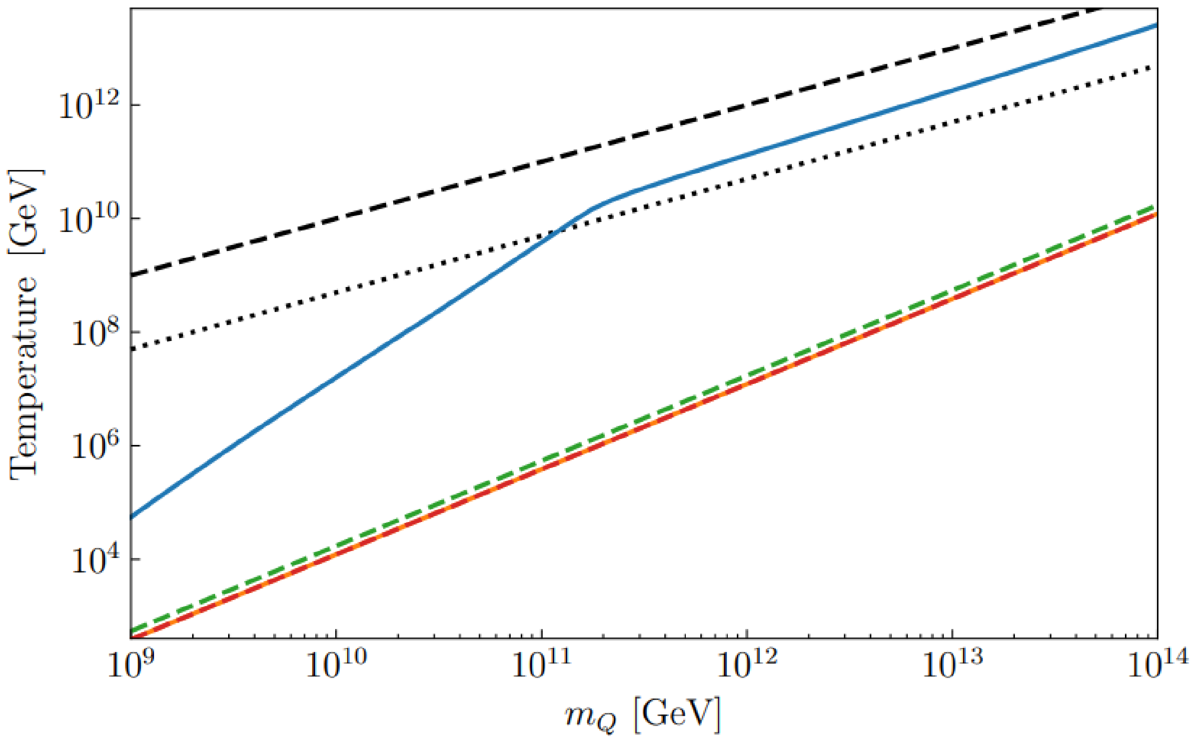


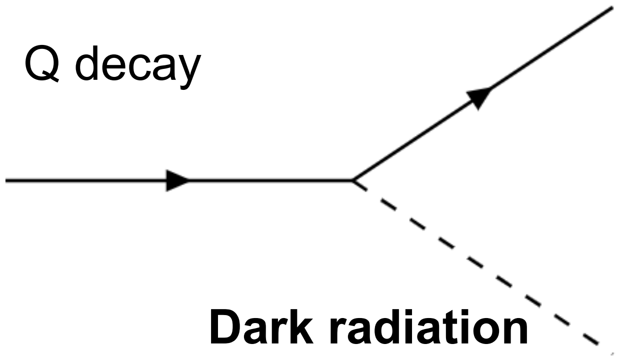
$T_{\mathrm{axion}}^{\mathrm{decoupling}} < T_Q^{\mathrm{decay}}$?

P. Graf et. al. (PRD) [arXiv:1008.4528] + more

Q decay after axion decoupling?

- The answer is model dependent





We perform a full numerical treatment

To assess the effect, we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

Entropy density evolution:

$$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3H s_{\rm R}^{\rm SM} + \frac{{\rm BR}_{\rm SM}\Gamma_Q}{T}\rho_Q \,,$$

Thermal axion energy density evolution:

$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR}_{\rm axion}\Gamma_Q\rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \,,$$

Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right] \,.$$

We perform a full numerical treatment

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Thermal axion energy density evolution:

$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR_{axion}}\Gamma_Q\rho_Q + \langle E_{\rm scat}^{\rm axion}\rangle\gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right) ,$$

Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3Hn_Q - \Gamma_Q n_Q - \langle\sigma\, v\rangle\left[n_Q^2 - (n_Q^{\rm eq})^2\right] .$$

"Simplified" branching ratio
If only $Q \to a + q$ the only decay channel, ${\rm BR_{SM}} = {\rm BR_{axion}} = 1/2$

We perform a full numerical treatment

To fully assess the effect we solve the coupled Friedmann-Boltzmann equations

$$\frac{3H^2 M_{\rm Pl}^2}{8\pi} = \rho_{\rm R}^{\rm SM} + \rho_{\rm DR}^{\rm axion} + \rho_Q$$

γ_a: Thermal axion production rate more on this later.

$$\left\langle E_{\rm scat}^{\rm axion} \right\rangle \sim 3T_{\rm SM}$$

Entropy density evolution:

$$\frac{{\rm d}s_{\rm R}^{\rm SM}}{{\rm d}t} = -3H s_{\rm R}^{\rm SM} + \frac{{\rm BR}_{\rm SM}\Gamma_Q}{T}\rho_Q \, ,$$

Thermal axion energy density evolution:

$$\frac{{\rm d}\rho_a}{{\rm d}t} = -4H\rho_a + {\rm BR}_{\rm axion}\Gamma_Q \rho_Q + \langle E_{\rm scat}^{\rm axion} \rangle \gamma_a \left(1 - \frac{n_a}{n_a^{\rm eq}}\right) \, ,$$

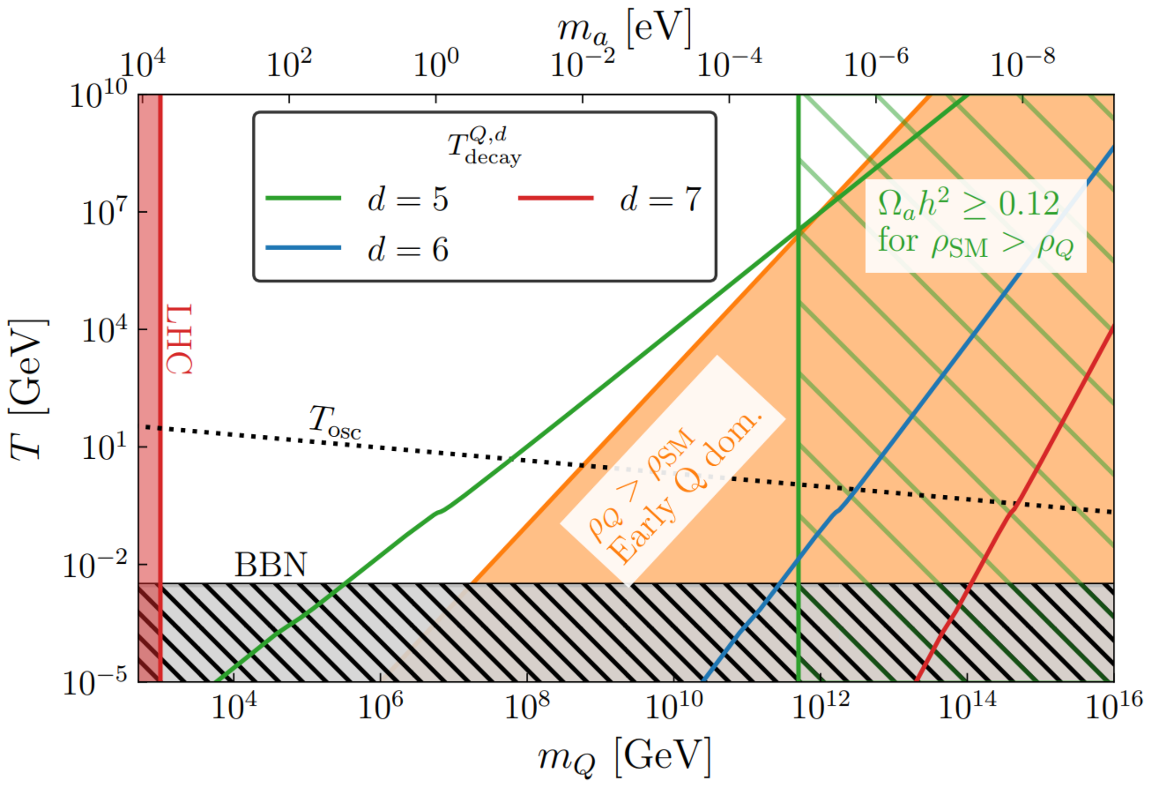
Heavy quark energy density evolution:

$$\frac{{\rm d}n_Q}{{\rm d}t} = -3H n_Q - \Gamma_Q n_Q - \langle \sigma \, v \rangle \left[n_Q^2 - (n_Q^{\rm eq})^2 \right] \, .$$

Heavy quark domination

- For these higher dimensional Q decay models, the heavy quarks will dominate the early universe.
- This alters the misalignment mechanism [*Steinhart* et. al. (1984) + *Lazarides* et. al. (1990)]
- We show T_{osc}, temperature when axion field oscillations begin

$$3H\left(T_{\mathrm{osc}}\right)=\tilde{m}_a\left(T_{\mathrm{osc}}\right)$$



Inflationary gravitational waves

For astrophysical gravitational waves, one works in linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

In cosmological settings, the background is expanding

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

The resulting wave equation is (problem 6.4 Baumann's book)

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 0$$

When treating the quantum fluctuations of tensor perturbations during inflation, one obtains the following power spectrum

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

Current constraints on IGW

The precise form of the power spectrum depends on the specifics of inflation.

$$P_T^{\text{prim.}}(k) = A_T(k)\left(\frac{k}{k_*}\right)^{n_T}$$

Tensor perturbations (GWs) alter the polarization of CMB photons

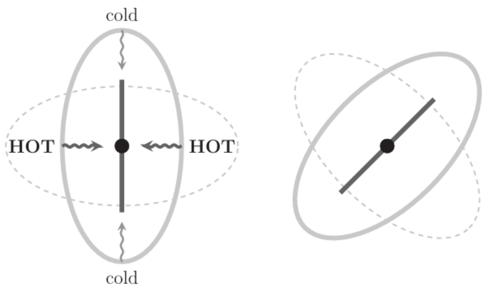


Fig. 7.35 Cartoon illustrating that the anisotropic stretching and compressing of space by a gravitational wave creates a temperature quadrupole and hence leads to CMB polarization. The two polarizations of the gravitational wave produce polarization of the CMB photons with a relative angle of 45°. This is why gravitational waves produce both E and B-modes, while density perturbations create only E-modes.

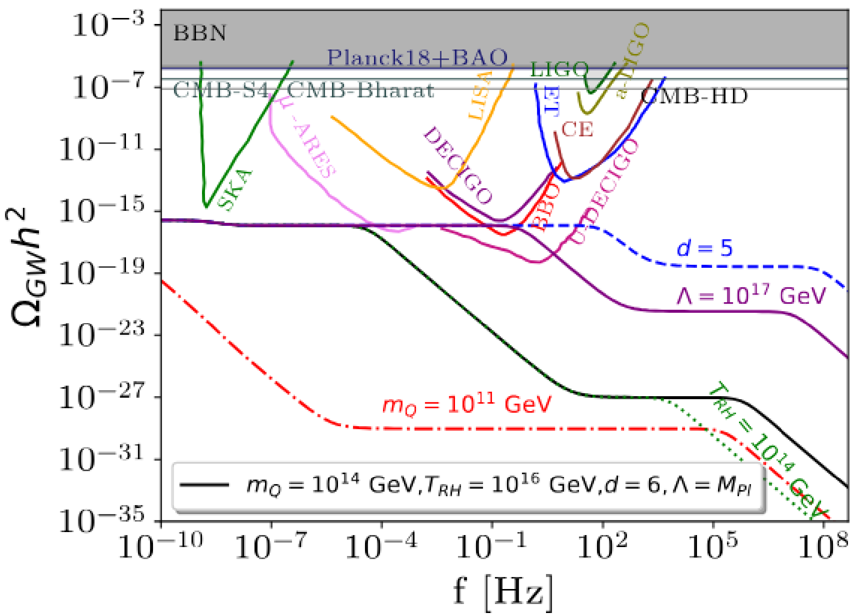
Current constraints on measurements of B-modes in the CMB constrain the ratio

$$r = \frac{A_T}{A_S} < 0.036$$

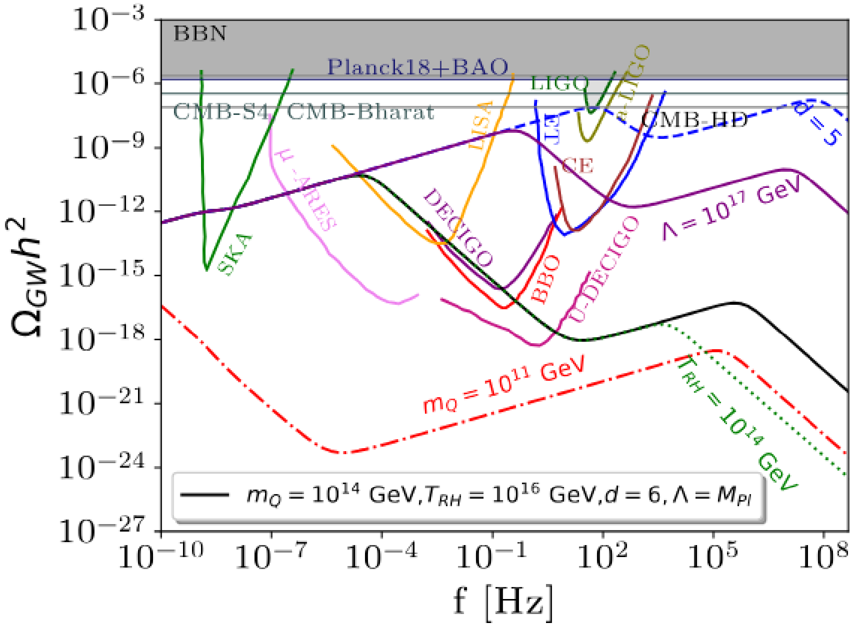
The tilt n_T is determined in vanilla slow-roll inflation to be $n_T = -\frac{r}{8}$, but there are numerous alternatives to this relation, string inflation… ekpyrotic… particle production at reheating

Forecasting IGW signal to learn about axion models

We take blue-tilted spectra and project the sensitivities of future experiments



(a) $n_T = 0$



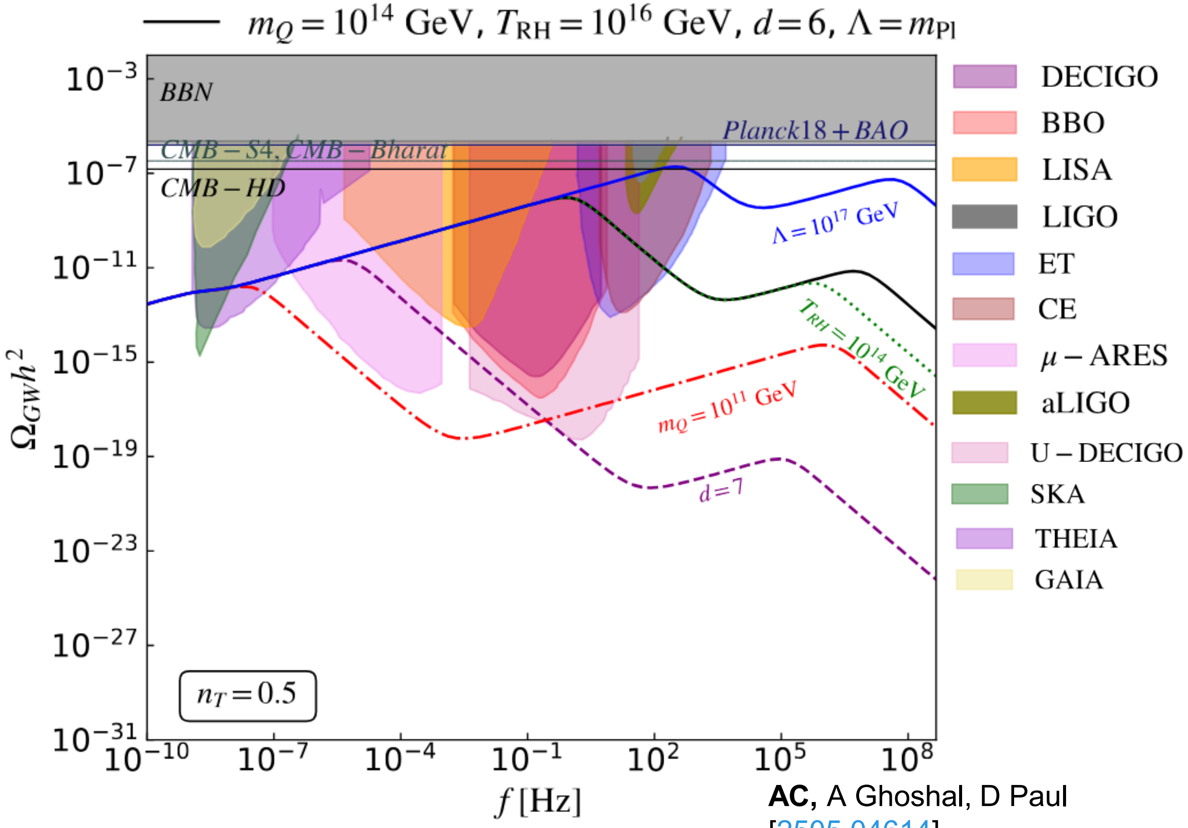
(b) $n_T = 0.5$

Maximum (blue-)tilt

- We take an optimistically blue-tilted power spectrum to assess the maximum sensitivity of GW experiments.
- The sensitivity plots shown on the right are just illustrative (from GWplotter for example)
- We perform our forecasts using the signal-to-noise ratio for each detector

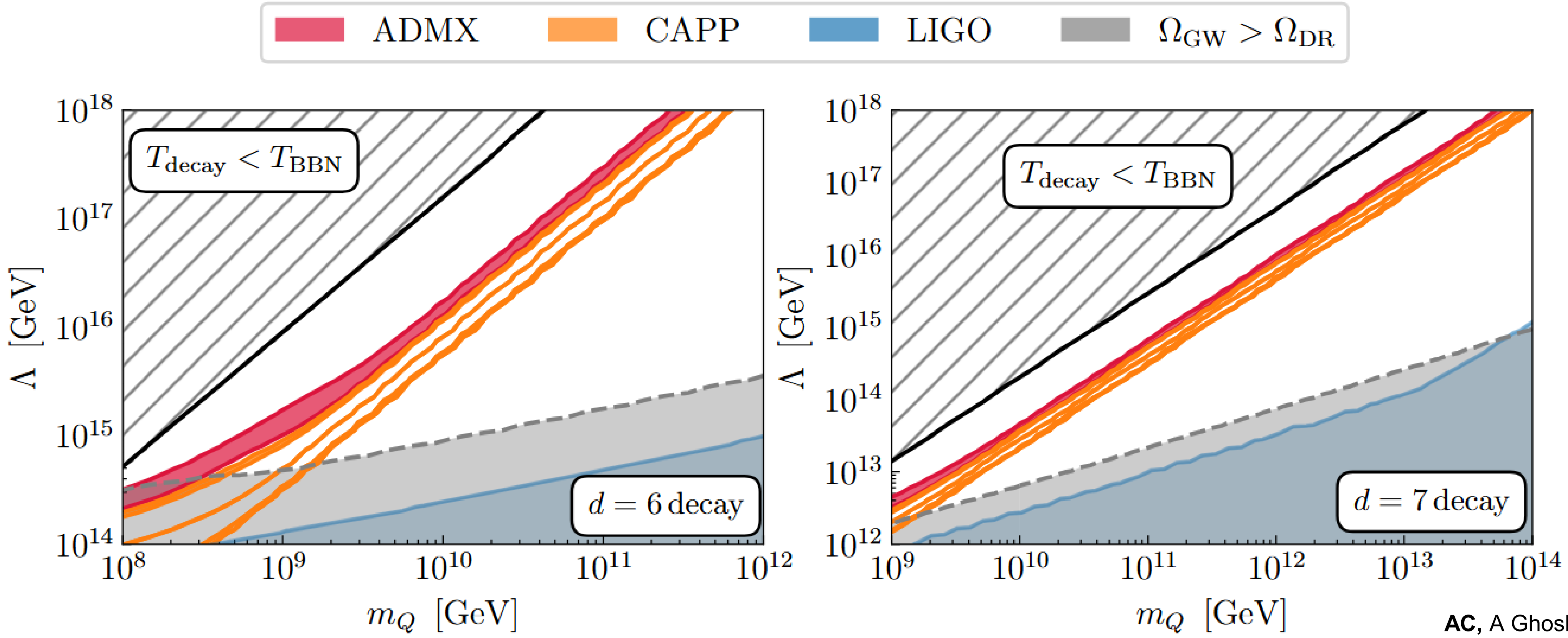
$$\mathrm{SNR} \equiv \sqrt{\tau_{\mathrm{obs}} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left(\frac{\Omega_{\mathrm{GW}}(f, \{\theta\})h^2}{\Omega_{\mathrm{GW}}^{\mathrm{noise}}(f)h^2} \right)^2}$$

Detectors	Frequency range	τ_{obs}
SKA	$[10^{-9} - 4 \times 10^{-7}]$ Hz	15 years
μ-ARES	$[10^{-7} - 1]$ Hz	4 years
LISA	$[10^{-4} - 1]$ Hz	4 years
BBO	$[10^{-3} - 7]$ Hz	4 years
ET	$[1 - 10^{3}]$ Hz	5 years



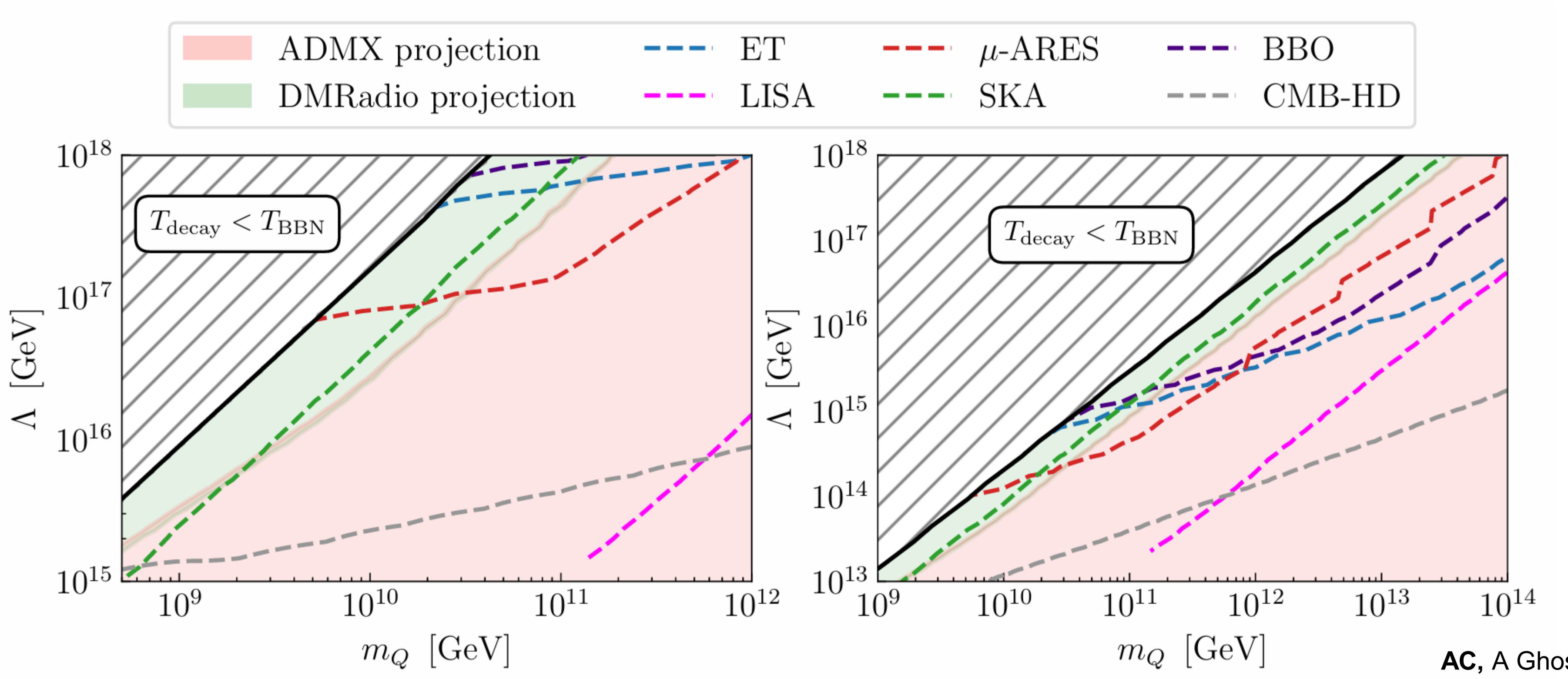
AC, A Ghoshal, D Paul [2505.04614]

Current GW-axion landscape



AC, A Ghoshal, D Paul
[2505.04614]

Future prospects



AC, A Ghoshal, D Paul
[2505.04614]