Number Theory and Minicharged Particles

June 17, 2026
The 16th Particle Physics Penomenology Workshop@NTHU

Junseok Lee
Particle Theory and Cosmology Group, Tohoku U.

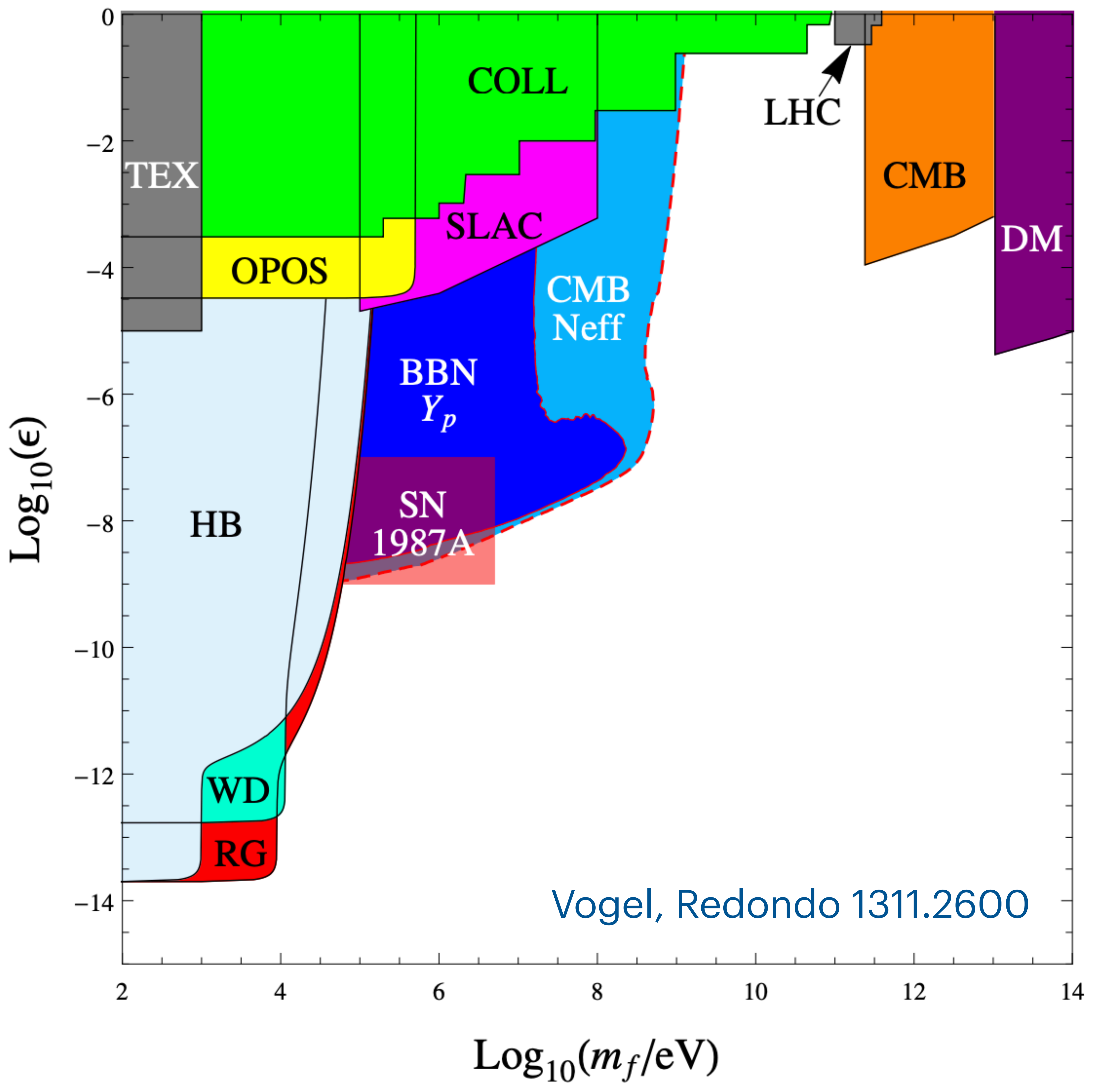
Based on **arXiv: 2603.12320**
collaboration with **Fuminobu Takahashi (Tohoku U.), and Yu-Dai Tsai (U. Manchester)**

Minicharged particle

Minicharged particle (mCP) is a particle Ψ with **electric charge** $Q_{\rm eff} \ll e$ and mass m_Ψ.

Small charge makes them weakly interacting but not completely invisible.

mCPs are testable by stellar cooling, laboratory searches, cosmological observations, …



We should ask the following question:

If Ψ is light enough to be probed by accelerators, colliders, stellar cooling, …, what keeps m_Ψ light?

What we will see in this talk:

Chirality protects light masses. Anomaly cancellation corresponds to the Prouhet-Tarry-Escott problem, a problem in number theory.

Extra $U(1)$ gives minicharges

Standard Model —— $U(1)_{\mathrm{H}}$ + fermion Ψ

$$\chi B_{\mu\nu} F'^{\mu\nu}$$

For an unbroken hidden $U(1)_{\mathrm{H}}$, one can change the basis to remove kinetic mixing term:

$$A'_\mu \to A'_\mu - \chi B_\mu \,.$$

A hidden fermion Ψ with

$$\mathscr{L}_{\mathrm{int}} \supset -g_{\mathrm{H}} q_{\mathrm{H}} A'_\mu \bar{\Psi} \gamma^\mu \Psi$$

obtains an effective electromagnetic charge,

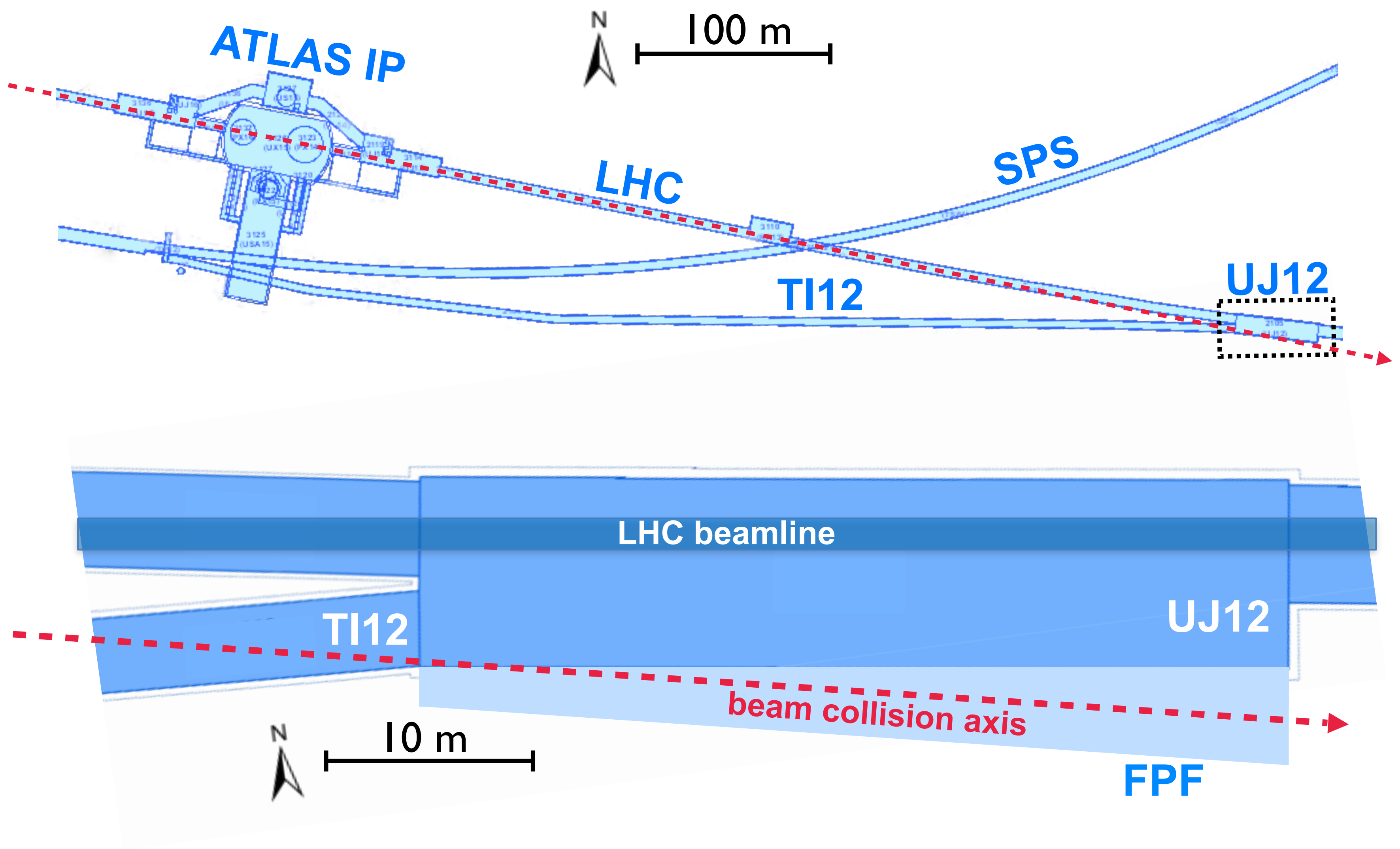
$$Q_{\mathrm{eff}} \simeq -\chi g_{\mathrm{H}} q_{\mathrm{H}} \cos\theta_W \,.$$

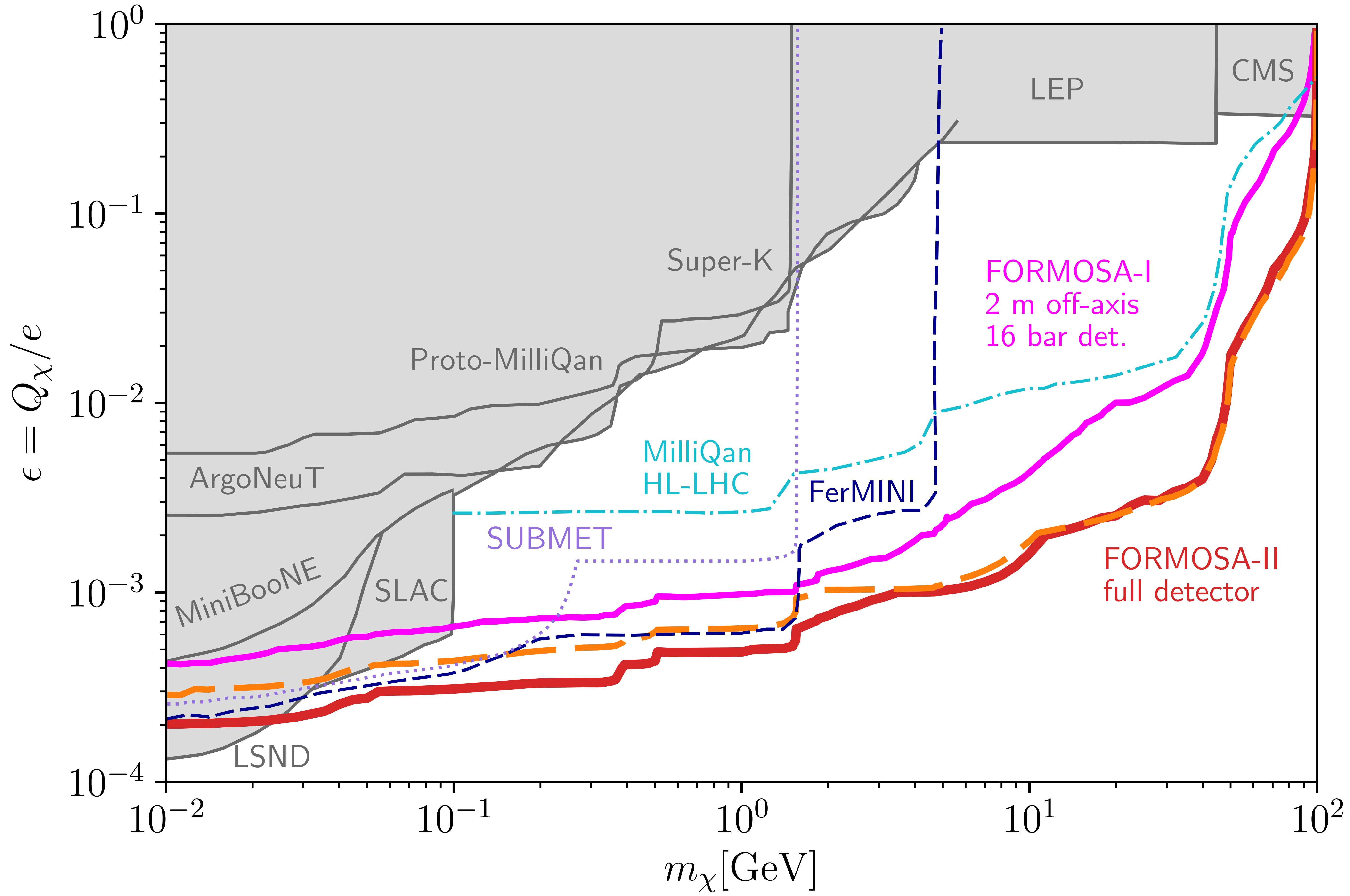
Minicharged Particle (mCP)

FORMOSA as an example

Foroughi-Abari, Kling, Tsai 2010.07941

Future scintillator-based experiments to be located in the far-forward region at the LHC.





Why is mCP light?

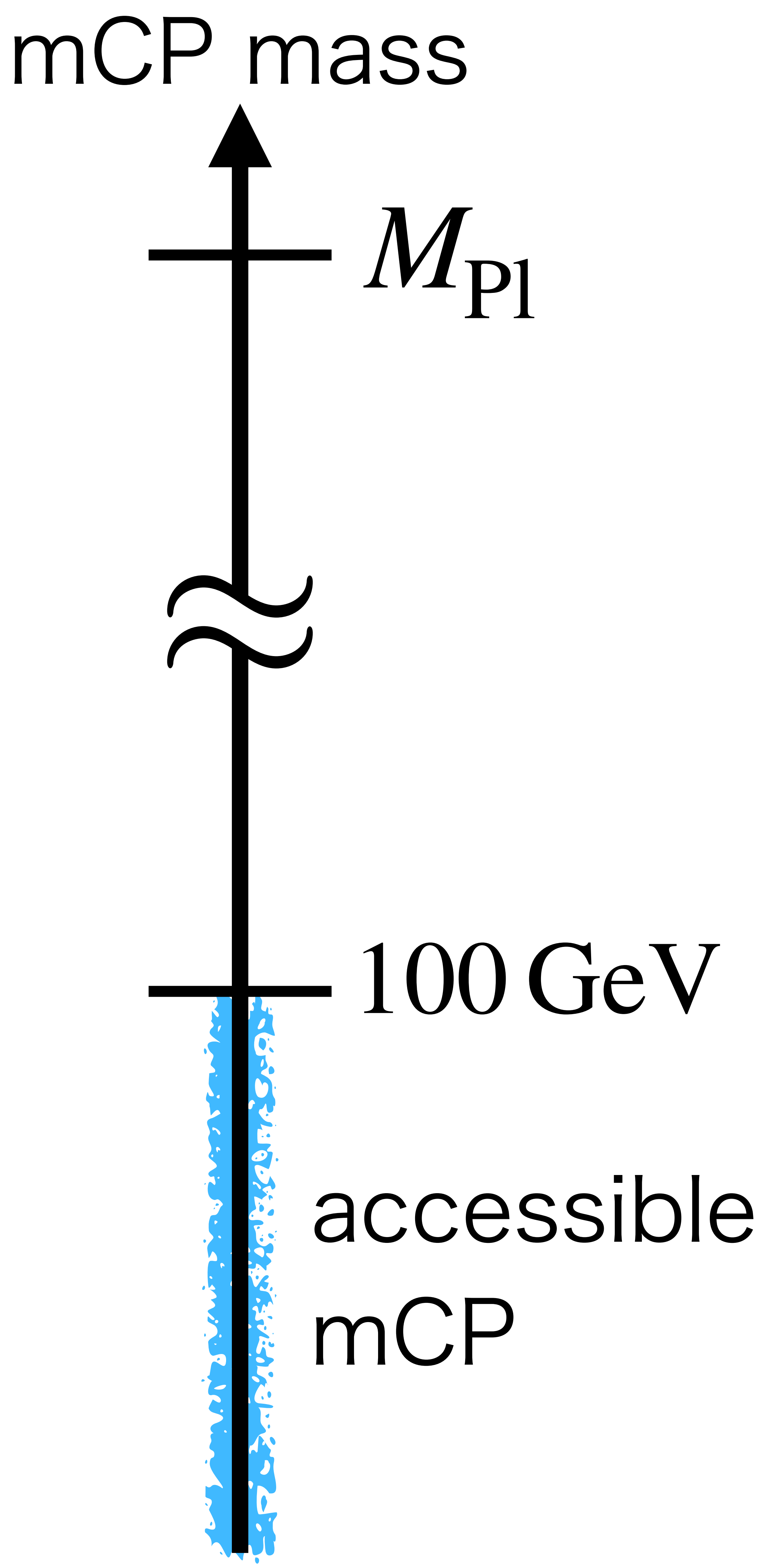
In the minimal setup, a Dirac mass is allowed:

$$-m_\Psi \bar{\Psi}\Psi\,.$$

If the cutoff is M_c, effective field theory suggests

$$m_\Psi \sim M_c\,,$$

unless the small mass is technically natural.



Strategy: symmetry protects small mass

Introduce a chiral gauge symmetry $U(1)_\mathrm{X}$, and forbid a mass term.

$$U(1)_\mathrm{H} \times U(1)_\mathrm{X}$$

$$\cancel{\psi_1\psi_2 + \mathrm{h.c.}}$$

		ψ_1	ψ_2
Vector-like	$U(1)_\mathrm{H}$	$+1$	-1
Chiral	$U(1)_\mathrm{X}$	a	$-b$

$$a \neq b$$

Strategy: symmetry protects small mass

Introduce a chiral gauge symmetry $U(1)_{\mathrm{X}}$, and forbid a mass term.

When $U(1)_{\mathrm{X}}$ is broken by ϕ, a small fermion mass is allowed.

$$U(1)_{\mathrm{H}} \times U(1)_{\mathrm{X}}$$

$$\cancel{\psi_1 \psi_2 + \mathrm{h.c.}}$$

$$y M_c \left(\frac{\phi^{(*)}}{M_c} \right)^{|a-b|} \psi_1 \psi_2 + \mathrm{h.c.}$$

($y = \mathcal{O}(1)$ complex coefficient)

		ψ_1	ψ_2	ϕ (Dark Higgs)
Vector-like	$U(1)_{\mathrm{H}}$	$+1$	-1	0
Chiral	$U(1)_{\mathrm{X}}$	a	$-b$	1

$$a \neq b$$

Anomaly cancellation

In general, we may introduce $2n$ Weyl fermions ψ_i .

$U(1)_\mathrm{X} - U(1)_\mathrm{X} - U(1)_\mathrm{X}$ anomaly

$$\sum_i a_i^3 = \sum_j b_j^3$$

$U(1)_\mathrm{X} - U(1)_\mathrm{X} - U(1)_\mathrm{H}$ anomaly

$$\sum_i a_i^2 = \sum_j b_j^2$$

$U(1)_\mathrm{X}$ – graviton – graviton anomaly

$$\sum_i a_i = \sum_j b_j$$

	ψ_i	ψ_j	ϕ
$U(1)_\mathrm{H}$	$+1$	-1	0
$U(1)_\mathrm{X}$	a_i	$-b_j$	1

$$i = 1, 2, \cdots, n$$
$$j = n+1, n+2, \cdots, 2n$$

$$a_i \neq b_j$$

Euler, Goldbach (1750), Prouhet (1851), Escott (1910), Tarry (1895),
Review: Borwein (Springer, `02)

Prouhet-Tarry-Escott problem

degree k and size n PTE problem:

Given a positive integer k, find two distinct sets of integer solutions $A = \{a_1, a_2, \cdots, a_n\}$ and $B = \{b_1, b_2, \cdots, b_n\}$ that satisfy

$$\sum_{i=1}^{n} a_i^{\ell} = \sum_{i=1}^{n} b_i^{\ell} \qquad \text{for } \ell = 1, 2, \cdots, k\,.$$

The smallest possible size is $n = k + 1$.

This can be proved using Newton's identities!

Such solutions are called **ideal solutions**.

Prediction I: at least four mCPs

	ψ_i	ψ_j	ϕ
$U(1)_{\mathrm{H}}$	$+1$	-1	0
$U(1)_{\mathrm{X}}$	a_i	$-b_j$	1

$i = 1, 2, 3, 4$

$j = 5, 6, 7, 8$

$a_i \neq b_j$

$\epsilon = \dfrac{\langle \phi \rangle}{M_c}$

$$\mathscr{L} \supset y_{ij} M_c \left(\frac{\phi^{(*)}}{M_c} \right)^{|a_i - b_j|} \psi_i \psi_j + \mathrm{h.c.}$$

($y_{ij} = \mathcal{O}(1)$ complex coefficients)

$\Rightarrow$

4×4 mass matrix

$$\boxed{M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}}$$

In this talk, we focus on the ideal case.

Four mCPs

More about PTE problem

There are solutions with good properties: **symmetric solutions**.

The solution is called symmetric when it remains unchanged under the sign flip, $\{a_i\} = \{-a_i\}$ and $\{b_i\} = \{-b_i\}$.

e.g.) $$A = \{-8, -1, 1, 8\}, \quad B = \{-7, -4, 4, 7\}$$

Up to a common shift, **many ideal solutions are symmetric**.

Symmetric solutions give paired exponents

For $a_i \in \{-a_1, a_1, -a_2, a_2\}$ and $b_i \in \{-b_1, b_1, -b_2, b_2\}$, the exponent in the mass eigenvalues appear in pairs.

$$\boxed{M_{ij} \simeq M_c \epsilon^{|a_i - b_j|}}$$

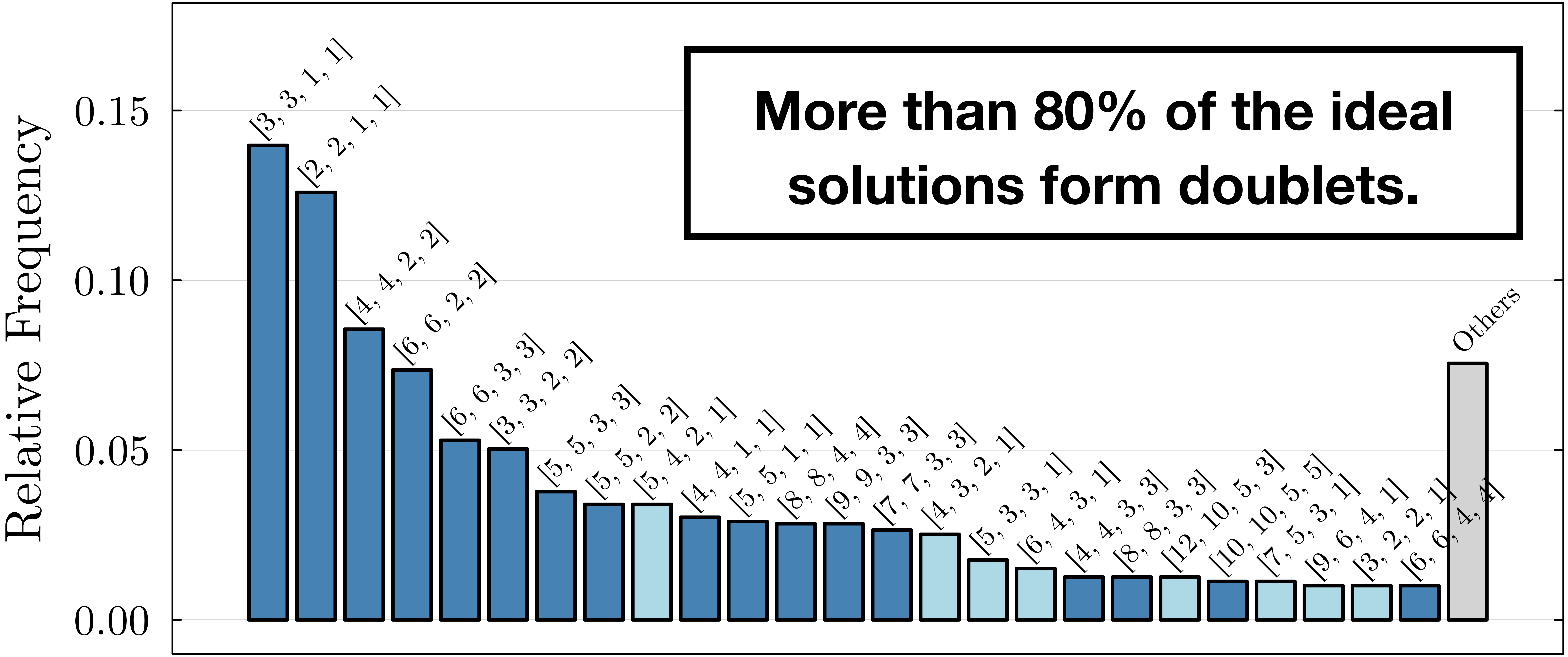
	$-a_1$	a_1	$-a_2$	a_2
$-b_1$	$\lvert a_1 - b_1 \rvert$	$\lvert a_1 + b_1 \rvert$	higher power	
b_1	$\lvert a_1 + b_1 \rvert$	$\lvert a_1 - b_1 \rvert$		
$-b_2$	higher power		$\lvert a_2 - b_2 \rvert$	$\lvert a_2 + b_2 \rvert$
b_2			$\lvert a_2 + b_2 \rvert$	$\lvert a_2 - b_2 \rvert$

Mathematical proof can be found in JL, Takahashi, Tsai 2603.12320

Prediction II: doublet structure

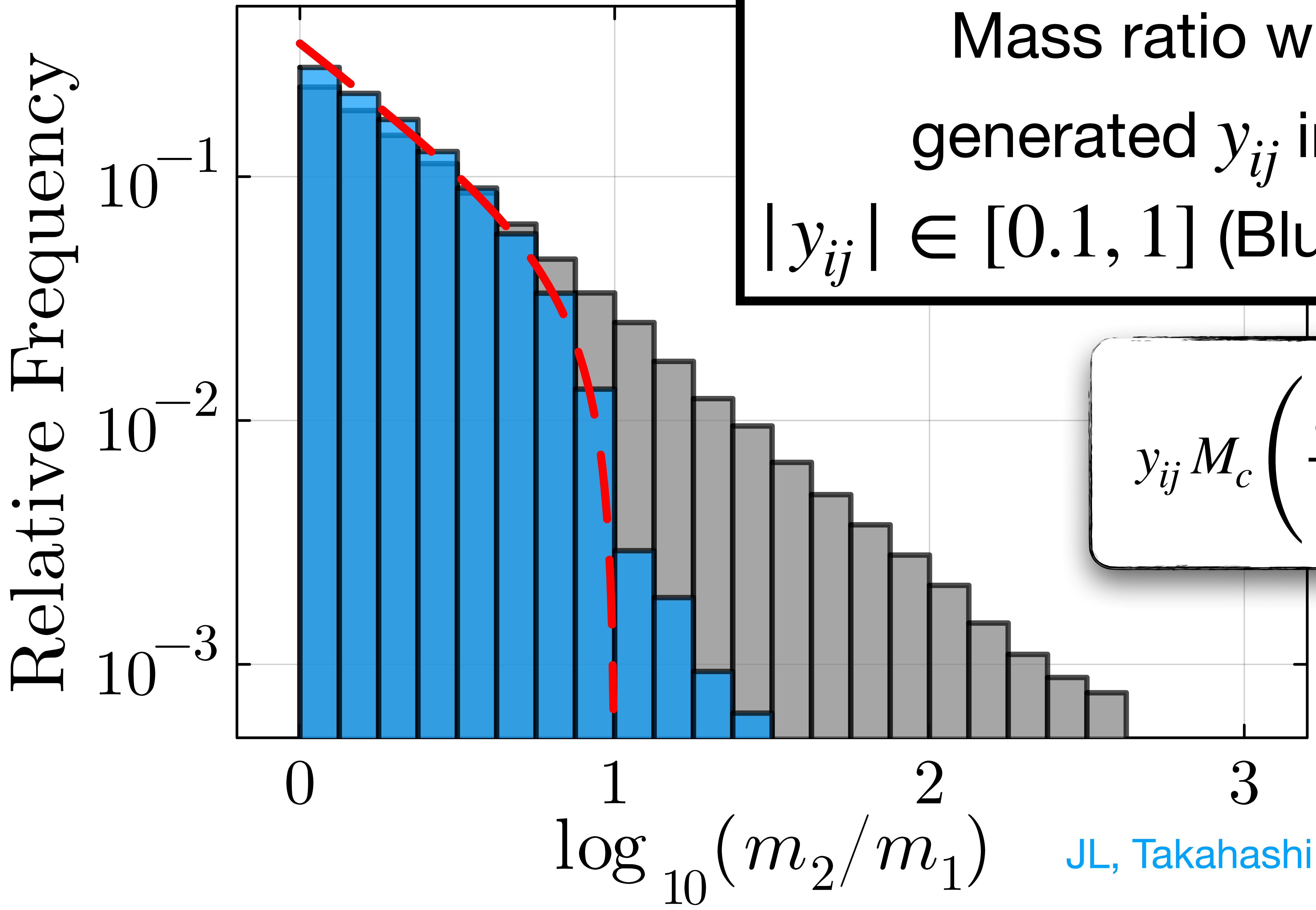
Degeneracy is in power of ϵ. $\mathcal{O}(1)$ coefficients split the pair.

mCPs likely form doublets



JL, Takahashi, Tsai 2603.12320

mCP mass ratio of the lightest doublet



Mass ratio with randomly generated y_{ij} in the range of $|y_{ij}| \in [0.1, 1]$ (Blue) and $[0, 1]$ (Gray)

$$y_{ij} M_c \left(\frac{\phi^{(*)}}{M_c}\right)^{|a_i - b_j|} \psi_i \psi_j + \mathrm{h.c.}$$

JL, Takahashi, Tsai 2603.12320

The lightest two states usually remain relatively close in mass.

Summary

- Light mCP masses can be protected by chiral $U(1)_{\mathrm{X}}$.
- Anomaly cancellation is equivalent to the degree-3 Prouhet-Tarry-Escott Problem.
- Minimal consistency predicts four mCPs and generically doublet spectra.

If one mCP is found, do not immediately fit it as a one-species model.
Look for a partner with the same minicharge and a nearby mass.

Back Up

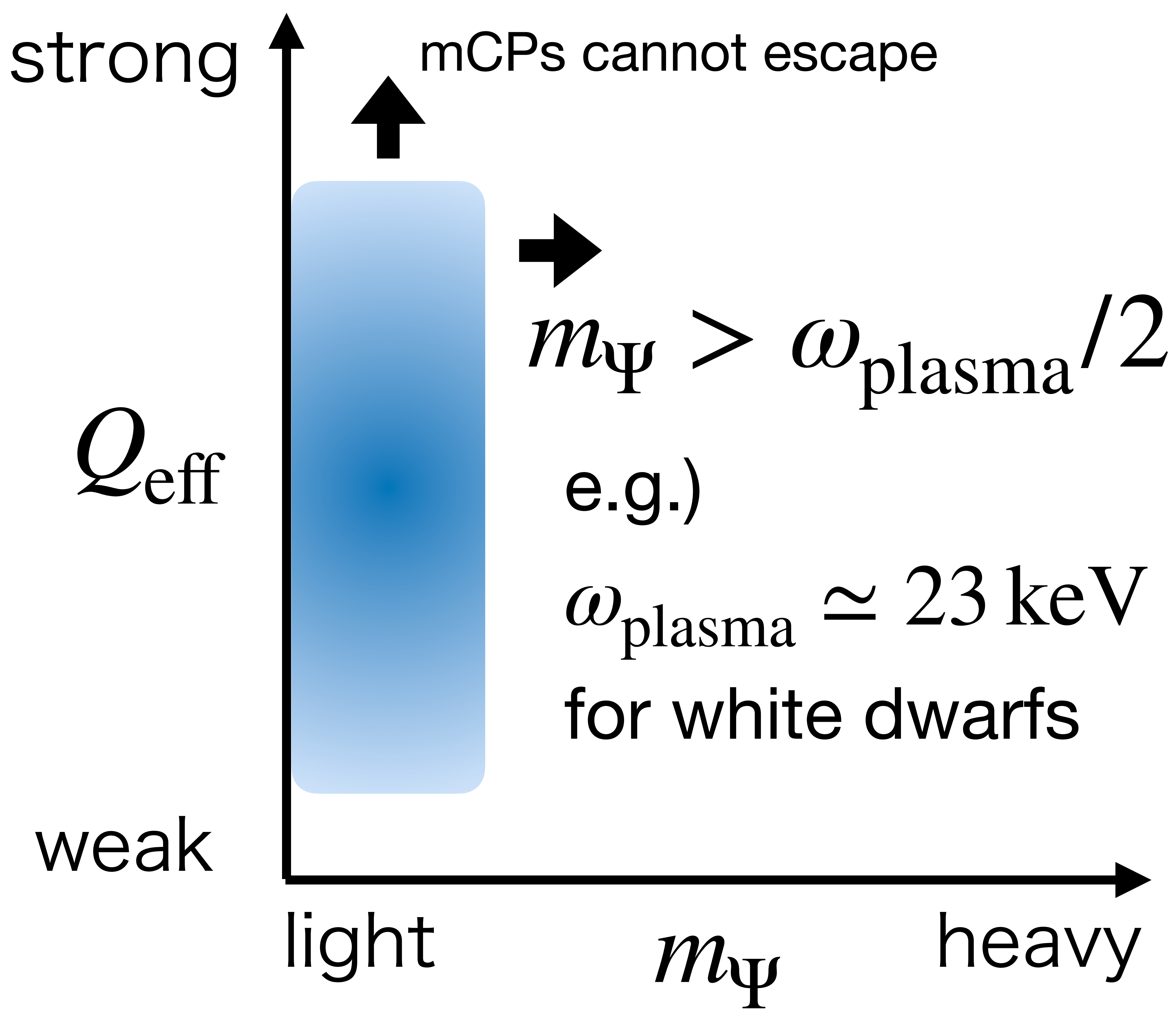
Stellar cooling

Davidson, Campbell, Bailey `91, Raffelt `96

In a stellar plasma, a photon can produce light mCP pairs.

$$\gamma^* \to \Psi\bar{\Psi}$$

Escaping mCPs carry away invisible energy and constrain light masses.



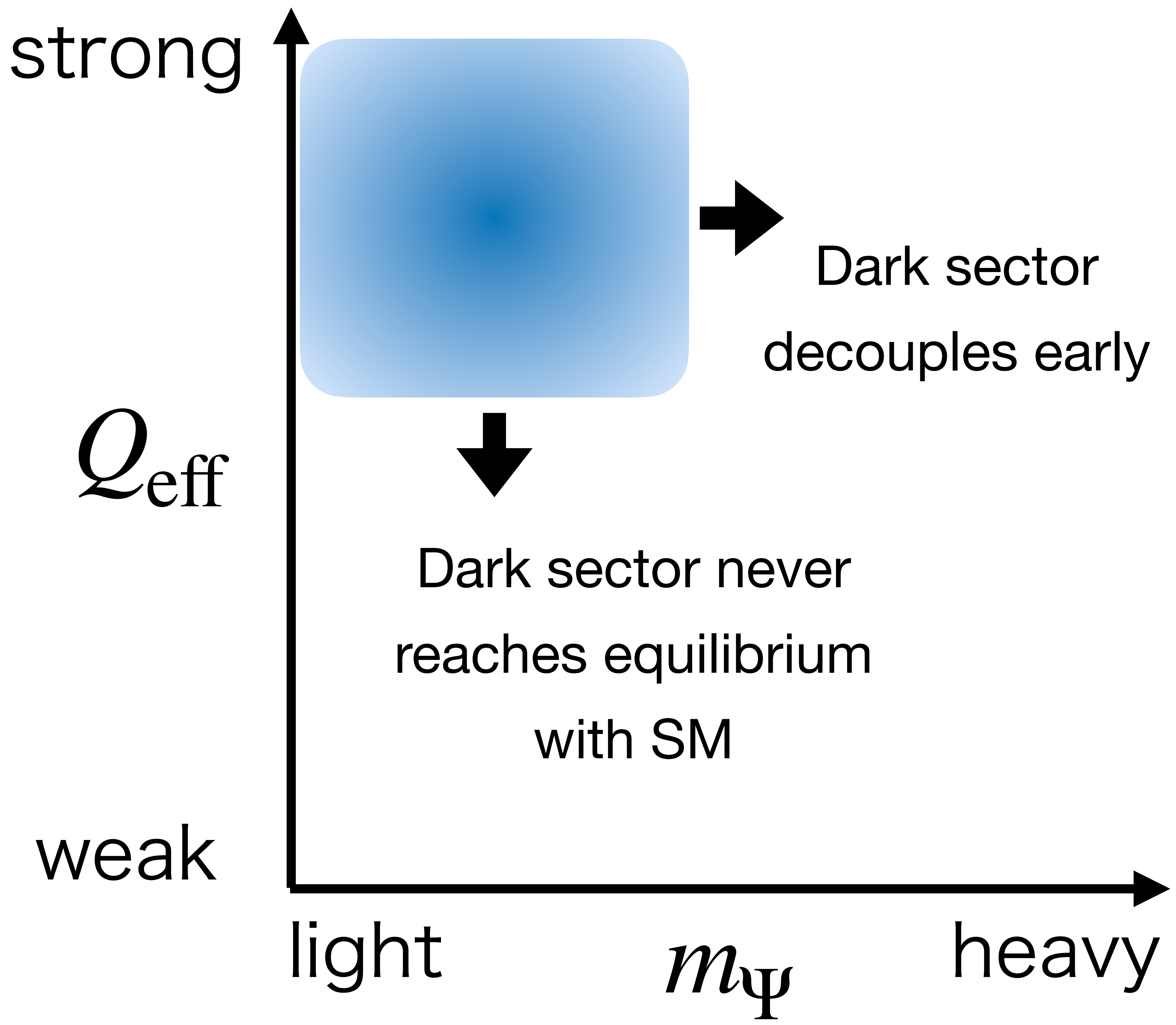
Cosmology

Davidson, Campbell, Bailey `91, Vogel, Redondo 1311.2600

Hidden sector particles are efficiently produced by

$$e^+e^- \to \Psi\bar{\Psi}, \quad \gamma^* \to \Psi\bar{\Psi}$$

in the early time, and may contribute to $N_{\rm eff}$ at the BBN and CMB epoch.



Laboratory searches

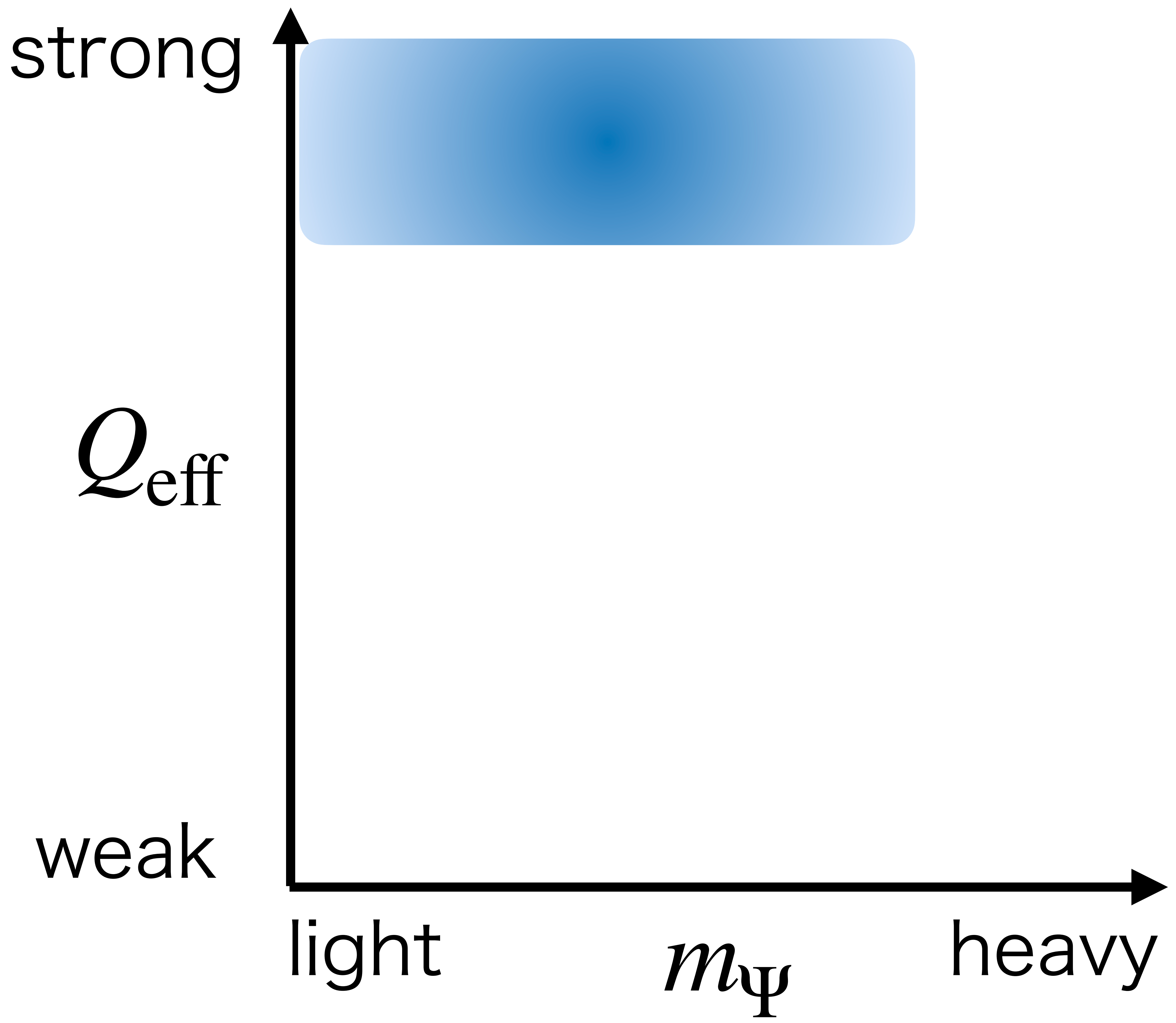
Various experiments constrain a large mixing regime.

Beam-dump

Collider

Positronium decay

…



Nota Bene: affine transformation

Mathematically,

$$A \to A' = \{ca_i + d\}, \qquad B \to B' = \{cb_i + d\}$$

$$(c, d : \text{rational numbers}, \quad c \neq 0)$$

maps a PTE solution to another solution, generating an equivalence class.

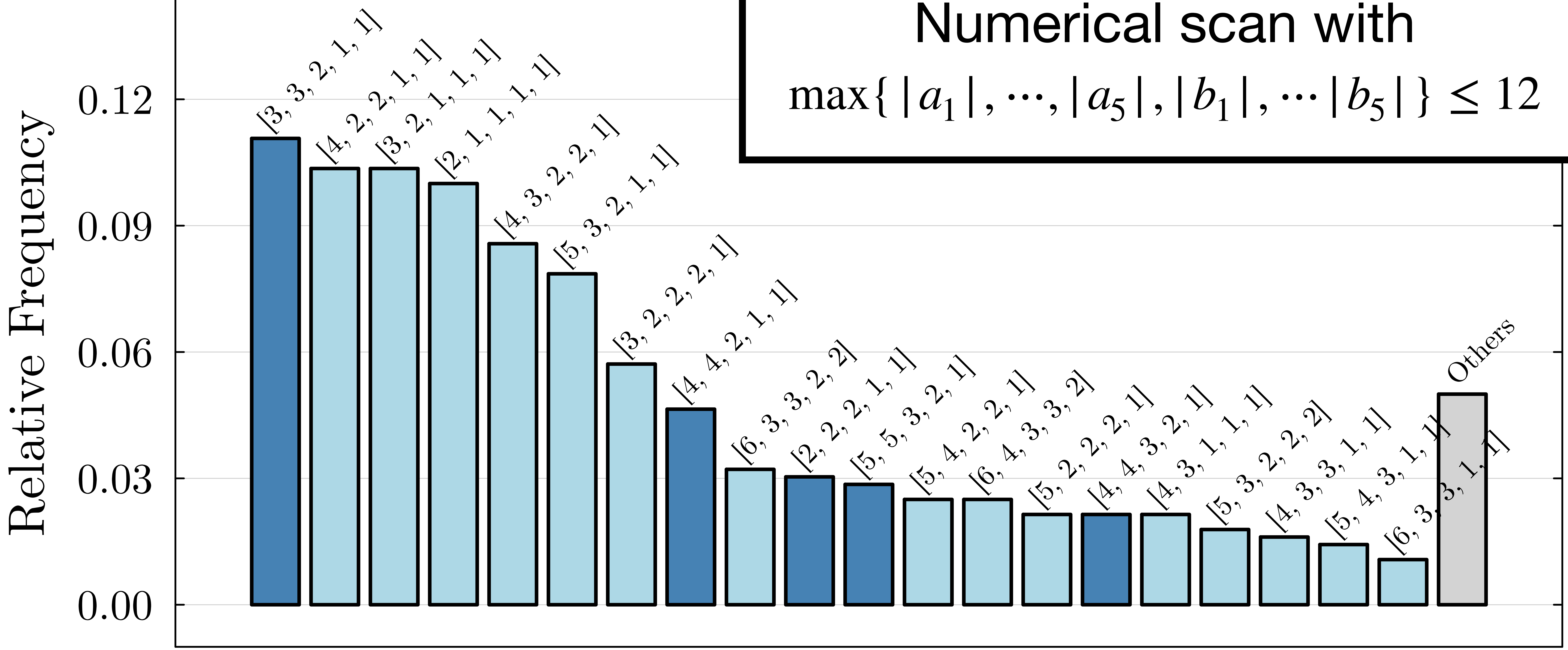
$\ast$ We extend the definition of a symmetric solution, allowing a shift.

e.g.) $A = \{0, 7, 9, 16\}$, $B = \{1, 4, 12, 15\}$ is a symmetric solution.

Affine-related PTE solutions are physically distinct.

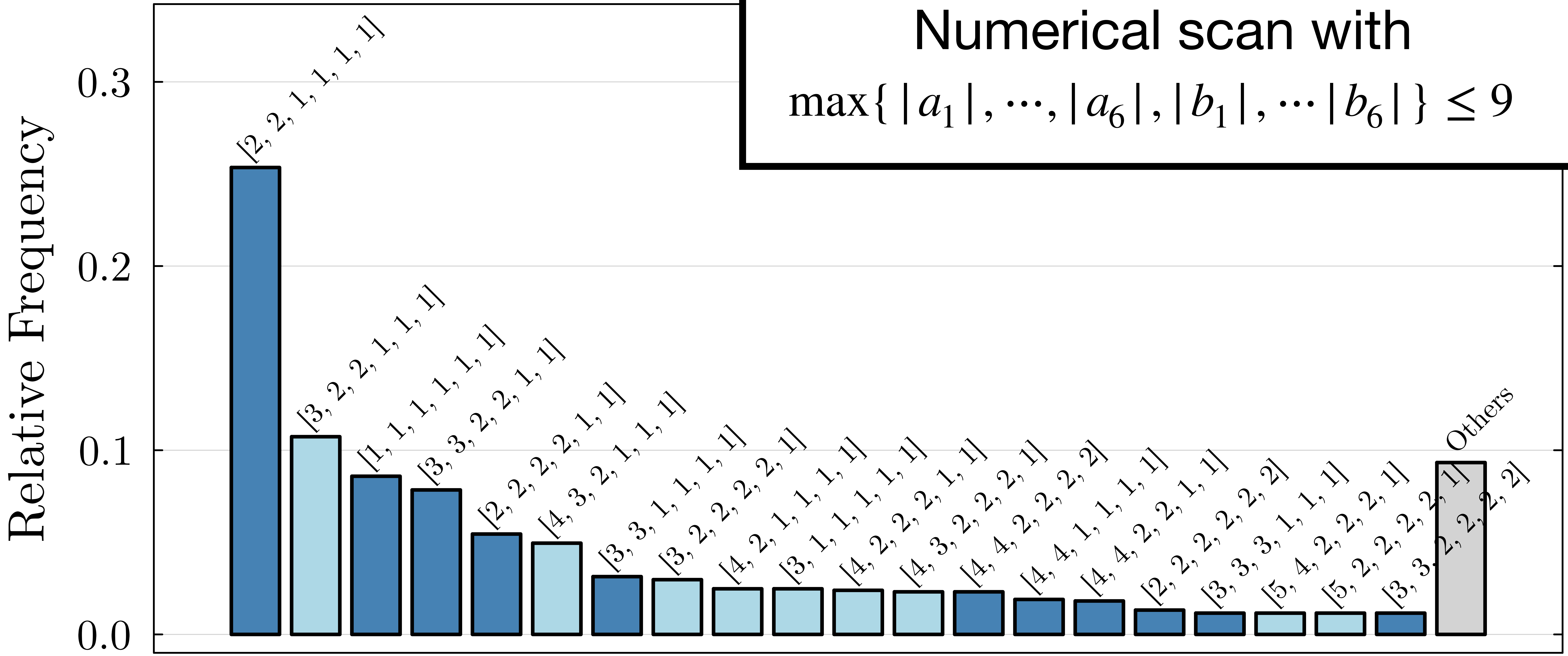
Numerical scan with

$$\max\{\,|a_1|,\cdots,|a_5|,|b_1|,\cdots|b_5|\,\}\leq 12$$



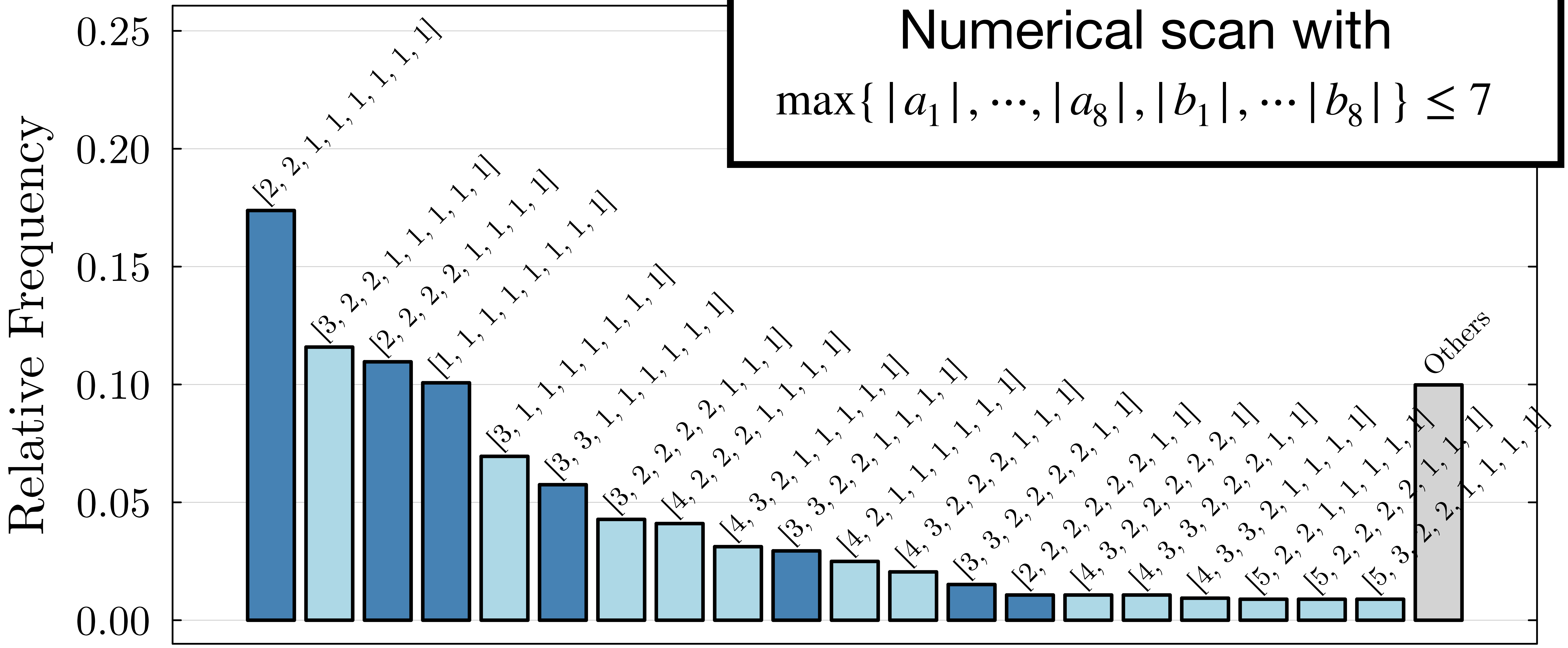
Numerical scan with

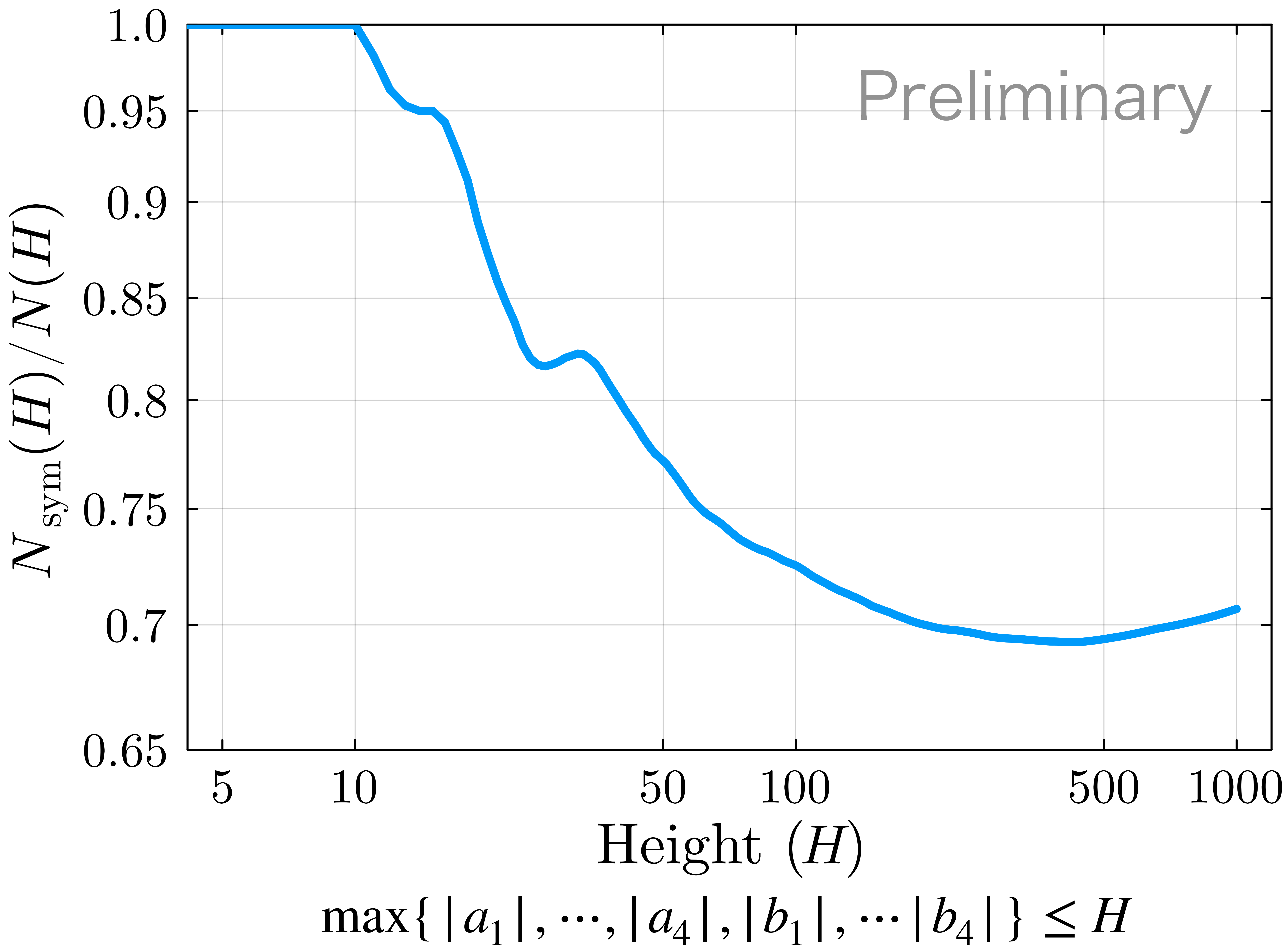
$\max\{\,|a_1|,\cdots,|a_6|,|b_1|,\cdots|b_6|\,\}\leq 9$



Numerical scan with

$$\max\{\,|a_1|,\cdots,|a_8|,|b_1|,\cdots|b_8|\,\}\leq 7$$




Preliminary
1.0
0.95
0.9
0.85
0.8
0.75
0.7
0.65
5
10
50
100
500
1000
$N_{\mathrm{sym}}(H)/N(H)$
Height (H)
$\max\{|a_1|, \cdots, |a_4|, |b_1|, \cdots |b_4|\} \leq H$