# Matrix Model of Quantum Black Hole

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# Outline

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# The Proposal: Quantum mechanical model of black hole

Proposal: large $N$ quantum mechanics of 3d quantum space

$$L = \mathrm{tr}\left[\frac{1}{2a_0^2 M_P}\dot{X}^{a2} + \frac{M_P}{N^2}([X^a, X^b]^2 + 4X^{a2}) + i\dot{\psi}^\dagger\psi - a_2\frac{M_P}{N^2}\psi^\dagger\sigma^a X^a\psi\right]$$

where $a = 1, 2, 3$, $X^a_{mn}, \psi_{mn}, \psi^\dagger_{mn}$ $N \times N$ traceless matrices.

1. $M_P$ mass scale, $a_0, a_2$ numerical coefficients.
2. $M_P$ will be fixed in terms of $G$, $a$'s will be partially fixed by matching physics.
3. Compare to SUSY BFSS: we have negative mass term and a vector fermion.

# Quantum physics of horizon from an unstable mode

1. The negative mass term implies a tachyonic mode at the quadratic level. However, it condense classically. Quantum mechanically, it is stabilized by 1-loop bosonic effects.
2. The unstable mode captures interesting physics associated with the infinite red shift of horizon:
   a. Decay: It leads to a "logarithmic" $\log N$ barrier potential between fuzzy spheres, and reproduces the Page result for black hole decay rate from semiclassical Hawking radiation.
   b. Thermality: It leads to a thermal behaviour of the quantum fuzzy sphere: two point functions that resembles a thermalized one with a tempeature $T \sim T_H$ (with Amigo Ho, to appear)
   c. Membrane: It leads to a "locking mechanism" which explains the EM membrane paradigm (to appear)

# Quantum Schwarzschild as Fuzzy Sphere

- EOM

$$-\frac{1}{M_0}\ddot{X}^a + \frac{4M_P}{N^2}\left([X^b,[X^a,X^b]] + 2X^a\right) + \frac{M_P}{N^2}\psi^\dagger\sigma^a\psi = 0.$$

  admits bosonic ($\psi = 0$) static solution given by $N \times N$ matrices obeying

$$[X^a, X^b] = i\epsilon^{abc}X^c, \quad \sum_a X^{a2} = \frac{N^2-1}{4}\mathbf{1},$$

  These are simply spin $J = (N-1)/2$ reps. of $SU(2)$. It describe a fuzzy sphere.

- Introduce dimensional coordinates $Y^a = 2l_P X^a$, the fuzzy sphere solution becomes

$$[Y^a, Y^b] = \frac{2iR}{\sqrt{N^2-1}}\epsilon_{abc}Y^c, \quad \sum_a Y^{a2} = R^2\mathbf{1},$$

  where $R = Nl_P$ is the radius of the fuzzy sphere for $N$ large.

# Fermionic states

- The fermionic states of the model can be obtained by quantizing

$$H_F = a_2 \frac{M_P}{N^2} \psi^\dagger \sigma^a X^a \psi$$

over the fuzzy sphere bkgd.

- In diagonalized form, we get from $\psi$ $2N^2$ oscillators

$$\psi_{mp\alpha} \rightarrow \begin{pmatrix} u_m^{(p)} \\ d_m^{(p)\dagger} \end{pmatrix} \qquad m = 1, \cdots N, \quad p = 1, \cdots, N_f = N, \quad \alpha = 1, 2.$$

and

$$H_F = \frac{a_2 M_P}{2N} \sum_{p,m=1}^{N} (u_m^{p\dagger} u_m^p + d_m^{p\dagger} d_m^p) = \frac{a_2 M_P}{2N}(r+s),$$

with

$$r + s = 0, \cdots, 2N^2,$$

- For a half filled Fermi sea ($r+s=N^2$), energy of the fermi system is

$$H_F = \frac{a_2 N M_P}{2}.$$

- The total energy of the fuzzy sphere system reproduces the Schwarzschild mass-radius relation

$$E = \frac{R}{2G}$$

if the mass scale is identified with the Newton constant

$$M_P \simeq \sqrt{\frac{2\hbar}{\pi G}}$$

# Microstate counting

- Let us consider the microstates counting. Note that $\psi \to e^{i\alpha}\psi$ is a global symmetry. This implies that $r - s$ is a conserved quantity.
- In particular, the ensembles of state with $r = s = N^2/2$ has a degeneracy of

$$\Omega = \binom{N^2}{N^2/2}\binom{N^2}{N^2/2} = 2^{2N^2}$$

in the leading large $N$ limit.
- These microstates give rises to the entropy $S = \log_2 \Omega_0$:

$$S = 2N^2 = \frac{A}{4G},$$

i.e. precisely the Bekenstein-Hawking entropy if the Planck length is given by $G$ as

$$l_P = \sqrt{\frac{2G}{\pi}}.$$

# Rotating black hole

- A similar analysis can be performed for the rotating Kerr BH
- Viewed in the Schwarzschild Cartesian coordinates, the horizon is an elliptical surface

$$\frac{x^2+y^2}{r_+^2+a^2}+\frac{z^2}{r_+^2}=1.$$

with $r_+ = M + \sqrt{M^2 - a^2}$.

- Bekenstein-Hawking entropy, mass and angular momentum

$$S=\frac{A}{4G}=\frac{\pi(r_+^2+a^2)}{G}, \quad M=\frac{r_+^2+a^2}{2Gr_+}, \quad J=a\frac{r_+^2+a^2}{2Gr_+}.$$

- These are reproduced by the rotating fuzzy sphere solution of the QM ! (Chu 2024)
- In particular, matching of the angular momentum requires

$$a_0=\frac{\pi}{3\hbar}.$$

This provides a nontrival check of our model: reproduction of Hawking decay rate of BH

# Outline

- Semi-classical QFT computation of Hawking radiation gives the decay rate of BH

$$\Gamma = -\frac{1}{M}\frac{dM}{dt} = -\frac{Q}{G^2 M^3}.$$

$Q = \hbar c^6/(15360\pi)$ for pure photon emission.

- The $1/M^3$ dependence is universal for a semi-classical black hole and is independent of the type of quantum field being considered.
- It gives also the Hawking temperature

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

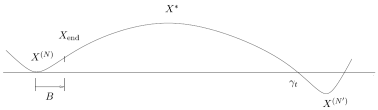
Results: Our model of BH provides a natural explanation of this process in terms of QM tunneling of fuzzy spheres.

# Monopole channel and barrier potential

- We have a collection of fuzzy sphere vacua

$$E = \frac{bNM_P}{2}$$

of different sizes (rank $N$). Decay by tunneling from $N$ to a small $N'$ is possible.




- However since there is an excess of Fermi states in the Fermi sea of $X^{(N)}$, it cannot decay to the small fuzzy sphere with a smaller Fermi sea unless the decay path has the right number of fermi zero modes to soak up the excess fermi states.

- Tunneling rate is extracted from the rentntion amplitude $\langle X_f, \psi_f | e^{-HT} | X_f, \psi_f \rangle$

$$= \int_{X(-T/2)=X_f}^{X(T/2)=X_f} [DX]\, e^{-S_{\rm bos}[X]} \int_{\psi(-T/2)=\psi_f}^{\psi(T/2)=\psi_f} [D\psi]\, e^{-\int \mathrm{tr}\psi^\dagger(\partial_\tau - c\sigma\cdot X)\psi}$$

- Saddle point approx over the bounce $X = X_b(\tau)$ gives

$$\Gamma = B\times F, \quad B = Ke^{-S_b/\hbar}\times F, \quad F = \int [D\psi D\psi^\dagger]\, e^{-\int \mathrm{tr}\psi^\dagger(\partial_\tau - c\sigma\cdot X_b)\psi}$$

- In general, the bounce admits femionic zero modes

$$(\partial_\tau - c\sigma\cdot X_b(\tau))\psi_0 = 0, \quad \text{BC:} \quad \dot{\psi}_0(0) = 0.$$

- As a result, $F$ factorizes to $F = F_0 \times F_1^2$,

$$F_0 = \int [D\psi_0 D\psi_0^\dagger], \quad \text{zero mode contribution}$$

$$F_1 = \langle \psi_t | e^{-iHt} | \psi_f \rangle, \quad |\psi_f = N^2\rangle, \quad |\psi_t = N'^2\rangle$$

- Denote $N' = N - n$, need $N^2 - N'^2 = 2Nn$ zero modes!

- Let us start with the natural (interpolating) path

$$X_a = X_a^{(N)} - \gamma(X_a^{(N)} - X_a^{(N')}), \quad 0 \le \gamma \le 1.$$

- Turns out the difference of two fuzzy spheres

$$A_a^{(k)} := X_a^{(N)} - X_a^{(N-n)},$$

  describes a Dirac monopole on $S_N^2$ with charge $g = n/2$
- We shows a version of index theorem for this monopole: the fuzzy monopole admits precisely $2nN$ zero modes for a decay $X^{(N)} \to X^{(N')}$.

Thus zero mode counting non-trivally select the path!

# Barrier potential

- Using the monopole path, we obtain the barrier potential

$$V(\gamma) = -\frac{2g^2 M_P}{N^2}\mathrm{tr}A^2\,\gamma^2 - \frac{g^3 M_P}{4N^2}\gamma^3 + \frac{g^4 M_P}{96N^3}\gamma^4, \quad -1 \le \gamma \le 1.$$

- It is nontrivial that $\mathrm{tr}A^2 = \frac{N\log N}{4} + O(1)$ has a non-analytic log-dependence instead of a poly. dep. in $N$.
  As a result, we obtain the bounce action

$$S_b = 2\int_{-1}^{\gamma_t} d\gamma\sqrt{2m(V(\gamma) - V(-1))} = \frac{3g^2\hbar}{2}\log N$$

- Computing the determinant $K$, we obtain

$$\Gamma \sim \frac{1}{N^3} \sim \frac{\hbar}{G^2 M^3}$$

- A nontrivial check of the model: $a_0 = \frac{\pi}{3\hbar}$ confirmed in two independent cal (Kerr BH and Hawking decay rate).

Hawking decay process of BH = QM tunneling of quantum space!

# Hawking temperature

- We have show also that a thermal-like probabilistic distribution arises in our fully quantum mechanical system of a large $N$ system!
- The Boltzman probabiliy distribution $e^{-}\omega/T$ is characterized by a "temperature"

$$T = T_H = \frac{1}{4\pi R}$$

  up to a numerical coefficient.
- However, in our description, Hawking radiation is not a thermal state. With a real time formulation of the tunneling process, one can go beyond the probabilistic description by determining the full wave function of the multi-patite Hawking radiation and there is no room for information loss.

# Outline

# Classical EM membrane

- BH membrane paradigm (Damour, Throne, Price, Macdonald (1982)) horizon properties of BH is reproduced with a fictitious membrane at the stretched horizon with certain EM (electrogmagnetic) or EnM (energy momentum) properties
- EM membrane: BH horizon is a Ohmic conductor with a surface conductivity of $\sigma_0 = 1/4\pi$

$$j_a = \frac{1}{4\pi} E_a, \quad a = \text{tangential direction}$$

- This is simply the statement of ingoing BC at the horizon

$$(\partial_t - \partial_{r_*}) A_a = 0, \quad A_a = A_a(t + r_*)$$

- Note that the current is ficitious and carries no independent physical information.
- The situation is drastically different for our quantum mechanical model of black hole.

# Quantum BH membrane paradigm

- In our model, a quantum black hole is made up of a fuzzy sphere together with a sea of fermi degrees of freedom residing on the horizon, which accounts microscopically for the BH entropy.
- The partons $u, d$ are $+/$- charged and dynamical.
- Moreover they see a topological Berry monopole $B$-field background. One way to see this is to note that a vector fermion $\psi_m$ is not globally defined on $S^2$, but is a section of a monopole bundle of $c_1 = N - 1$.
- In fact, in the north patch,

$$\psi(\Omega) := \langle \Omega | \psi \rangle = \frac{P_{N-1}(z)}{(1+|z|^2)^{(N-1)/2}}, \quad P_{N-1} \text{ poly of degree} \leq N-1$$

- These are holomorphic sections $D_{\bar{z}}\psi = 0$ with

$$A_{\bar{z}} = i\frac{N-1}{2}\frac{z}{1+|z|^2}.$$

As a result, the partons are simply LLL in the magnetic field.

# Membrane current in QBH

- As a result, under a worldvolume (wv) electric field on the fuzzy sphere (horizon), the charged partons acquires a chiral Hall response

$$j_a^{(H)} = \sigma_H \epsilon_{ab} E_b, \quad a, b = \theta, \varphi$$

or in freq domain $E_b = E_b(\omega) e^{-i\omega t}$,

$$j_a^{(H)}(\omega) = \sigma_H \epsilon_{ab} E_b(\omega)$$

The Hall conductance $\sigma_H \neq 0$ for a charged BH $Q := q(r - s) \neq 0$.

- In addition, the mass term of matrix model give rises to a London inductance current. In frequency domain,

$$j_a^{(L)}(\omega) = \frac{iD}{\omega} E_a(\omega)$$

- Moreover, taking into account of local tachyonic dissipation effect in our model, we get the Ohm's law (exact $1/4\pi$ by fixing the normalization of the parton charge $q$).

# Quantum membrane current and modified BC

- The quantum current

$$j_a = j_a^{(0)} + j_a^{(L)} + j_a^{(H)}$$

  is physical: follows from the fermionic quantum spacetime; and it has interesting physical consequences (other than repackaging).

- Diagonalizing the Hall term, we get chiral electric response

$$j_a^{(\pm)}(\omega) = \sigma_\pm(\omega) E_a(\omega)$$

$$\sigma_\pm(\omega) = \sigma_0 + \frac{iD}{\omega} + \pm i\sigma_H$$

# Reflective horizon

- One can show that the modified conductance leads to reflective BC at the horizon

$$\partial_u A_\pm = R_\pm \partial_v A_\pm , \quad \partial_u = \partial_t - \partial_{r^*} ,$$

with the reflective coefficient

$$R_\pm(\omega) = \frac{-iD_\pm/2}{\omega + iD_\pm/2}, \quad D_\pm := D \pm \sigma_H \omega .$$

- As a result, BH is not just not black, we predict that it is also reflective, with a characteristic frequency and chiral response.

# Applications

1. Echo (Cardoso etal, .... ): We provide fundamental template for the reflective coefficient and the echo phenomena.
2. Quasi normal modes: the quantum reflective nature of horizon will leave imprints on the QNMs. recent detection?
3. Systematic and fundamental Black Hole Spectroscopy.

# Outline

Summarizing, we have proposed a concrete model of quantum black hole. It has reproduced macroscopic properties, entropic properties of black hole. Decay process of black hole can be described consistently in terms of tunneling. Moreover a thermal distribution emerges. It also provides quantum degrees of freedom that gives the membrane of membrane paradigm a life!

So far a model producing interesting BH horizon physics.

I. Fundamental issues:

- emergence of gravity? metric? holography?
- other properties of Hawking radiation: Page curve? information?
- black hole singularity?

II. Consequence of the quantum model of black hole horizon:

- possible origin of "thermalization" of BH (to appear)
- possible origin of quantum chaos of BH? (ongoing)
- final state? PBH? remanants?

Time for BH phenomenology!

Thank you for listening!