Preheating & Higher-Dimensional Operators in Higgs-Starobinsky Inflation After ACT/SPT Data

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The 16th Particle Physics Phenomenology Workshop, 15-18, June 2026, NTHU

Inflation and CMB-BAO tension

- Flatness and horizon problems, Large scale structures and, CMB anisotropis
- Starobinsky or R^2 inflation: one of the simplest models
- Pure R^2 inflation may be in tension with ACT/SPT data

Single-field attractor

$$n_{s*} = 1 - \frac{2}{\mathcal{N}_*};\ \text{with}\ \ \mathcal{N}_* \in [50, 60] \to n_s* \in [0.9600, 0.9667]$$

0.9657 ± 0.0040 (Planck)

CMB+BAO tension

$n_s = 0.9752 \pm 0.0030$ P-ACT-LB2 (2503.14452)

$n_s = 0.9728 \pm 0.0027$ CMB-SPA+DESI (2506.20707)

R^2-Higgs inflation with dimesnion-six modification

Action in Jordan frame (TM PRD'25; TM PLB; 2606.11929, H. Gonuguntla, TM, A. Samanta)

$$S_J = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}f(R_J,\Phi) - g_J^{\mu\nu}(\nabla_\mu\Phi)^\dagger\nabla_\nu\Phi - V(\Phi,\Phi^\dagger) - \frac{1}{4}g_J^{\mu\rho}g_J^{\nu\sigma}B_{\mu\nu}B_{\rho\sigma} - \frac{1}{4}g_J^{\mu\rho}g_J^{\nu\sigma}W^i_{\mu\nu}W^i_{\rho\sigma}\right]$$

$$f(R_J,\Phi) = R_J + \frac{\xi_R}{2M_P^2}R_J^2 + \frac{2\xi_H}{M_P^2}|\Phi|^2R_J + \frac{1}{3M_P^4\xi_c}R_J^3$$

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$$V(\Phi,\Phi^\dagger) = \lambda|\Phi|^4$$

Potential in Einstein Frame

***R*^3^ modification**

$$V_E = \frac{1}{\Theta^2}\left[\lambda|\Phi^\dagger\Phi|^2 + \frac{M_{\rm P}^4\xi_c^2}{48}(\xi_R - \tilde{\zeta})^2(\xi_R + 2\tilde{\zeta})\right],$$

$$\text{with } \tilde{\zeta} = \left\{\xi_R^2 + \frac{4}{\xi_c}\left[\Theta - 1 - \frac{2\xi_H(\Phi^\dagger\Phi)}{M_{\rm P}^2}\right]\right\}^{1/2}$$

Dimension-six nonminimal coulipns

$$V_E = e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}}\left[\lambda\,|\Phi|^4 + \frac{M_{\rm P}^4}{4\xi_R}\frac{\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm P}}} - 1 - 2\xi_H\frac{|\Phi|^2}{M_{\rm P}^2} - \frac{|\Phi|^4}{M_{\rm P}^4\xi_2}\right)^2}{\left(1 + \frac{2|\Phi|^2}{M_{\rm P}^2\xi_1\xi_R}\right)}\right]$$

Inflationary dynamics

Background and perturbation

$$\phi^I(x^\mu) = \varphi^I(t) + \delta\phi^I(x^\mu)$$

$$\phi^I \in \{\phi, h, \phi_2, \phi_3, \phi_4\}$$

$$\text{and } \varphi^I(t) = \{\varphi(t), h_0(t)\}$$

Background dynamics

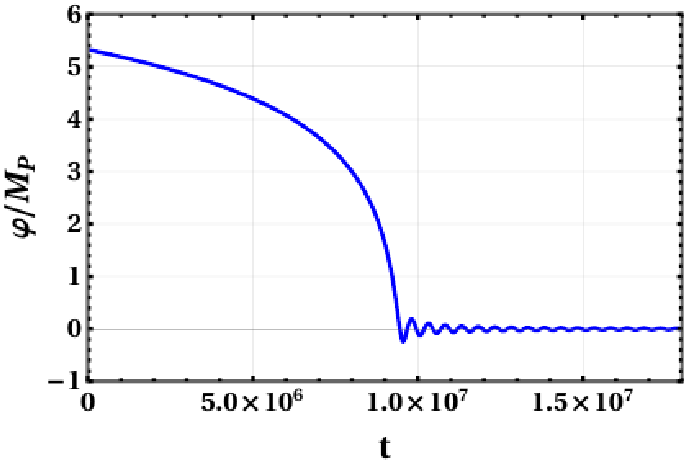
$$\mathcal{D}_t\dot{\varphi}^I + 3H\dot{\varphi}^I + G^{\phi J}V_{E,J}(\varphi^I) = 0$$

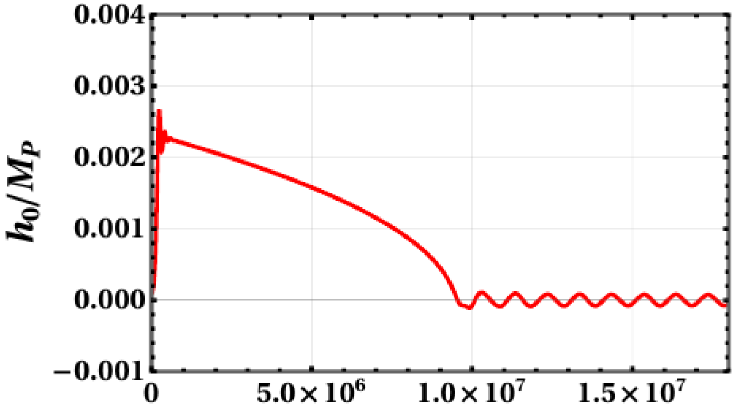
Hubble parameter

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_\mathrm{P}^2}\left(\frac{1}{2}G_{IJ}\dot{\varphi}^I\dot{\varphi}^J + V_0(\varphi^I)\right)$$

Background energy density

$$\rho_{\rm inf} = \frac{1}{2}G_{IJ}\dot{\varphi}^I\dot{\varphi}^J + V_0(\varphi^I)$$





Benchmark parameter values

R^3

BP	ξ_R	ξ_H	ξ_c	$\phi_0(t_{\rm in})$ $[M_{\rm P}]$	$h_0(t_{\rm in})$ $[M_{\rm P}]$
a	2.19×10^9	1.5	1×10^{-14}	5.305	8×10^{-5}
b	2.3×10^9	10	8×10^{-15}	5.35	3×10^{-7}

$\Phi^2 R^2$

BP	ξ_R	ξ_H	ξ_1	$\varphi(t_{\rm in})$ $[M_{\rm P}]$	$h_0(t_{\rm in})$ $[M_{\rm P}]$
a	2.12×10^9	1	2×10^{-9}	5.35	10^{-4}
b	2.32×10^9	10	10^{-8}	5.4	10^{-5}

$\Phi^4 R$

BP	ξ_R	ξ_H	ξ_1	$\varphi(t_{\rm in})$ $[M_{\rm P}]$	$h_0(t_{\rm in})$ $[M_{\rm P}]$
1	2.18×10^9	1.5	5×10^{-6}	5.37	10^{-6}
2	2.17×10^9	10	7×10^{-6}	5.36	10^{-3}

Dynamics of Perturbation

EoMs of gauge invariant perturbation

$$\mathcal{D}_t^2 Q^I + 3H\mathcal{D}_t Q^I - \frac{\partial^2}{a^2} Q^I + \mathcal{M}^I{}_J Q^J + \mathcal{F}_{(\phi_I)} = 0 \qquad \mathcal{F}_{(\phi)} = \mathcal{F}_{(h)} = 0$$

$$\mathcal{M}^I{}_L = G^{IJ}(\mathcal{D}_L \mathcal{D}_J V_E) - \mathcal{R}^I{}_{JKL}\dot{\varphi}^J \dot{\varphi}^K - \frac{1}{M_{\rm P}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\varphi}^I \dot{\varphi}_L\right)$$

Power spectrum

$$\mathcal{P}_{\mathcal{R}}(t;k) = \frac{k^3}{2\pi^2}|\mathcal{R}|^2 \qquad \mathcal{P}_{\mathcal{S}}(t;k) = \frac{k^3}{2\pi^2}|\mathcal{S}|^2$$

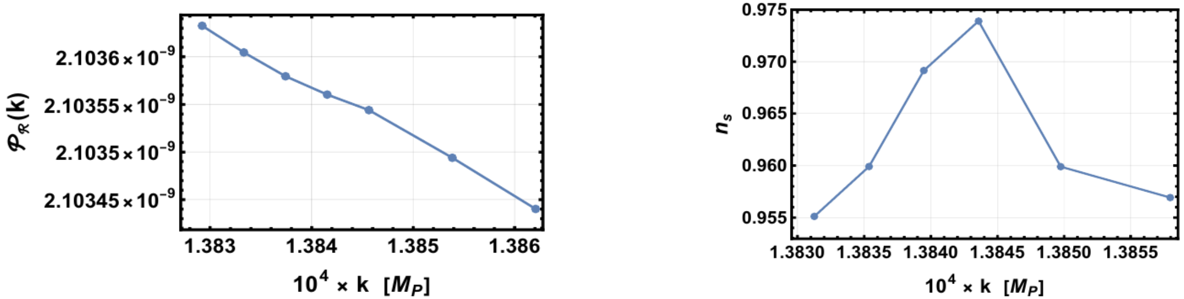
$$Q_\sigma = \hat{\sigma}_I Q^I, \qquad Q_s = \hat{\omega}_I Q^I$$

$$\mathcal{R} = \frac{H}{\dot{\sigma}} Q_\sigma, \qquad \mathcal{S} = \frac{H}{\dot{\sigma}} Q_s \qquad \dot{\sigma} = \sqrt{G_{IJ}\dot{\varphi}^I \dot{\varphi}^J},\ \hat{\sigma}^I = \dot{\varphi}^I/\dot{\sigma}$$

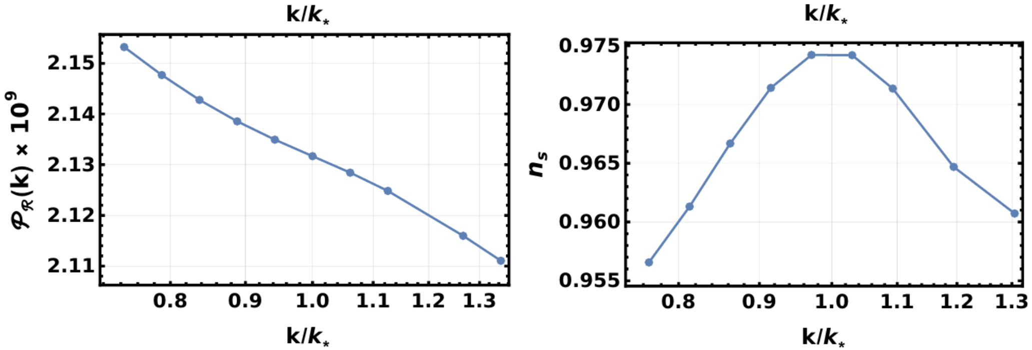
Spectral index

$$n_s = 1 + \frac{d\ln \mathcal{P}_{\mathcal{R}}(k)}{d\ln k}$$

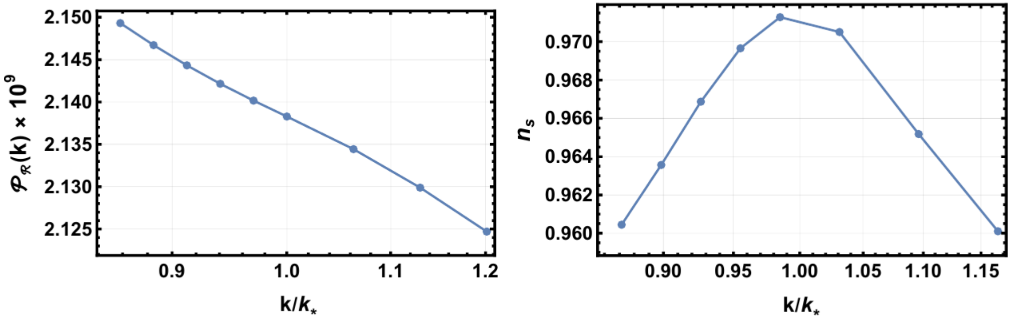
Power spectrum and Spectral index



R^3



$\Phi^2 R^2$

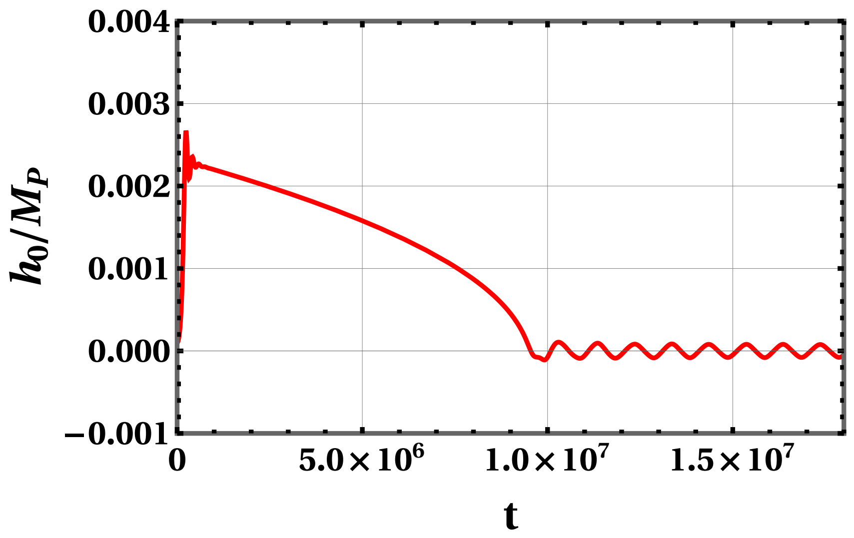


$\Phi^4 R$

Preheating in scalar sector

Preheating in a nutshell

- Time dependent space-time background
- Nonperturbative particle production
- May thermalize Universe



EoMs

$$\mathcal{D}_\tau^2 \widetilde{X}^\phi + \left(k^2 + a^2 m^2_{\mathrm{eff},(\phi)}\right)\widetilde{X}^\phi = 0$$

$$\mathcal{D}_\tau^2 \widetilde{X}^h + \left(k^2 + a^2 m^2_{\mathrm{eff},(h)}\right)\widetilde{X}^h = 0$$

$$\mathcal{D}_\tau^2 \widetilde{X}^{\phi_2} + \left(k^2 + a^2 m^2_{\mathrm{eff},(\phi_2)}\right)\widetilde{X}^{\phi_2} + a\,\frac{g_Z}{2}\left[\left(\sqrt{\frac{2}{3}}\frac{\varphi' h_0}{M_{\mathrm{P}}} - \frac{a'}{a}h_0 - 2h_0'\right)Z_0 + h_0\,\eta^{\alpha\nu}D_\alpha Z_\nu\right] = 0$$

Unitary gauge

$$\text{Unitary gauge: } X^{\phi_2} = 0 \quad \longrightarrow \quad D_\mu Z^\mu = \frac{1}{h_0}\left(\sqrt{\frac{2}{3}}\frac{\varphi' h_0}{M_{\mathrm{P}}} - \frac{a'}{a}h_0 - 2h_0'\right)Z_0$$

Quantization

Decomposition

$$\hat{\widetilde{X}}^{\phi} = \left[\left(v_{1k}(\tau)e_1^{\phi}(\tau)\hat{a}_1(\mathbf{k}) + v_{2k}(\tau)e_2^{\phi}(\tau)\hat{a}_2(\mathbf{k})\right) + \left(v_{1k}^*(\tau)e_1^{\phi}(\tau)\hat{a}_1^{\dagger}(-\mathbf{k}) + v_{2k}^*(\tau)e_2^{\phi}(\tau)\hat{a}_2^{\dagger}(-\mathbf{k})\right)\right],$$

$$\hat{\widetilde{X}}^{h} = \left[\left(y_{1k}(\tau)e_1^{h}(\tau)\hat{a}_1(\mathbf{k}) + y_{2k}(\tau)e_2^{h}(\tau)\hat{a}_2(\mathbf{k})\right) + \left(y_{1k}^*(\tau)e_1^{h}(\tau)\hat{a}_1^{\dagger}(-\mathbf{k}) + y_{2k}^*(\tau)e_2^{h}(\tau)\hat{a}_2^{\dagger}(-\mathbf{k})\right)\right],$$

$$\hat{\widetilde{X}}^{\phi_2} = s_k(\tau)e^{\phi_2}(\tau)\hat{a}(\mathbf{k}) + s_k^*(\tau)e^{\phi_2}(\tau)\hat{a}^{\dagger}(-\mathbf{k})$$

EoMs for modes

$$v_{1k}'' + \omega_{(\phi)}^2\, v_{1k} \simeq 0,$$

$$y_{2k}'' + \omega_{(h)}^2\, y_{2k} \simeq 0,$$

$$s_k'' + \frac{2m_Z^2\Upsilon}{\mathcal{K}_Z}s_k' + \left[k^2 + a^2M_{\phi_2}^2 + \frac{2m_Z^2\Upsilon^2}{\mathcal{K}_Z}\right]s_k = 0$$

$$m_Z^2 = (g_Z^2/4)e^{-\sqrt{\frac{2}{3}}\frac{\phi_0}{m_{\rm P}}}h_0^2,\ M_{\phi_2}^2 = m_{{\rm eff},(\phi_2)}^2 + m_Z^2$$

$$\mathcal{K}_Z = \frac{k^2}{a^2} + m_Z^2, \quad \Upsilon(\tau) = \frac{\phi_0'}{\sqrt{6}M_{\rm P}} - \frac{a'}{a} - \frac{h_0'}{h_0}$$

Energy Densities of Perturbations

Quantum energy density (vacuum subtracted)

$$\rho^{q}_{(I)} = \frac{1}{a^4}\int\left(\frac{d^3k}{(2\pi)^3}\rho_k^{(I)} - \frac{k^3}{4\pi^2\Delta_{(I)}}dk\right)$$

$$\Delta_{(\phi)} = 1,\ \Delta_{(h)} = 1,\ \Delta_{(\phi_2)} = \exp\int_{-\infty}^{\tau}\frac{2m_Z^2\Upsilon}{\mathcal{K}_Z}\,d\tau',$$

Energy density per mode

$$\rho_k^{(\phi)} = \frac{1}{2}G_{\phi\phi}\left(|v'_{1k}|^2 + \omega^2_{(\phi)}|v_{1k}|^2\right)e_1^{\phi}e_1^{\phi} = \frac{1}{2}\left(|v'_{1k}|^2 + \omega^2_{(\phi)}|v_{1k}|^2\right),$$

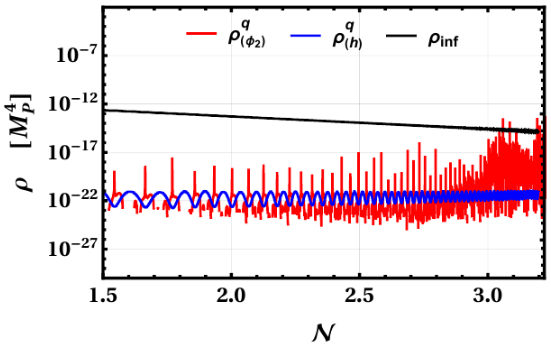
$$\rho_k^{(h)} = \frac{1}{2}G_{hh}\left(|y'_{2k}|^2 + \omega^2_{(h)}|y_{2k}|^2\right)e_2^{h}e_2^{h} = \frac{1}{2}\left(|y'_{2k}|^2 + \omega^2_{(h)}|y_{2k}|^2\right),$$

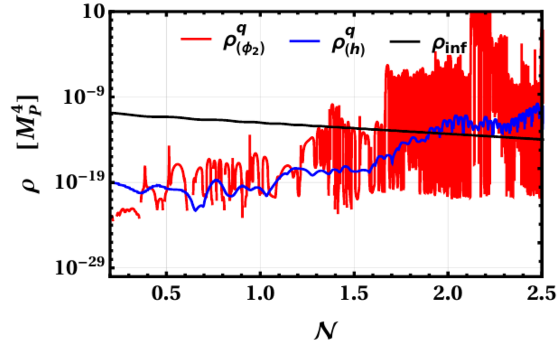
$$\rho_k^{\rm int} = \mathcal{O}(h^2) \sim 0.$$

$$\rho_k^{(\phi_2)} = \frac{1}{2}\left\{\left(1 - \frac{m_Z^2}{\mathcal{K}_Z}\right)|s'_k|^2 + \left[k^2 + a^2 m^2_{\text{eff},(\phi_2)} - \frac{m_Z^2}{\mathcal{K}_Z}\,\Upsilon^2\right]|s_k|^2 - \frac{m_Z^2}{\mathcal{K}_Z}\,\Upsilon\left(s'_k s_k^* + s_k^{*\prime} s_k\right)\right\}.$$

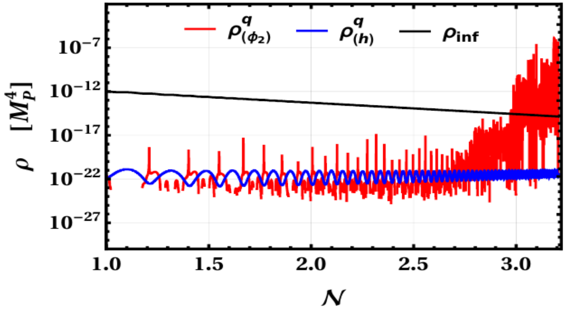
Preheating

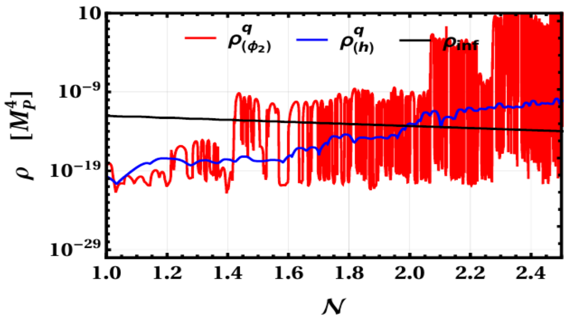
R^3



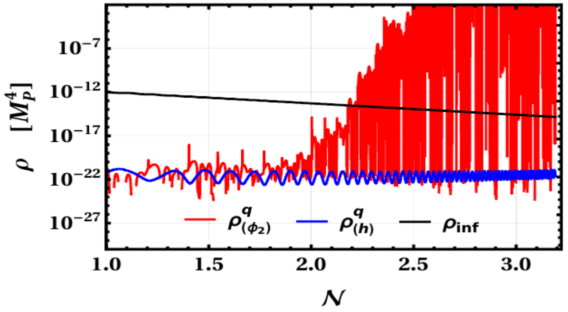


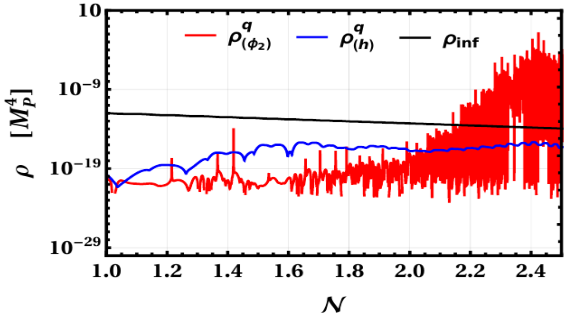
$\Phi^2 R^2$





$\Phi^4 R$





Matching CMB scale

$$k_{\rm ref} = k_* = a(t_*)H(t_*) = \frac{a(t_*)}{a(t_{\rm end})}\frac{a(t_{\rm end})}{a(t_{\rm pre})}\frac{a(t_{\rm pre})}{a_0}a_0 H(t_*)$$

$$\mathcal{N}_* = -\ln\left[\frac{H_*}{k_{\rm ref}/a_0}\frac{T_0}{T_{\rm pre}}\frac{g_0^{1/3}}{g_{\rm pre}^{1/3}}\right] + \mathcal{N}_{\rm pre}$$

$$k_{\rm ref}/a_0 = 0.05\ {\rm Mpc}^{-1}$$

Model dependent parameters

$T_{\rm pre},\ \mathcal{N}_{\rm pre},\ H_*,\ N_*$

$$\rho_{\rm inf}\big|_{\mathcal{N}=\mathcal{N}_{\rm pre}} \equiv \rho_{\rm pre} = \frac{g_{\rm pre}\pi^2}{30}\,T_{\rm pre}^4$$

Matching CMB scale

$$\mathcal{N}_* = -\ln\left[\frac{H_*}{k_{\rm ref}/a_0}\frac{T_0}{T_{\rm pre}}\frac{g_0^{1/3}}{g_{\rm pre}^{1/3}}\right] + \mathcal{N}_{\rm pre}$$

BPs	Operators	$\mathcal{N}_*$	$\mathcal{N}_{\rm pre}$	$\mathcal{T}_{\rm pre}$ [GeV]
BPa	R^3	-53.7	-54.8	2.2×10^{14}
BPb		-54.5	-55.3	7.8×10^{14}
BPa	$\Phi^2 R^2$	-54.94	-54.16	2.3×10^{14}
BPb		-55.31	-56.89	7.5×10^{14}
BP1	$\Phi^4 R$	-55.22	-55	4.2×10^{14}
BP2		-55.07	-55.03	4.4×10^{14}

All good at CMB-front

Parameter	*Planck*	SPT-3G D1	ACT DR6	SPT+ACT	SPT+*Planck*	CMB-SPA
Sampled						
$10^4\theta_s^\star$	104.184 ± 0.029	104.171 ± 0.060	104.157 ± 0.030	104.158 ± 0.025	104.176 ± 0.026	104.162 ± 0.023
$100\,\Omega_b h^2$	2.238 ± 0.014	2.221 ± 0.020	2.257 ± 0.016	2.247 ± 0.013	2.230 ± 0.011	2.2381 ± 0.0093
$100\,\Omega_c h^2$	11.98 ± 0.11	12.14 ± 0.16	12.26 ± 0.17	12.22 ± 0.12	12.050 ± 0.089	12.009 ± 0.086
n_s	0.9657 ± 0.0040	0.951 ± 0.011	0.9682 ± 0.0069	0.9671 ± 0.0058	0.9636 ± 0.0035	0.9684 ± 0.0030
$\log(10^{10} A_s)$	3.042 ± 0.011	3.054 ± 0.015	3.038 ± 0.012	3.042 ± 0.011	3.046 ± 0.010	3.0479 ± 0.0099
τ_{reio}	0.0535 ± 0.0056	0.0506 ± 0.0059	0.0513 ± 0.0060	0.0514 ± 0.0059	0.0538 ± 0.0054	0.0559 ± 0.0055
Derived						
H_0 [km/s/Mpc]	67.41 ± 0.49	66.66 ± 0.60	66.51 ± 0.64	66.59 ± 0.46	67.07 ± 0.38	67.24 ± 0.35
Age [Gyr]	13.797 ± 0.022	13.826 ± 0.027	13.797 ± 0.021	13.805 ± 0.016	13.812 ± 0.017	13.805 ± 0.014
$10^9 A_s e^{-2\tau_{reio}}$	1.883 ± 0.010	1.915 ± 0.021	1.884 ± 0.013	1.889 ± 0.011	1.8890 ± 0.0092	1.8843 ± 0.0060
Ω_Λ	0.6854 ± 0.0067	0.6753 ± 0.0091	0.670 ± 0.010	0.6722 ± 0.0072	0.6810 ± 0.0054	0.6833 ± 0.0051
Ω_m	0.3145 ± 0.0067	0.3246 ± 0.0091	0.330 ± 0.010	0.3277 ± 0.0072	0.3189 ± 0.0054	0.3166 ± 0.0051
r_d [Mpc]	147.13 ± 0.25	146.92 ± 0.47	146.20 ± 0.46	146.43 ± 0.34	147.06 ± 0.23	147.07 ± 0.22
σ_8	0.8099 ± 0.0051	0.8158 ± 0.0058	0.8171 ± 0.0055	0.8169 ± 0.0042	0.8132 ± 0.0042	0.8137 ± 0.0038

(2506.20707)

Summary

- **CMB+BAO data: Many single-field attractor models are disfavored**
- **Starobinsky and single-field regime of Starobinsky-Higgs are in tension**
- **Indication of higher-dimensional terms?**
- **Goldstone and Higgs preheating: help alleviate the tension**

Thank you!!

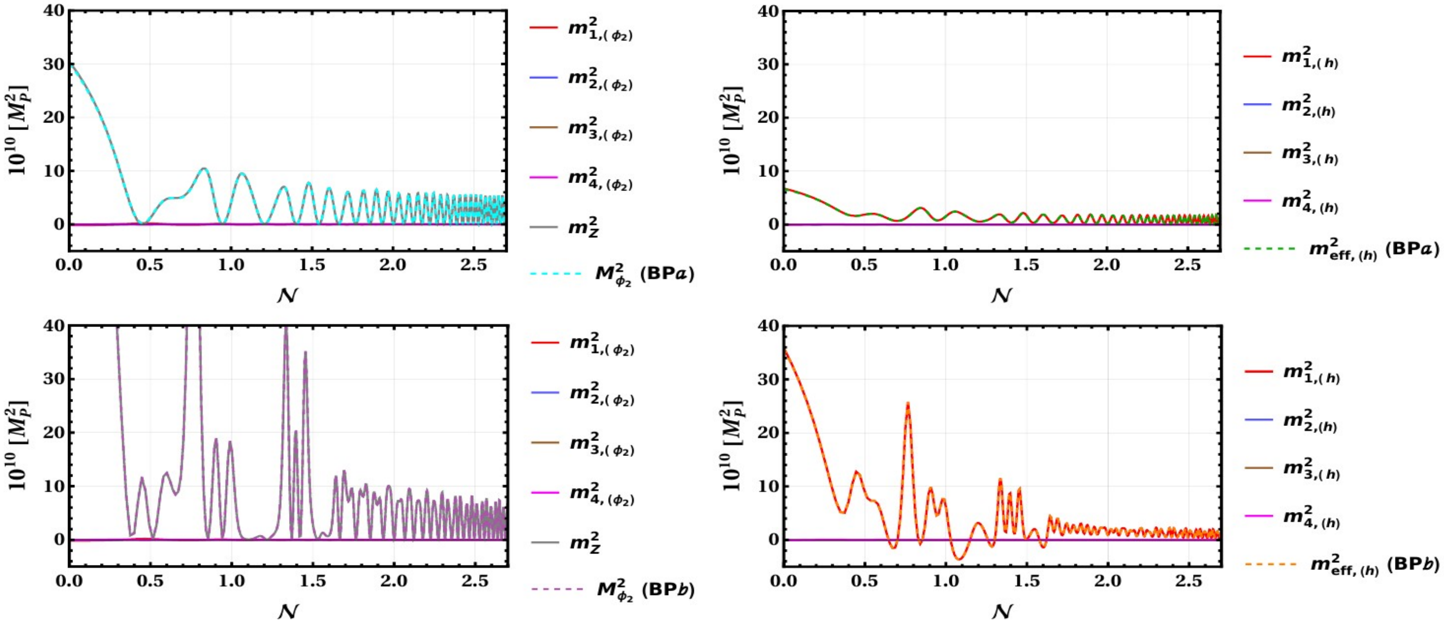
Additional slides

Effective masses

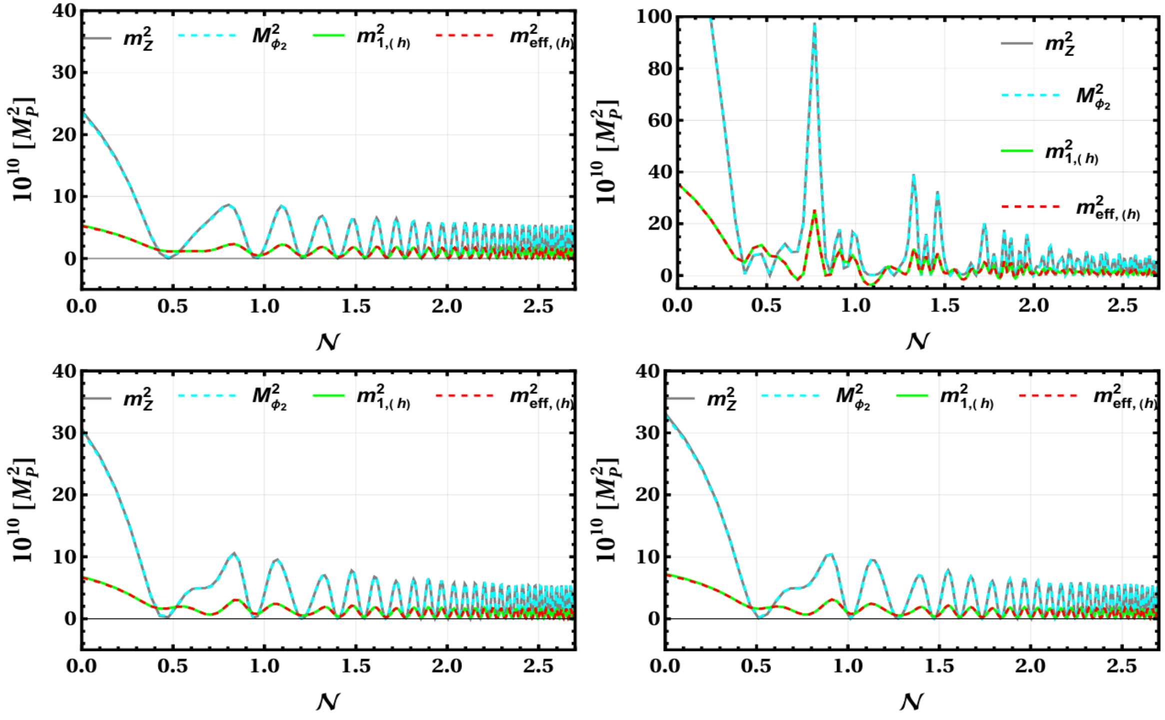
$$m^2_{\mathrm{eff},(I)}(\tau) = \mathcal{M}^I{}_I - \frac{1}{6}R_E G^I{}_I = G^{(I)J}(\mathcal{D}_{(I)}\mathcal{D}_J V_E) - \mathcal{R}^{(I)}{}_{JK(I)}\dot{\varphi}^J\dot{\varphi}^K - \frac{1}{M_{\mathrm{P}}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\varphi}^{(I)}\dot{\varphi}_{(I)}\right) - \frac{R_E}{6}$$

$$M^2_{\phi_2} = m^2_{\mathrm{eff},(\phi_2)} + m^2_Z$$

$$s''_k + \frac{2m_Z^2\Upsilon}{\mathcal{K}_Z}s'_k + \left[k^2 + a^2 M^2_{\phi_2} + \frac{2m_Z^2\Upsilon^2}{\mathcal{K}_Z}\right]s_k = 0 \qquad y''_{2k} + \omega^2_{(h)}\, y_{2k} \simeq 0$$



R^3



$\Phi^2 R^2$

$\Phi^4 R$

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n_s	0.9657 ± 0.0040	0.951 ± 0.011	0.9682 ± 0.0069	0.9671 ± 0.0058	0.9636 ± 0.0035	0.9684 ± 0.0030
$\log(10^{10}\,A_s)$	3.042 ± 0.011	3.054 ± 0.015	3.038 ± 0.012	3.042 ± 0.011	3.046 ± 0.010	3.0479 ± 0.0099
τ_{reio}	0.0535 ± 0.0056	0.0506 ± 0.0059	0.0513 ± 0.0060	0.0514 ± 0.0059	0.0538 ± 0.0054	0.0559 ± 0.0055
Derived						
H_0 [km/s/Mpc]	67.41 ± 0.49	66.66 ± 0.60	66.51 ± 0.64	66.59 ± 0.46	67.07 ± 0.38	67.24 ± 0.35
Age [Gyr]	13.797 ± 0.022	13.826 ± 0.027	13.797 ± 0.021	13.805 ± 0.016	13.812 ± 0.017	13.805 ± 0.014
$10^9\,A_s e^{-2\tau_{reio}}$	1.883 ± 0.010	1.915 ± 0.021	1.884 ± 0.013	1.889 ± 0.011	1.8890 ± 0.0092	1.8843 ± 0.0060
Ω_Λ	0.6854 ± 0.0067	0.6753 ± 0.0091	0.670 ± 0.010	0.6722 ± 0.0072	0.6810 ± 0.0054	0.6833 ± 0.0051
Ω_m	0.3145 ± 0.0067	0.3246 ± 0.0091	0.330 ± 0.010	0.3277 ± 0.0072	0.3189 ± 0.0054	0.3166 ± 0.0051
r_d [Mpc]	147.13 ± 0.25	146.92 ± 0.47	146.20 ± 0.46	146.43 ± 0.34	147.06 ± 0.23	147.07 ± 0.22
σ_8	0.8099 ± 0.0051	0.8158 ± 0.0058	0.8171 ± 0.0055	0.8169 ± 0.0042	0.8132 ± 0.0042	0.8137 ± 0.0038

CMB-SPA+DESI

Parameter	SPT-3G D1 + DESI	CMB-SPA + DESI
Sampled		
$10^4\theta_s^\star$	104.227 ± 0.056	104.180 ± 0.022
$100\,\Omega_b h^2$	2.218 ± 0.022	2.2452 ± 0.0089
$100\,\Omega_c h^2$	11.749 ± 0.079	11.813 ± 0.058
n_s	0.949 ± 0.012	0.9728 ± 0.0027
$\log(10^{10}\,A_s)$	3.066 ± 0.014	3.0574 ± 0.0094
τ_{reio}	0.0559 ± 0.0056	0.0625 ± 0.0050

Inflationary dynamics

Background and perturbation

$$\phi^I(x^\mu) = \varphi^I(t) + \delta\phi^I(x^\mu)$$
$$\text{and } \varphi^I(t) = \{\varphi(t), h_0(t)\}$$

Background dynamics

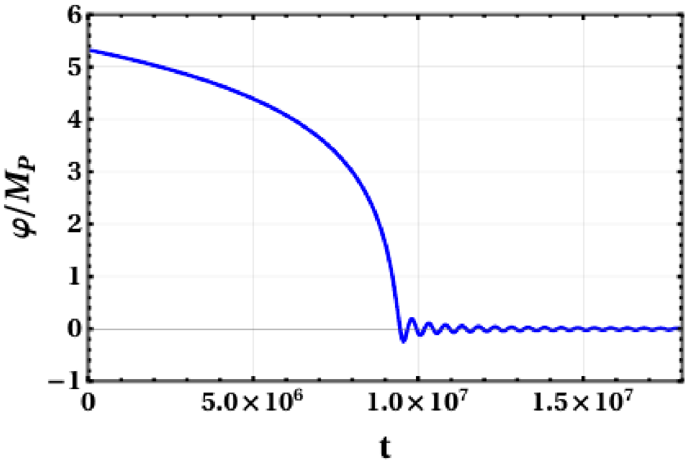
$$\mathcal{D}_t\dot{\varphi} + 3H\dot{\varphi} + G^{\phi J}V_{0,J} = 0,$$
$$\mathcal{D}_t\dot{h}_0 + 3H\dot{h}_0 + G^{hJ}V_{0,J} = 0,$$

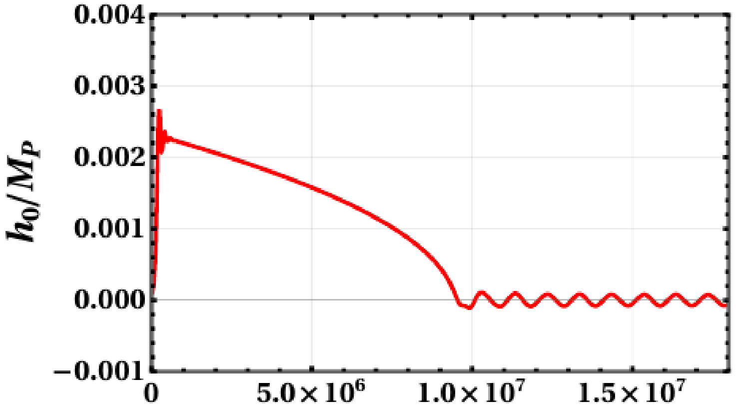
Hubble parameter

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\rm P}^2}\left(\frac{1}{2}G_{IJ}\dot{\varphi}^I\dot{\varphi}^J + V_0(\varphi^I)\right)$$

Background energy density

$$\rho_{\rm inf} = \frac{1}{2}G_{IJ}\dot{\varphi}^I\dot{\varphi}^J + V_0(\varphi^I)$$





Choice of vacuum

Hamiltonian

$$\langle \hat{\mathcal{H}} \rangle = \int \frac{d^3k}{(2\pi)^3} \left[\rho_k^{(\phi)} \delta^3(\mathbf{0}) + \rho_k^{(h)} \delta^3(\mathbf{0}) \right]$$

Bunch-Davies (BD) vacuum (at early time)

$$v_{1k} = c_1^{\phi} e^{-ik\tau} + c_2^{\phi} e^{ik\tau}, \quad y_{2k} = c_1^{h} e^{-ik\tau} + c_2^{h} e^{ik\tau}$$

Minimization of Hamiltonian

$$c_1^{\phi} = \frac{1}{\sqrt{2k}}, \qquad c_2^{\phi} = 0, \qquad c_1^{h} = \frac{1}{\sqrt{2k}}, \qquad c_2^{h} = 0.$$

$$v_{1k}^{\mathrm{BD}} = \frac{1}{\sqrt{2k}} e^{-ik\tau}, \qquad y_{2k}^{\mathrm{BD}} = \frac{1}{\sqrt{2k}} e^{-ik\tau}$$