Cosmo. Correlators with Double Massive Exchanges

Bootstrap Equation and Phenomenology

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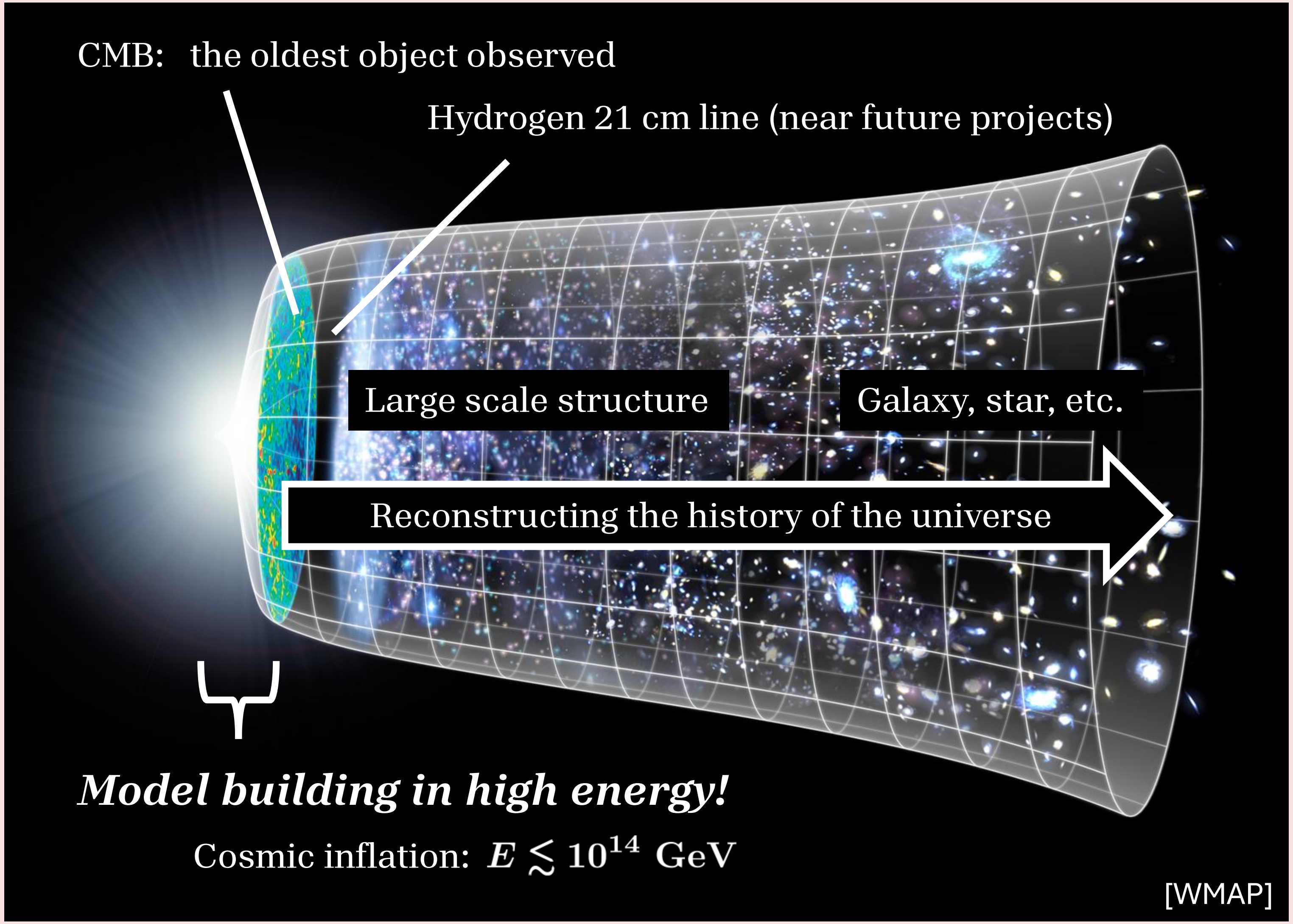
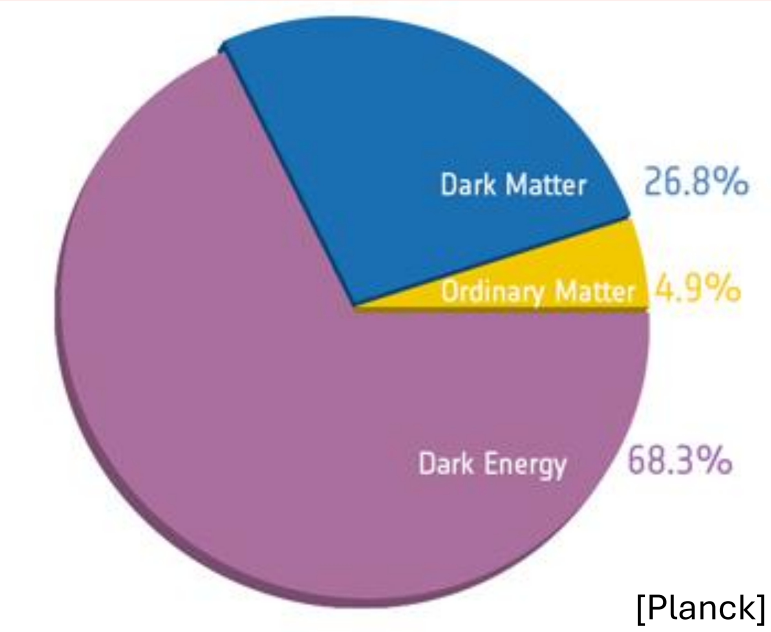
Based on

JHEP09(2024)176 with S. Aoki, L. Pinol, M. Yamaguchi, Y. Zhu

Cosmology for high energy physics





Plenty of unresolved problems

- ✓ Dark matter
- ✓ Dark energy
- ✓ Cosmic ray
- ✓ The dawn
- ⋮

Interplay with particle physics

Observables for inflationary cosmology

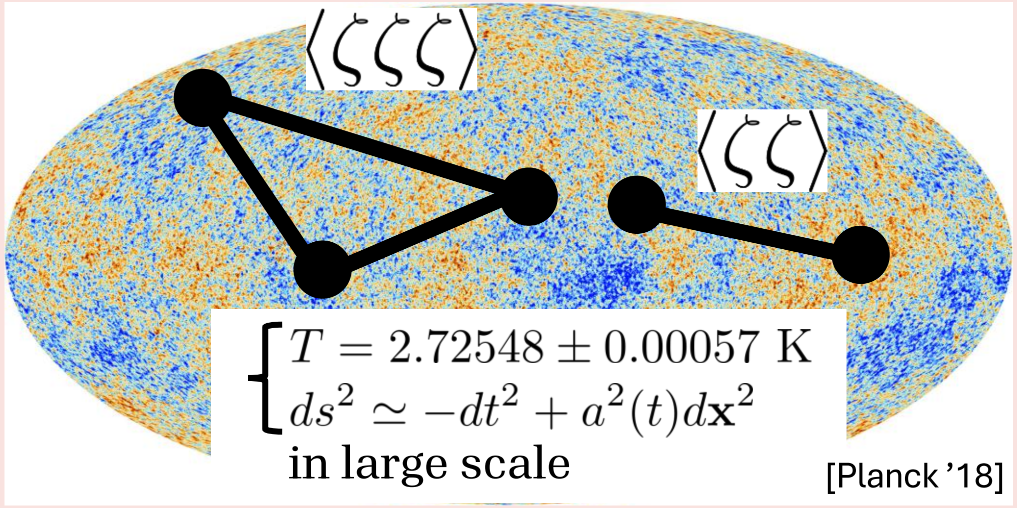
◆ **2pt. correlation function (power spectrum)**

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle_{\text{inf. end}} = (2\pi)^3 \delta^3(\boldsymbol{k}+\boldsymbol{k}') \frac{2\pi^2}{k^3} P_\zeta$$

$$P_\zeta \simeq \frac{H^2}{8\pi^2 \epsilon} \left(\frac{k}{k_*} \right)^{n_s - 1} \quad n_s \simeq 0.965, \quad \frac{dn_s}{d\log k} \simeq 0.002$$

[Planck '18]

Consistent with slow-roll inflation
Free propagation is dominant



◆ **3pt. correlation function (bispectrum)**

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle_{\text{inf. end}} = (2\pi)^7 \delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3) \frac{P_\zeta^2}{(k_1 k_2 k_3)^2} S\left(\frac{k_1}{k_3}, \frac{k_2}{k_3} \right)$$

✓ 3pt.: effects of interactions ➡ ***Probe for BSM physics*** and inflation models

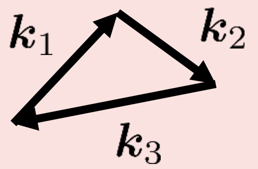
Crash course of inflationary bispectrum

◆ Definition and formulation

➢ Convention: shape function and $f_{\rm NL}$

$$\langle\zeta_1\zeta_2\zeta_3\rangle = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{P_\zeta^2}{(k_1k_2k_3)^2}S = (2\pi)^7\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\frac{3P_\zeta^2}{10k_2^3k_3^3}f_{\rm NL} + (k_1\to k_2,k_3)$$

➢ Triangle of momentum



Momentum conservation $\delta^3(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)$

$M \gg H$

✓ Equilateral $k_1 = k_2 = k_3$: Peak of bispectrum. Sensitivity to UV.

✓ Squeezed/local $k_1 \ll k_2 \simeq k_3$: Less sensitive to UV theory.
$\equiv k_{\rm L}$ $\equiv k_{\rm S}$

◆ Single field inflation: gravitational floor $(\delta\phi\leftrightarrow\zeta)$

➢ Interactions terms: Non-linearity of general relativity. [Maldacena astro-ph/0210603]

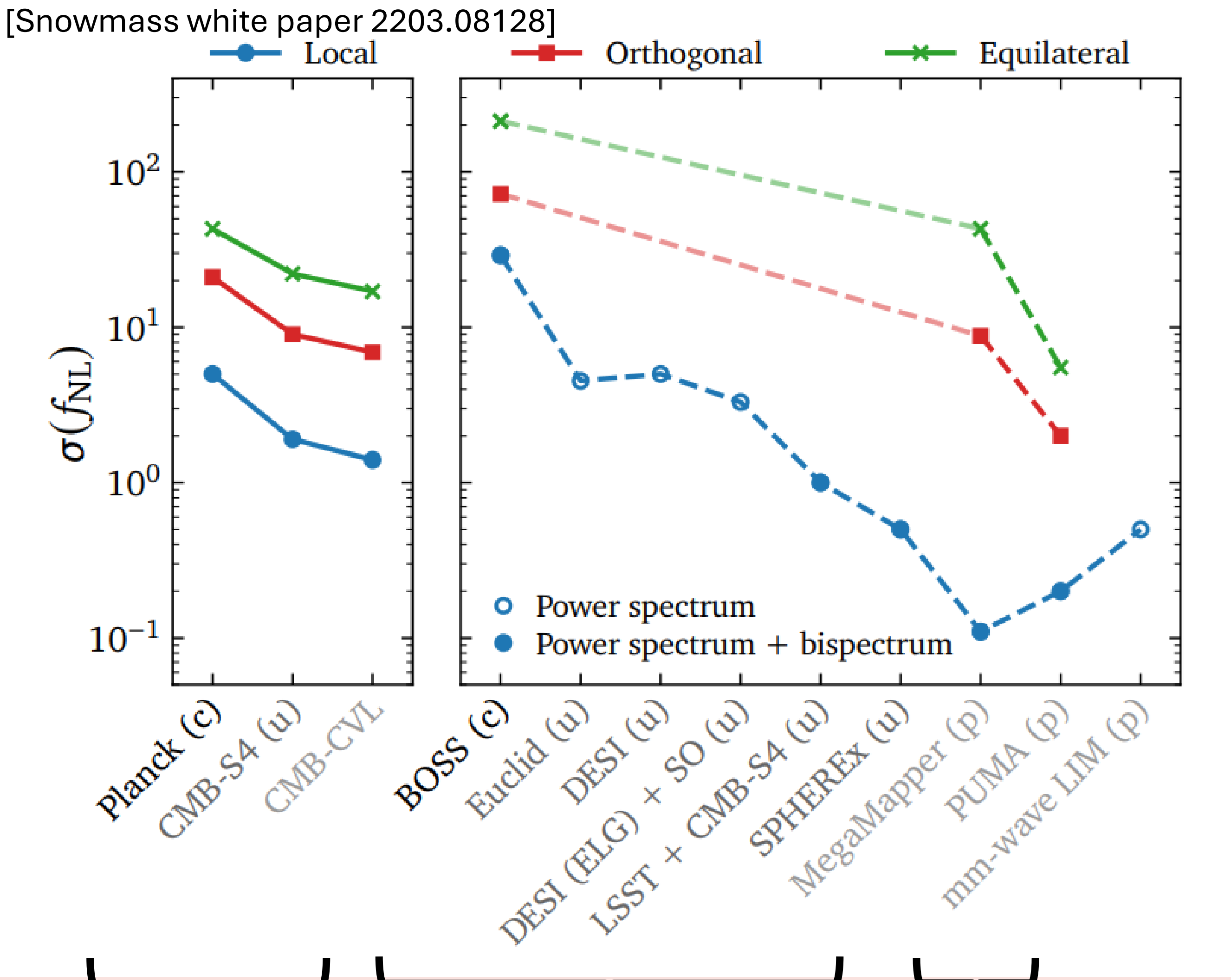
$$\mathcal{L}_{\rm EH} = \sqrt{-g}R + ({\rm GHY}) \xrightarrow{\text{Cubic order of }\zeta} \mathcal{L}_3^\zeta = a^3\epsilon^2\zeta\dot\zeta^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot\zeta\partial\zeta\partial\chi + \partial_t\left(-\frac{\epsilon\eta}{2}a^3\zeta^2\dot\zeta\right) + \cdots$$

➢ Bispectrum: $\displaystyle\lim_{k_{\rm L}\ll k_{\rm S}} S = \frac{1-n_s}{4}\frac{k_{\rm S}}{k_{\rm L}} \sim 10^{-2}\frac{k_{\rm S}}{k_{\rm L}}$ or $\displaystyle f_{\rm NL}^{\rm local} \equiv \lim_{k_{\rm L}\ll k_{\rm S}} f_{\rm NL} = \frac{5}{12}(1-n_s) \sim 10^{-2}$

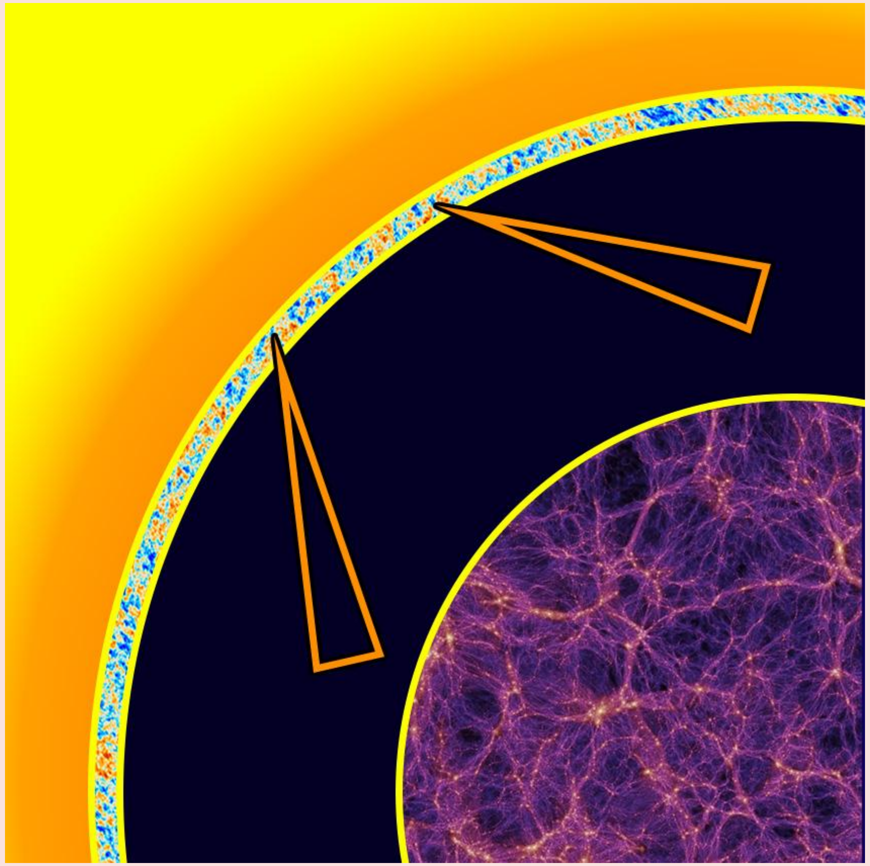
Future Observations

(c): completed
(u): upcoming
(p): projected

[Snowmass white paper 2203.08128]



CMB | Galaxy survey | 21 cm line



CMB | Dark age (21 cm line) | Galaxies

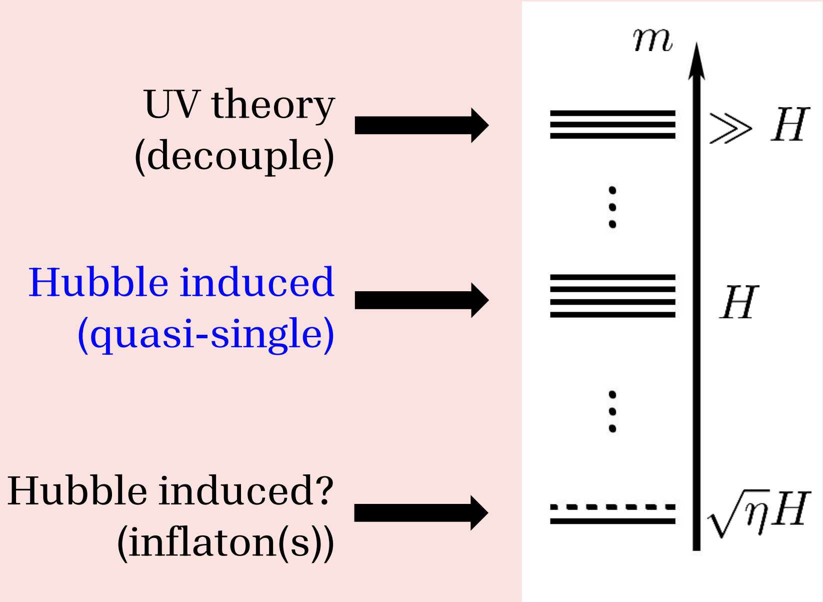
21cm-21cm-CMB cross-correlation

$\sigma(f_{NL}^{local}) \sim 6 \times 10^{-3}$ [Orlando et al. '23]

Particles during inflation: Beyond single field

- **Mass spectra** [Copeland et al. astro-ph/9401011, Chen et al. 1604.07841 etc.]
  - Loop resummation $\Delta m^2 \propto H^2$
  - SUGRA $\mathcal{L} \supset e^K V(\phi) \simeq V + 3cH^2\sigma^2$
  - Non-minimal coupling $\mathcal{L} \supset \xi\sigma^2 R \simeq 12\xi H^2\sigma^2$
  - $m \sim H$ may produce observable effects

$$f_{\rm NL}^{\rm local} \sim (\text{coupling}) \times e^{-\pi\mu} \times \mathcal{O}(1) \quad \text{vs.} \quad f_{\rm NL,21cm}^{\rm local} \sim 10^{-2}$$



- **Massive particles during inflation**
  - De Sitter symmetry in 3+1-dim $\sim$ conformal symmetry in 3-dim $(SO(1,4))$

➡ Scaling behavior is universally determined

$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

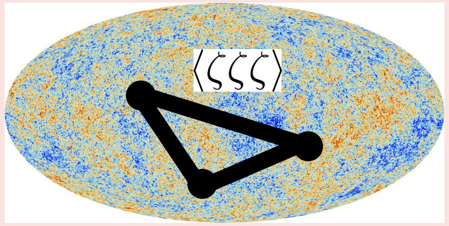
e.g., scalar field $\lim_{k\tau \to 0} \sigma_{\mathbf{k}}(\tau) \sim (-k\tau)^{3/2+i\mu} + e^{-\pi\mu}(-k\tau)^{3/2-i\mu}$

$$ds^2 = \frac{1}{H^2\tau^2}(-d\tau^2 + d\mathbf{x}^2), \quad -\infty < \tau < 0$$

The heavy fields oscillate with ***wavelength being their mass parameter.***

Cosmological Collider physics

[Chen & Wang 0911.3380, Noumi et al. 1211.1624, Arkani-Hamed & Maldacena 1503.08043 etc.]



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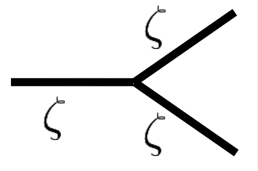

[Maldacena astro-ph/0210603]

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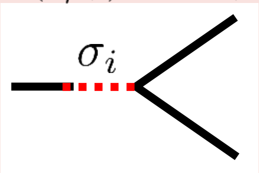
$\sigma(\partial_\mu\phi)^2 \propto \sigma\delta\dot{\phi}$



[Arkani-Hamed et al. 1811.00024, Qin & Xianyu 2208.13790, 2301.07047]

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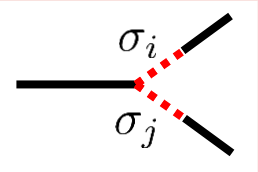

[Xianyu & Zang 2309.10849, **Aoki et al. 2404.09547**]

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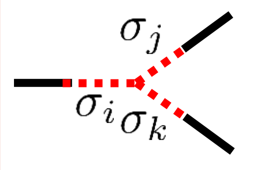


[Xianyu & Zang 2511.08677]

$+ \cdots$

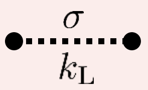
Loops, ...
[Xianyu and Zhang 2211.03810, Aoki et al. 2605.28054]

◆ **Qualitative behavior**

$$S \sim \left(\frac{k_L}{k_S}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_L}{k_S} + \delta\right)$$

Squeezed: $k_L \ll k_S$

$$\mu = \sqrt{\left(\frac{m_\sigma}{H}\right)^2 - \frac{9}{4}}$$



$\Rightarrow e^{-\pi\mu}\left(\frac{k_L}{k_S}\right)^{1/2+i\mu}$

✓ Dictionary for particles of $m \sim H \lesssim 10^{11}$ GeV

Supersymmetry, [Baumann & Green 1109.0292]
RH neutrino, [Chen et al. 1805.02656]
CP violation, [Liu et al. 1909.01819]
gauge theory, [Maru & Okawa 2101.10634]
extra dimension, ... [Reece et al. 2204.11869]

✓ Expected as a target of (near) future observations

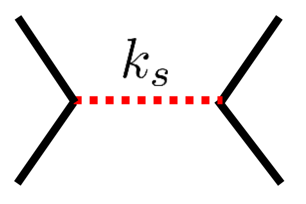
Observational templates are also discussed actively. [Snowmass white paper 2203.08128 for a review]

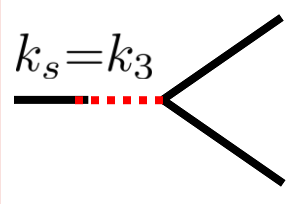
Difficulty in analytical computations

◆ **Seed integral**

$$\langle 0|\zeta_1\zeta_2\zeta_3(\tau)|0\rangle \propto \frac{1}{8k_1k_2k_3^4}\lim_{k_4\to 0}\sum_{\mathrm{a,b}=\pm}\mathcal{I}_{\mathrm{ab}}^{0,-2} + (k_3\to k_1,k_2) + (\text{higher order Feynman diagrams})$$

$$\mathcal{I}_{\mathrm{ab}}^{p_1p_2} = -\mathrm{ab}k_s^{5+p_{12}}\int_{-\infty}^{0} d\tau_1\, d\tau_2\, \underbrace{(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\mathrm{i}\mathrm{a}k_{12}\tau_1+\mathrm{i}\mathrm{b}k_{34}\tau_2}}_{\text{Scale factor and propagators of }\zeta} \underbrace{D_{\mathrm{ab}}(k_s;\tau_1,\tau_2)}_{\text{Propagators of }\sigma} \qquad \mathrm{a,b}=\pm \quad \begin{matrix} +:\mathrm{T} \\ -:\bar{\mathrm{T}} \end{matrix}$$

$\mathcal{I} =$ 

$\xrightarrow{k_4\to 0}$ 

* In our paper, trispectrum is also computed.

- Analytical computation: $D_{++}(k_s;\tau_1,\tau_2) \sim \theta(\tau_1-\tau_2)H_{i\mu}^{(1)}(-k_s\tau_1)H_{i\mu}^{(1)*}(-k_s\tau_2)$ is hard.
  No special fn. is developed for the integral …
- Numerical computation: Seed integral is highly oscillatory.

➡ We need some smart ways to reduce the seed integral to simpler templates.

Analytical method: De Sitter bootstrap equations

[Arkani-Hamed et al. 1811.00024, Qin & Xianyu 2208.13790, 2301.07047]

◆ De Sitter symmetry ~CFT

$$\text{Translation}\quad P_i=\partial_i,\quad \text{Rotation}\quad J_{ij}=x_i\partial_j-x_j\partial_i,\quad \text{Dilatation}\quad D=-\tau\partial_\tau-x_i\partial_i,$$

$$\text{dS boosts}\quad K_i=\left(2x^jx_i+(\tau^2-x^2)\delta_i^j\right)\partial_j+2x_i\tau\partial_\tau$$

- Ward identity: Symmetry $\hat{S}$ ⟶ $\langle 0|[\hat{S},\hat{\mathcal{O}}]|0\rangle=0$ (assuming $\hat{S}\,|0\rangle=0$)

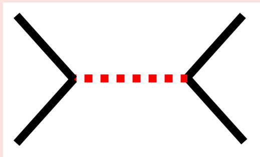
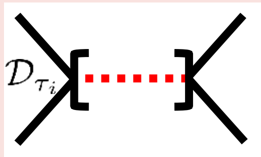
◆ Bootstrap equations for seed integrals

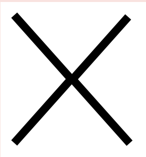
- Equations of motion: quadratic Casimir operator $\nabla_\mu\nabla^\mu$

$$(\nabla^2+a^2m^2)\sigma=0 \quad \Rightarrow \quad \mathcal{D}_{\tau_i}\widetilde{D}^{\sigma}_{\text{ab}}(k_s\tau_1,k_s\tau_2)=-\mathrm{ia}H^2(k_s\tau_1)^2(k_s\tau_2)^2\delta_{\text{ab}}\delta(k_s\tau_1-k_s\tau_2)$$

$$\mathcal{D}_{\tau_i}=\tau_i\partial_{\tau_i}(\tau_i\partial_{\tau_i})-3\tau_i\partial_{\tau_i}+k_s^2\tau_i^2+\mu^2+\frac{9}{4},\quad \mu=\sqrt{\frac{m^2}{H^2}-\frac{9}{4}},\quad \widetilde{D}=k_s^3D$$

- Dilatation: $\tau\partial_\tau(\cdots)=k\partial_k(\cdots)$ ⟶ $\mathcal{D}_\tau\widetilde{D}=\mathcal{D}_k\widetilde{D}$

⟶ $\widetilde{\mathcal{D}}_{k_s}\Big[$  $\Big]$ ~ 

~  ⟶ $\mathcal{I}^{p_1p_2}_{\text{ab}}\sim {}_2F_1,\ \sum_n^\infty\left(\frac{k_i}{\sum_j k_j}\right)^n\ {}_3F_2$

Boundary conditions: Mellin-Barnes representation

[Qin, Xianyu 2208.13790, 2301.07047]

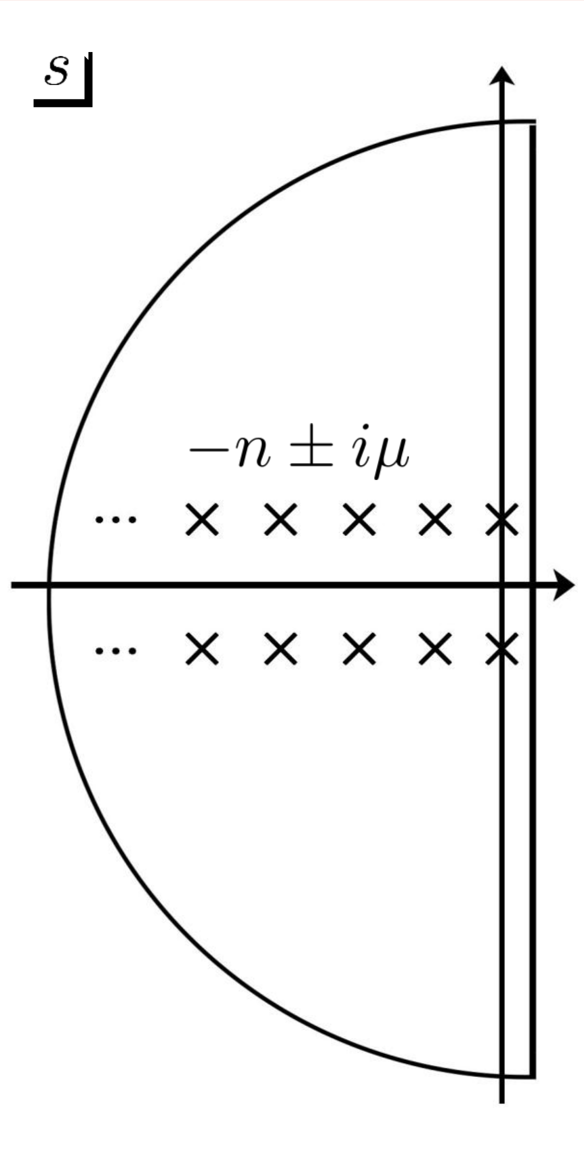
- **Bootstrap:** Boundary conditions are not fixed.

- **Direct integration using MB rep.**

$$H^{(1)}_{i\mu}(-k\tau) = \frac{1}{\pi}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\left(\frac{-k\tau}{2}\right)^{-2s} e^{(2s-1-i\mu)\pi i/2}\Gamma(s-i\mu)\Gamma(s+i\mu)$$

$$\Rightarrow \mathcal{I} \sim \int d\tau_1 d\tau_2 e^{ik_{12}\tau_1 + ik_{34}\tau_2}(-\tau_1)^{p_1}(-\tau_2)^{p_2} H^{(1)}_{i\mu}(-k_s\tau_1) H^{(1)*}_{i\mu}(-k_s\tau_2)\theta(\tau_1-\tau_2)$$

$$\sim \sum_{\substack{n_1,n_2 \\ s_i = -n_i \pm i\mu}} \mathcal{A}_{n_1,n_2}(k,k')\mathrm{Res}[\Gamma(s_1 \pm i\mu)]\mathrm{Res}[\Gamma(s_2 \pm i\mu)]$$



- ✓ MB rep.: double sum. but boundary conditions are chosen in mode fn.
- ✓ Bootstrap: single sum. but boundary conditions are not fixed.

Matching them in some limits and obtaining simple expression (e.g., $k_s \to 0$)

Single exchange diagram

[Qin, Xianyu 2208.13790, 2301.07047]

◆ Exact solution

$$\mathcal{I}_{\pm\mp}^{p_1p_2} = \frac{-e^{\mp i\frac{\pi}{2}\bar{p}_{12}}\left[1+\cosh(2\pi\mu)\right]}{2\sinh^2(2\pi\mu)} \times\left\{2^{\pm i\mu}\left(\frac{u_1}{2}\right)^{\frac{5}{2}+p_1\pm i\mu}{}_2\mathcal{F}_1\left[\begin{matrix}\frac{5}{2}+p_1\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\middle| u_1\right]-(\mu\to-\mu)\right\} \times\left\{2^{\pm i\mu}\left(\frac{u_2}{2}\right)^{\frac{5}{2}+p_2\pm i\mu}{}_2\mathcal{F}_1\left[\begin{matrix}\frac{5}{2}+p_2\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\middle| u_2\right]-(\mu\to-\mu)\right\},$$

$$\mathcal{I}_{\pm\pm}^{p_1p_2} = \frac{\mp i e^{\mp i\frac{\pi}{2}p_{12}}\pi}{\Gamma\left[\frac{1}{2}-i\mu,\frac{1}{2}+i\mu\right]\sinh^2(2\pi\mu)} \times\left\{\frac{e^{\pi\mu}\cosh\left[\pi(-\mu)\right]}{2^{\mp i\mu}}\left(\frac{u_1}{2}\right)^{\frac{5}{2}+p_1\pm i\mu}{}_2\mathcal{F}_1\left[\begin{matrix}\frac{5}{2}+p_1\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\middle| u_1\right]-(\mu\to-\mu)\right\} \times\left\{2^{\pm i\mu}\left(\frac{u_2}{2}\right)^{\frac{5}{2}+p_2\pm i\mu}{}_2\mathcal{F}_1\left[\begin{matrix}\frac{5}{2}+p_2\pm i\mu,\frac{1}{2}\pm i\mu\\1\pm 2i\mu\end{matrix}\middle| u_2\right]-(\mu\to-\mu)\right\} + \frac{e^{\mp i\frac{\pi}{2}p_{12}}\Gamma(p_{12}+5)}{2^{p_{12}+5}}\sum_{n=0}^{\infty}u_1^{n+p_{12}+5}\left(1-\frac{1}{u_2}\right)^n\binom{n+p_{12}+4}{n} \times\frac{1}{\mu^2+\left(\frac{5}{2}+n+p_2\right)^2}{}_3\mathrm{F}_2\left[\begin{matrix}1,3+n+p_2,5+n+p_{12}\\\frac{7}{2}+n+p_2-i\mu,\frac{7}{2}+n+p_2+i\mu\end{matrix}\middle| u_1\right].$$

◆ Squeezed limit

$$\lim_{k_1\ll k_2\simeq k_3} S=\frac{\mu\lambda_1/H^2}{32\pi P_\zeta^{1/2}}\mathrm{Re}\left(\frac{2\pi^{3/2}\cos\left[\frac{\pi}{2}\left(i\mu-\frac{1}{2}\right)\right]}{\sin(2\pi i\mu)\,\Gamma(1+i\mu)}\cos\left[\frac{\pi}{4}(3+2i\mu)\right]\Gamma\left(\frac{5}{2}+i\mu\right)\left(\frac{k_1}{4k_2}\right)^{\frac{1}{2}+i\mu}\right)+\cdots\propto\left(\frac{k_{\mathrm{L}}}{k_{\mathrm{S}}}\right)^{1/2}e^{-\pi\mu}\cos\left(\mu\log\frac{k_{\mathrm{L}}}{k_{\mathrm{S}}}+\delta\right)$$

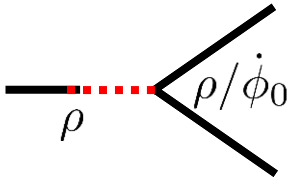
➢ $f_{\mathrm{NL}}^{\mathrm{local}}\sim(\text{coupling})\times\underbrace{e^{-\pi\mu}\times\mathcal{O}(1)}_{\sim\mathcal{I}}$ vs. $f_{\mathrm{NL,21cm}}^{\mathrm{local}}\sim 10^{-2}$

Why double-exchange?: Size of signals

[Pinol et al. 2312.06559, **Aoki et al. 2404.09547**]

◆ Single-exchange (SE)

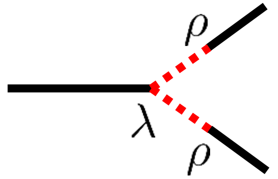
$$\frac{\rho}{\dot\phi_0}(\partial_\mu\phi)^2\sigma \longrightarrow \rho\ \delta\phi'\sigma + \frac{\rho}{\dot\phi_0}\ (\delta\phi')^2\sigma$$



$$S_{\rm SE} \sim \frac{\rho^2}{\dot\phi_0} \times P_\zeta^{-1/2}$$

◆ Double-exchange (DE)

$$\rho\ \delta\phi'\sigma + \lambda\ \delta\phi'\sigma^2$$

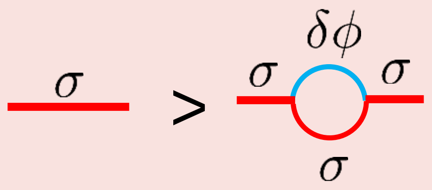


$$S_{\rm DE} \sim \lambda\frac{\rho^2}{H^2} \times P_\zeta^{-1/2}$$

◆ Constraints

✓ Perturbativity $\lambda \lesssim 1$

✓ Naturalness $\lambda \lesssim P_\zeta^{1/4}$

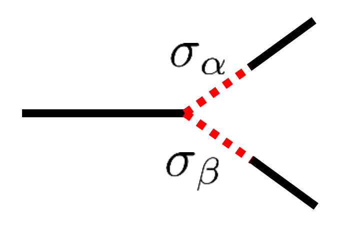


$$\frac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda\frac{\dot\phi_0}{H^2} \sim \lambda P_\zeta^{-1/2} \lesssim P_\zeta^{-1/4} \sim 10^2$$ **Naturally larger than single-exchange**

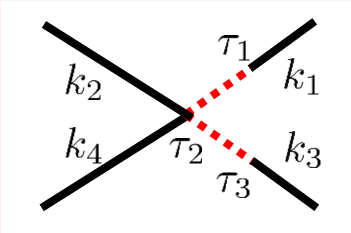
Method: Bootstrap Equations and MB Representations

[Aoki et al. 2404.09547]

$$\mathcal{L}_{\rm int} = a^3 \sum_{\alpha} \rho_\alpha \sigma_\alpha \delta\phi' + a^3 \sum_{\alpha,\beta} \lambda_{\alpha\beta}\sigma_\alpha\sigma_\beta\delta\phi'$$



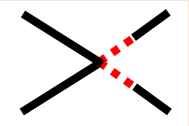
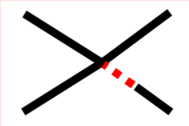
Seed integral



($k_4 \to 0$: bispectrum)

$$\mathcal{I}^{p_1p_2p_3}_{\mathrm{abc},\alpha\beta} = H^{-4} k_{24}^{9+p_{123}} (-\mathrm{iabc}) \int_{-\infty}^{0} \mathrm{d}\tau_1\, \mathrm{d}\tau_2\, \mathrm{d}\tau_3\, (-\tau_1)^{p_1} (-\tau_2)^{p_2} (-\tau_3)^{p_3}$$
$$\times\, e^{\mathrm{i}ak_1\tau_1 + \mathrm{i}bk_{24}\tau_2 + \mathrm{i}ck_3\tau_3} D^{\alpha}_{\mathrm{ab}}(k_1;\tau_1,\tau_2)\, D^{\beta}_{\mathrm{bc}}(k_3;\tau_2,\tau_3)$$

◆ Bootstrap equations

$\mathcal{D}$ [] $\sim$ 

$$\mathcal{I} \sim F_4,\ \sum_{n}^{\infty} \left(\frac{k_i}{\sum_j k_j}\right)^n ({}_3F_2 + {}_2F_1)$$

Analytical expression for arbitrary momentum configuration

◆ Bispectrum in squeezed limit

$$\langle \delta\phi_{k1}\delta\phi_{k2}\delta\phi_{k3}\rangle' \xrightarrow[k_3\to 0]{} \sum_{\alpha,\beta} \frac{\rho_\alpha\rho_\beta\lambda_{\alpha\beta}H}{(k_1k_2k_3)^2} \cdot \mathrm{Re}\left\{\left[\left[\mathrm{i}\frac{\pi^{3/2}}{2^{4+2\mathrm{i}\mu_\alpha}} \mathrm{sech}(\pi\mu_\beta)\left[1+\tanh(\pi\mu_\alpha)\right] \times \Gamma\left[\begin{matrix} -\mathrm{i}\mu_\alpha \\ -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta \end{matrix}\right]\right.\right.\right.$$
$$\left.\left.\left. \times {}_3\mathcal{F}_2\left[\left.\begin{matrix} -\frac{3}{2}-\mathrm{i}\mu_\alpha, -1-\mathrm{i}\mu_\alpha-\mathrm{i}\mu_\beta, -1-\mathrm{i}\mu_\alpha+\mathrm{i}\mu_\beta \\ -\frac{1}{2}-\mathrm{i}\mu_\alpha, -\frac{1}{2}-\mathrm{i}\mu_\alpha \end{matrix}\right| 1\right] + \mathcal{O}\left(e^{-2\pi\mu_\alpha}, e^{-2\pi\mu_\beta}\right)\right] \left(\frac{k_1}{k_3}\right)^{\frac{1}{2}+\mathrm{i}\mu_\alpha} + \mathcal{O}\left(\frac{k_1}{k_3}\right)\right\}$$

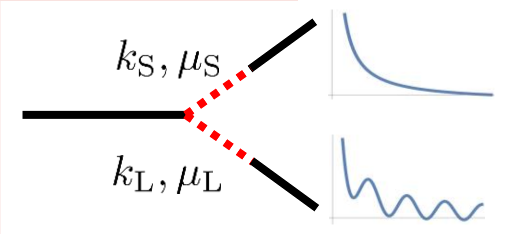
Observational Signals in Bispectrum

[Aoki et al. 2404.09547]

Consistency check: CosmoFlow
[Pinol et al. 2312.06559]

- **Squeezed limit** $k_3 \ll k_1 \simeq k_2$
  - Qualitatively the same as single-exchange since only a long mode oscillates.

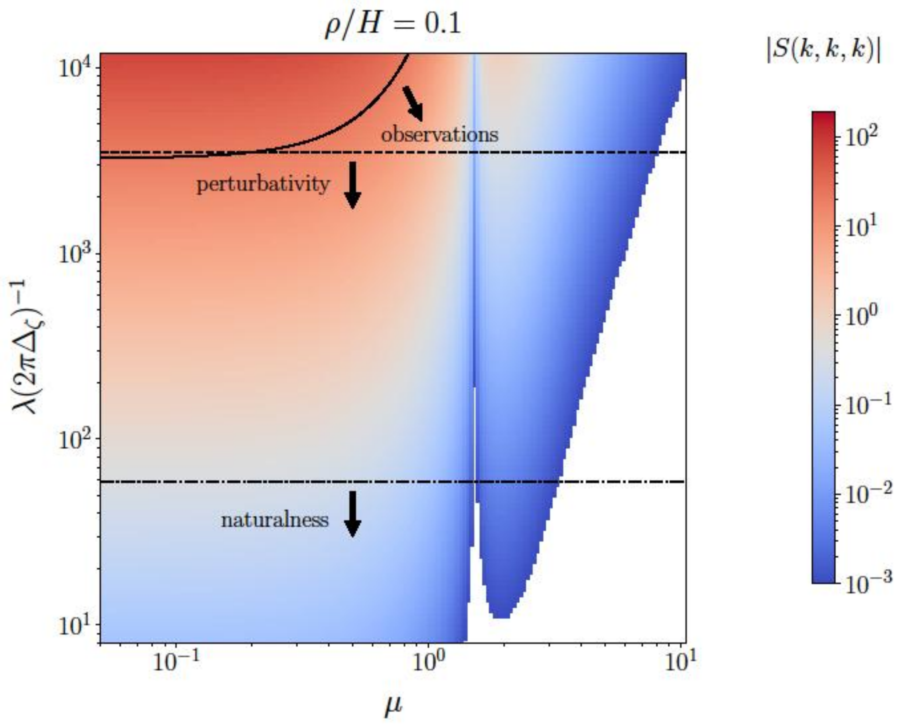
$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu_{\rm L}} \cos\left(\mu_{\rm L} \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$



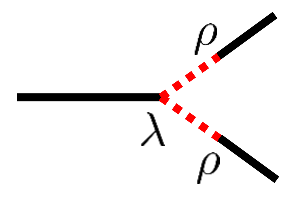
  - Quantitatively,

$$S^{\rm local}_{\rm DE} \sim (1+\tanh\pi\mu_{\rm L})\Gamma\left[\begin{matrix} -i\mu_{\rm L} \\ -1-i(\mu_{\rm L}-\mu_{\rm S}),\ -1-i(\mu_{\rm L}+\mu_{\rm S})\end{matrix}\right] {}_3\mathcal{F}_2[\cdots|1]\left(\frac{k_{\rm L}}{4k_{\rm S}}\right)^{1/2+i\mu_{\rm L}}$$

$$S^{\rm local}_{\rm SE} \sim \frac{2\pi^{3/2}\cos\left[\frac{\pi}{2}\left(i\mu-\frac{1}{2}\right)\right]}{\sin(2\pi i\mu)\,\Gamma(1+i\mu)} \cos\left[\frac{\pi}{4}(3+2i\mu)\right]\Gamma\left(\frac{5}{2}+i\mu\right)\left(\frac{k_1}{4k_2}\right)^{\frac{1}{2}+i\mu}$$



- **Size in equilateral limit** $k_1 = k_2 = k_3 = k$



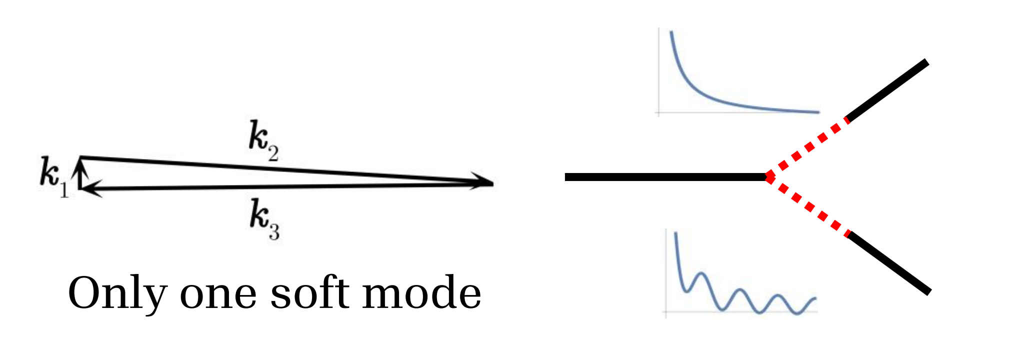
$$S^{\rm eq}_{\rm DE} \sim \frac{\lambda}{P_\zeta^{1/2}}\frac{\rho^2}{H^2}\frac{1}{(1/4+\mu_{\rm S}^2)(1/4+\mu_{\rm L}^2)}$$

  - Even under naturalness condition, it can be observable.

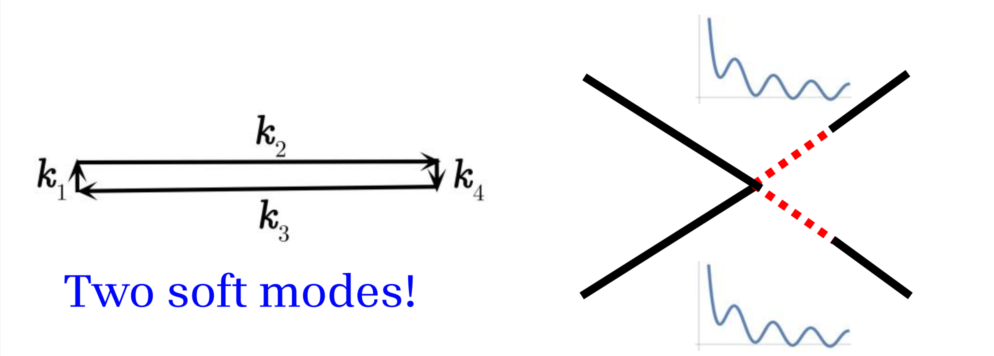
Difference between SE and DE: Trispectrum

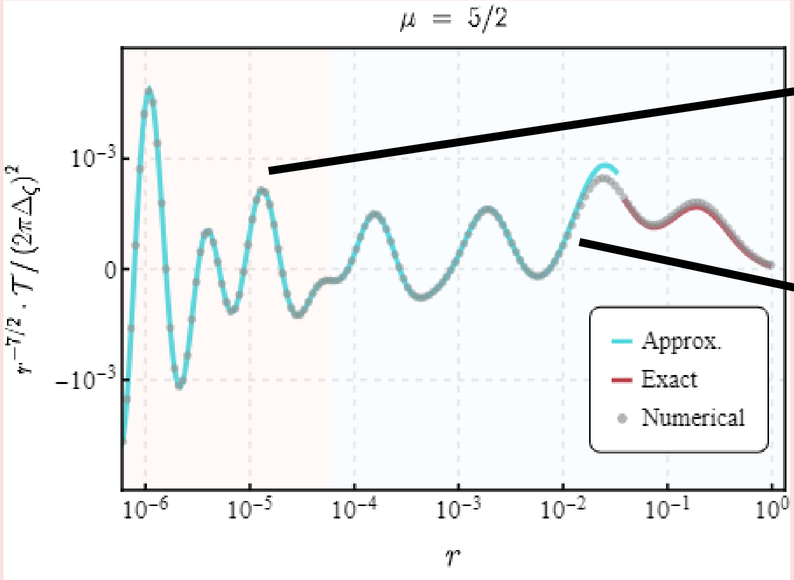
[Aoki et al. 2404.09547]

◆ Bispectrum



◆ Trispectrum

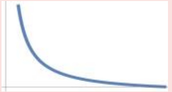




$$\mu_\alpha^{3/2}\mu_\beta^{3/2} e^{-\pi(\mu_\alpha+\mu_\beta)} \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3+i(\mu_\alpha+\mu_\beta)}$$

No such signals in SE

$$\frac{\mu_\alpha^{3/2}}{\mu_\beta^2} e^{-\pi\mu_\alpha} \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{7/2+i\mu_\alpha}$$

*  $\sim \mu^{3/2} e^{-\pi\mu} \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3/2+i\mu}$,  $\sim \frac{1}{\mu^2} \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^2$

Summary

◆Cosmological Collider physics

$$S \sim \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{1/2} e^{-\pi\mu} \cos\left(\mu \log\frac{k_{\rm L}}{k_{\rm S}} + \delta\right)$$

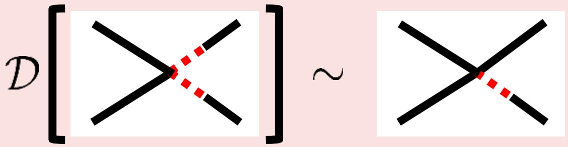
Goal: mass spectrum of particles during inflation
Task: preparing precise observational templates

◆Double-exchange vs. single-exchange

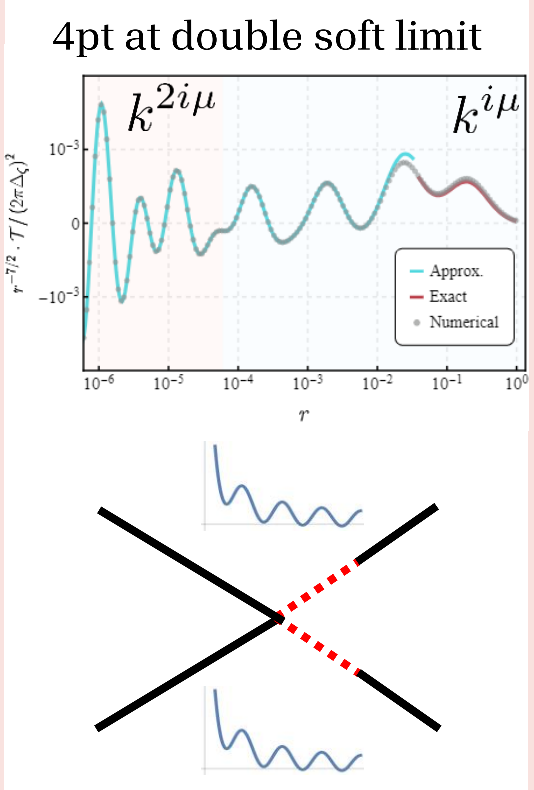
- Double-exchange is naturally large

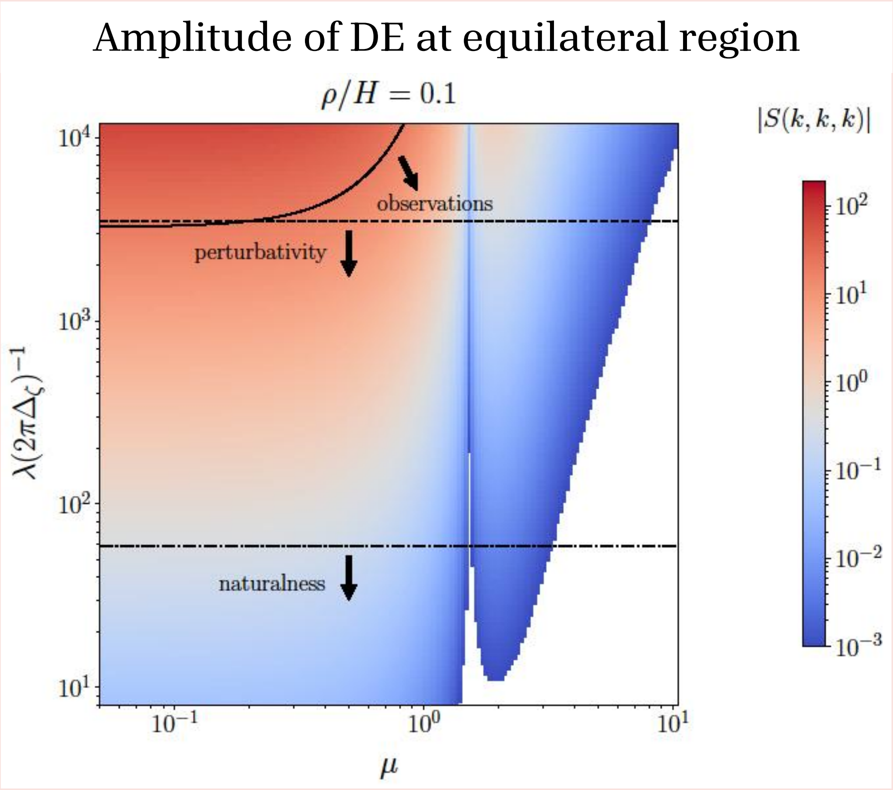
$$\frac{S_{\rm DE}}{S_{\rm SE}} \sim \lambda P_\zeta^{-1/2}\frac{1}{\mu^2} \lesssim P_\zeta^{-1/4}\frac{1}{\mu^2}$$

- Method: bootstrap equations



with MB rep. for bd conditions





Back-up

Difference between SE and DE: Phase Information

[Aoki et al. 2404.09547]

◆ Phase information

➢ SE and single isocurvature DE

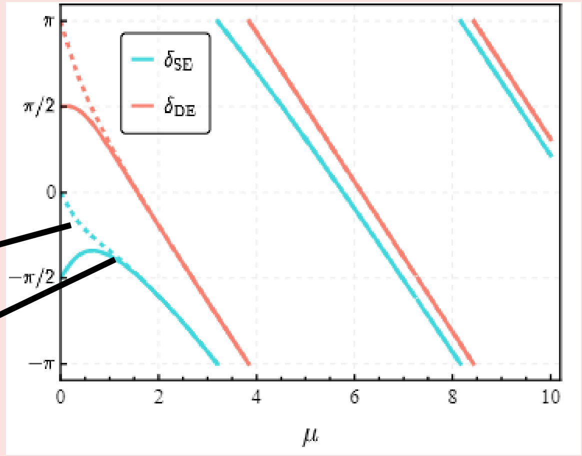
$$S^{\text{single}}_{\text{DE,CC}} = \frac{\rho^2}{H^2}\frac{\lambda}{2\pi P_\zeta^{1/2}}\text{Re}\left[\left(\frac{k_L}{k_S}\right)^{1/2+i\mu}\mathcal{A}_{\text{DE}}(\mu)e^{i\delta(\mu)}\right]$$

$$S_{\text{SE,CC}} = \frac{\rho^2}{\dot\phi}\frac{1}{2\pi P_\zeta^{1/2}}\text{Re}\left[\left(\frac{k_L}{k_S}\right)^{1/2+i\mu}\mathcal{A}_{\text{SE}}(\mu)e^{i\delta(\mu)}\right]$$

Consistency between phase and wavelength

Leading of μ

Full order



➢ DE with multiple isocurvature modes

$$S^{\text{multi}}_{\text{DE,CC}} = \sum_{\alpha,\beta}^{N}\frac{\rho_\alpha\rho_\beta}{H^2}\frac{\lambda_{\alpha\beta}}{2\pi P_\zeta^{1/2}}\text{Re}\left[\left(\frac{k_L}{k_S}\right)^{1/2+i\mu_\alpha}\mathcal{A}_{\mu_\alpha,\mu_\beta}e^{i\delta_{\mu_\alpha,\mu_\beta}}\right]$$

$$= \sum_{\alpha}^{N}\frac{\rho_\alpha}{H}\text{Re}\left[\left(\frac{k_L}{k_S}\right)^{1/2+i\mu_\alpha}\mathcal{B}_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}e^{i\vartheta_{\mu_\alpha,\mu_\beta,\lambda_{\alpha\beta},\rho_\beta}}\right]$$

$$\left(a\sin\theta + b\sin(\theta+\Delta\theta) = \sqrt{a^2+b^2+2ab\cos\Delta\theta}\,\sin(\theta+\alpha)\right)$$

✓ Information in squeezed limit

# of observables		$\leq$	# of parameters	
✓ Amplitude	N		✓ ρ_α	N
✓ Wavelength	N		✓ μ_α	N
✓ Phase	N		✓ $\lambda_{\alpha\beta}$	$N(N+1)/2$