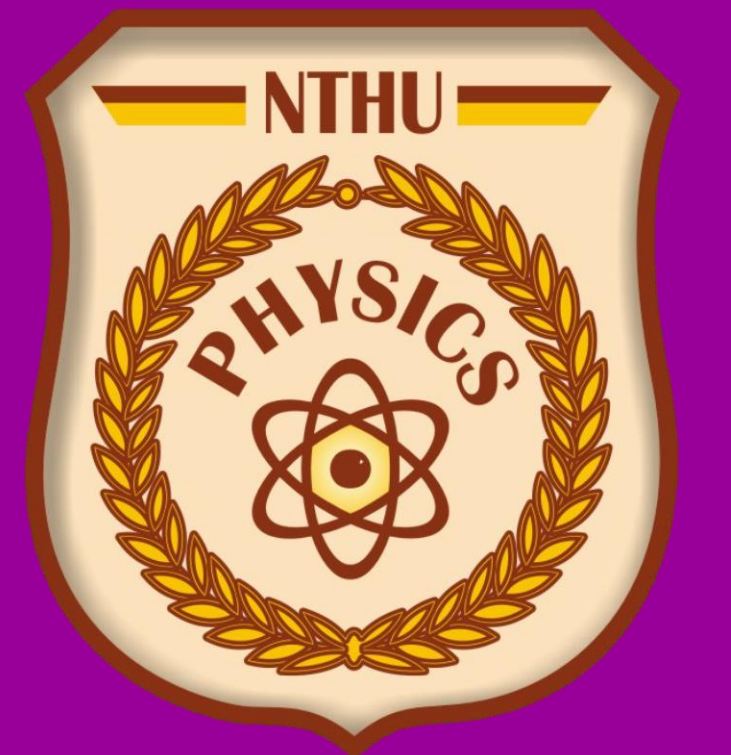




Properties and Implications of Milling Behavior in Active Particles with Anisotropic Vision



Hsin-Yu Chen (陳欣佑) and Kuo-An Wu (吳國安)

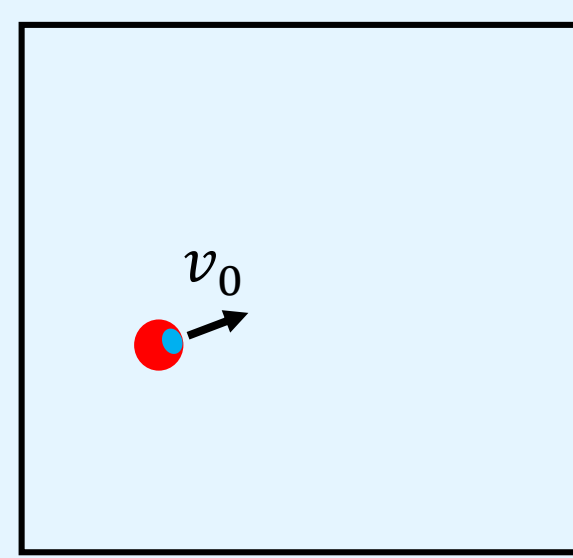
Department of Physics, National Tsing Hua University, Taiwan

Collective behaviors of living matter are a common phenomenon in nature, observed in flocks of birds, schools of fish, and bacterial communities, among others. Milling behavior—such as the vortex-like motion observed in reindeer herds, fish schools, and army ant colonies—represents a striking example of collective dynamics in nature. Animals form a milling structure spontaneously and maintain it continuously. Based on the Vicsek model (1995) [1] and the idea from Costanzo *et al* [2], we propose a minimal model that exhibits milling behavior in active Brownian particle (ABP) simulations. The vision angle of particles, depicting how broad in front an individual pays attention to and responds to, is a key factor in alignment for milling to form. It shows that the milling forms on a relatively narrow range of the vision angle; on the other hand, the system prefers flocking states with larger vision angles and aster states with smaller vision angles.

Model for Active Brownian Particles with Alignment

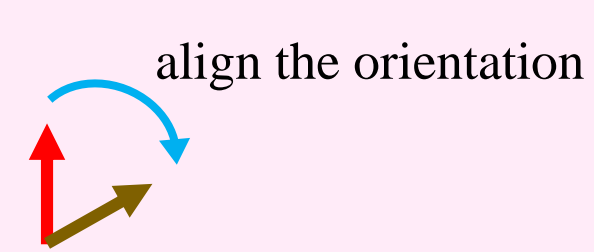
$$\begin{aligned}\dot{\mathbf{r}}_i(t) &= \frac{1}{\gamma} \left[F_a \mathbf{e}_{\theta_i} + \sum_{j \neq i} (-\nabla_i U(\mathbf{r}_{ij})) \right] + \sqrt{2D_t} \boldsymbol{\eta}_i \\ &= v_0 \mathbf{e}_{\theta_i} + \frac{1}{\gamma} \sum_{j \neq i} \mathbf{F}(\mathbf{r}_{ij}) + \sqrt{2D_t} \boldsymbol{\eta}_i \\ \dot{\theta}_i(t) &= \frac{Q}{M} \sum_{\substack{j \in [-\phi/2, \phi/2] \\ j \in [0, R]}} \sin(\theta_j - \theta_i) + \sqrt{2D_r} \xi_i\end{aligned}$$

1. Constant self-propelled speed v_0



2. Tend to align with their neighbors.

- Follow the herd
- Avoid collision

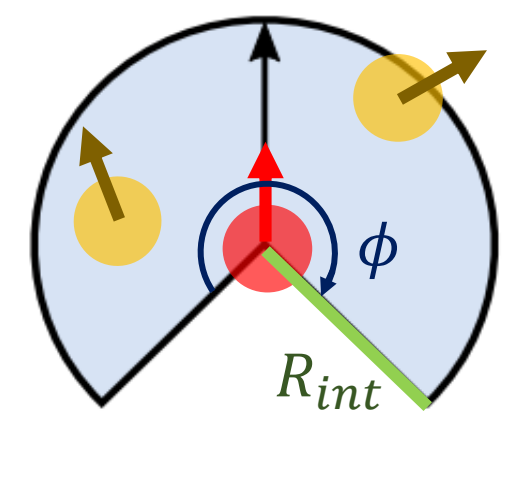


Neighbor selection

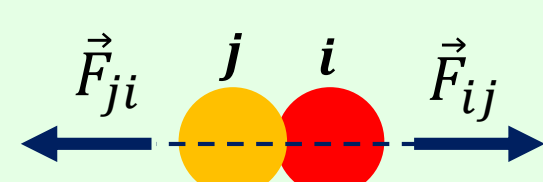
Vision range:

- Vision angle ϕ
- Vision distance R_{int}

→ Anisotropic Vision



3. Volume exclusion (steric repulsion)



WCA potential (only repulsion)

$$U(\mathbf{r}_{ij}) = \begin{cases} 4\epsilon \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^6 \right] + \epsilon & \text{for } r_{ij} \leq 2\frac{1}{2}\sigma \\ 0 & \text{for } r_{ij} > 2\frac{1}{2}\sigma \end{cases}$$

(σ : diameter of the particle.)

Our particles are under an overdamped condition, with strong self-propulsion, strong alignment strength, and low diffusion, similar to hard disks, so they cannot penetrate each other.

Important parameters:

- ϵ : Strength of interparticle potential.
- Pe : Péclet number.
- Q : Alignment strength.
- R_{int} : Vision distance.
- ϕ : Vision angle.
- N : Number of particle in a system.

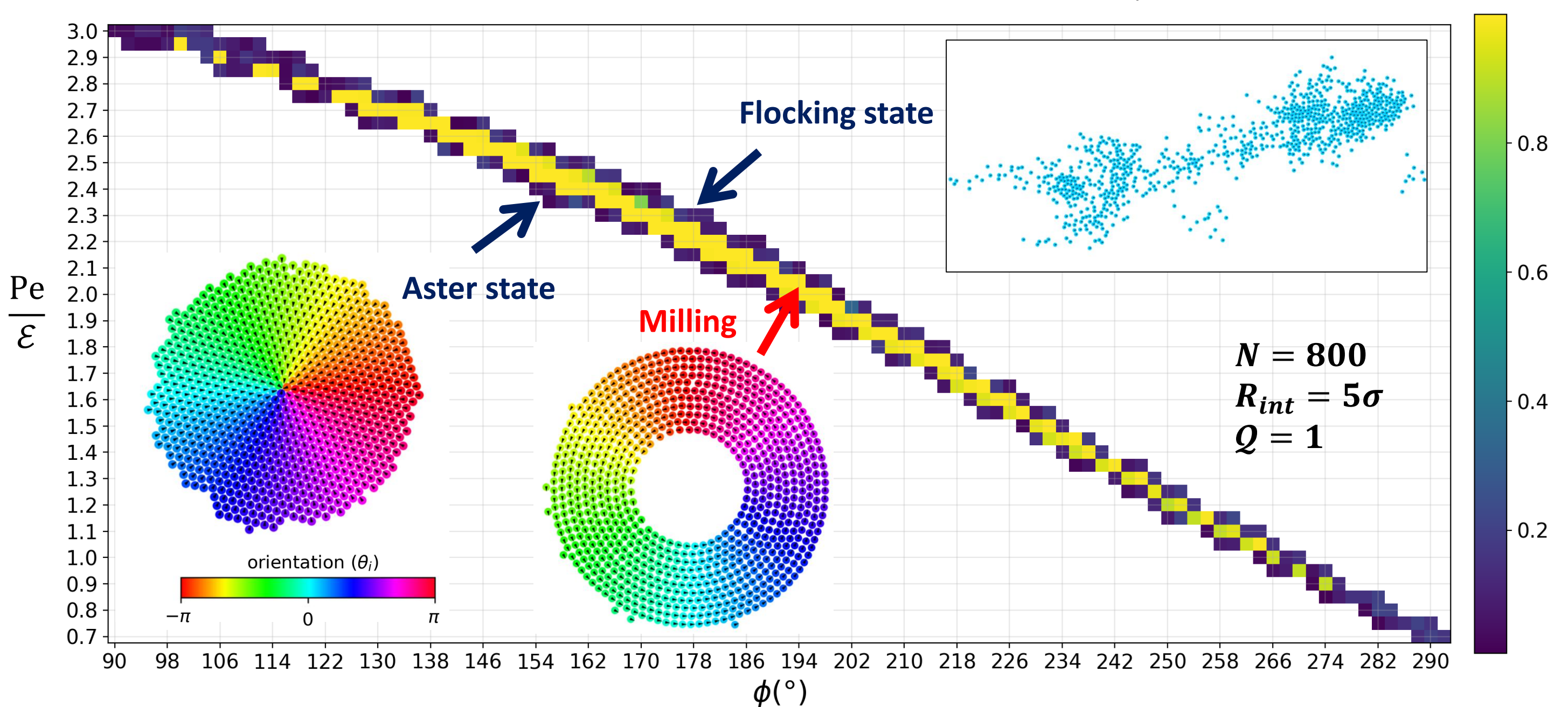
Dimensionless equations:

$$\begin{aligned}\dot{\mathbf{r}}_i(\hat{t}) &= \frac{Pe}{\epsilon} \mathbf{e}_{\theta_i} + \sum_{j \neq i} \frac{1}{r_{ij}} 4 \left[12 \left(\frac{1}{r_{ij}} \right)^{12} - 6 \left(\frac{1}{r_{ij}} \right)^6 \right] \frac{\mathbf{r}_{ij}}{r_{ij}} + \sqrt{\frac{2}{\epsilon}} \boldsymbol{\eta}_i \\ \dot{\theta}_i(\hat{t}) &= \frac{Q}{M} \frac{1}{M} \sum_{\substack{j \in \text{Sector} \\ j \in [0, R]}} \sin(\theta_j - \theta_i) + \sqrt{\frac{6}{\epsilon}} \xi_i\end{aligned}$$

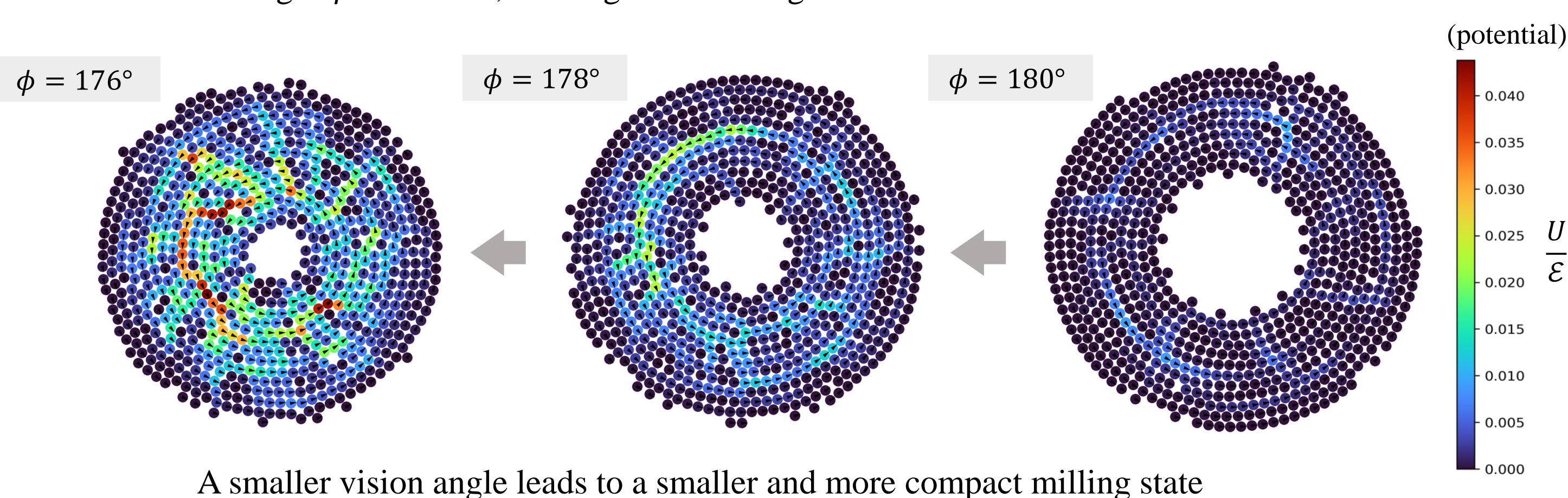
Phase Diagram

Vortex order parameter $0 \leq M \leq 1$

$$M = \frac{1}{N} \left| \sum_i \frac{(\mathbf{r}_i - \mathbf{r}_{CM}) \times \mathbf{v}_i}{|\mathbf{r}_i - \mathbf{r}_{CM}| |\mathbf{v}_i|} \right|$$

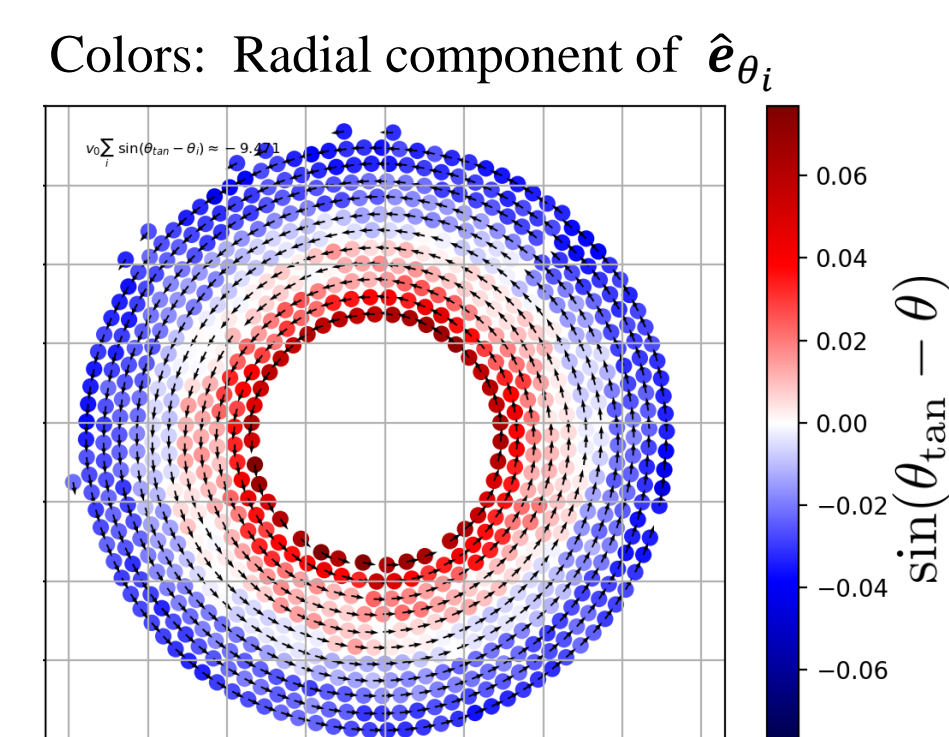


- For higher activity (Pe) (self-propelled speed), particles need smaller vision angle to maintain a milling structure.
- Smaller vision angle ϕ gives rise to a better alignment and results in a higher angular velocity of orientation $\dot{\theta}$.
- As the vision angle ϕ decreases, milling \rightarrow aster.
- As the vision angle ϕ increases, milling \rightarrow flocking.



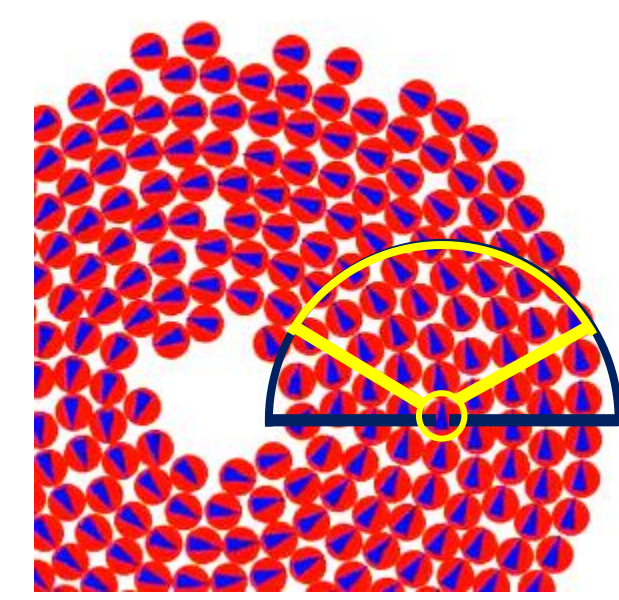
A smaller vision angle leads to a smaller and more compact milling state

How can the milling state be stable?



- The inner particles face slightly outward, while the outer particles face slightly inward, resulting in a force balance in the radial direction for milling structures.

Effects of individuals with different vision angles



“Sandwiched” Milling State

Circular Motion for a fixed radius:

$\dot{\theta}$ (angular velocity of orientation) from alignment effect should match the orbital angular velocity $\omega = \frac{v}{r}$.

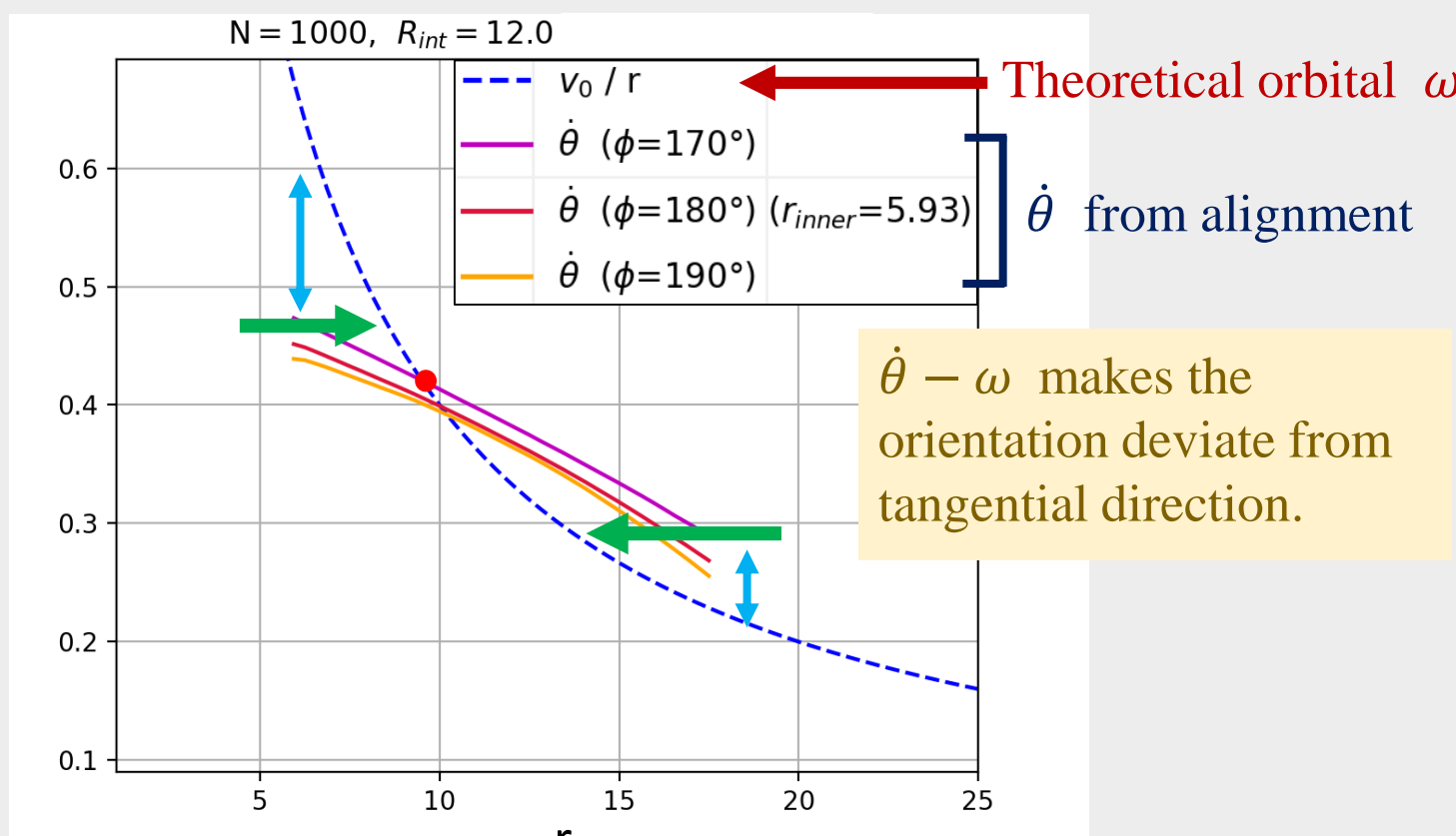
Perspective 1 (see green arrows shown in the figure):

- Outer particles with $\dot{\theta} > \omega$ prefer smaller r .
- Inner particles with $\dot{\theta} < \omega$ prefer larger r .

Perspective 2 (see cyan arrows shown in the figure):

- Outer particles with $\dot{\theta} - \omega > 0$ turn their orientations more than needed for orbiting, and then face inward.
- Inner particles with $\dot{\theta} - \omega < 0$ turn their orientations less, and then face outward.

Theoretical calculation of $\dot{\theta}$ using perfect hexagonal lattice with tangential orientations



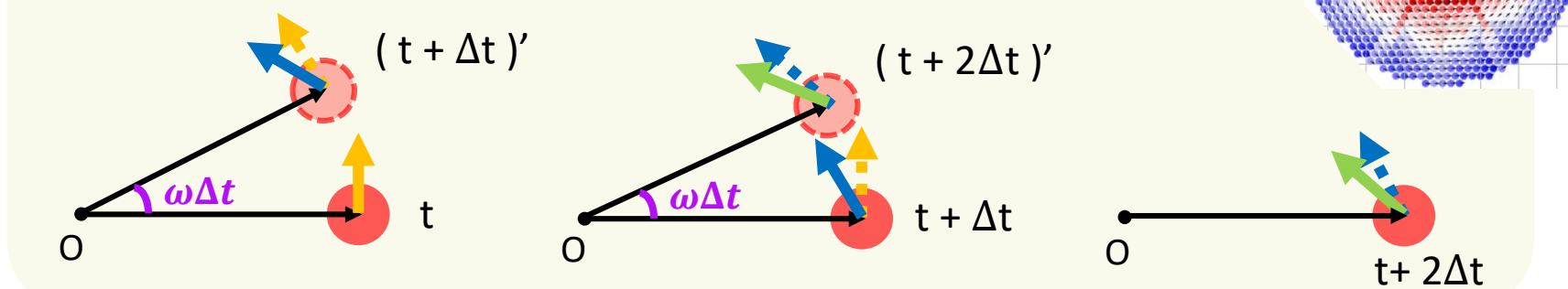
Theoretical Prediction for Geometry of Milling State

By considering (1) the resultant tangential speed of active particles due to shear motion and (2) the compactness of particles due to compressive radial stress, the theoretical prediction of the milling state geometry is as follows.

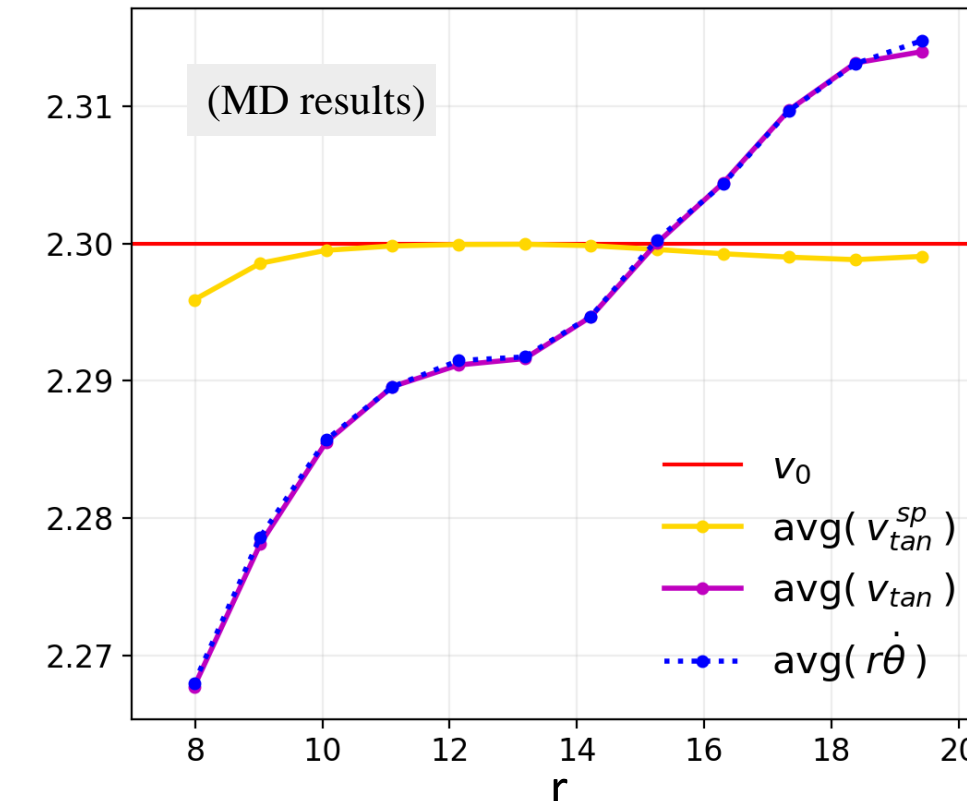
$$\dot{\theta}_i = \left[\frac{Q}{M} \sum_{\substack{j \in [-\phi/2, \phi/2] \\ j \in [0, R]}} \sin(\theta_j - \theta_i) \right] - \frac{v_{tan}(r_i, \theta_j)}{r_i}$$

original alignment term $\dot{\theta}$ ω

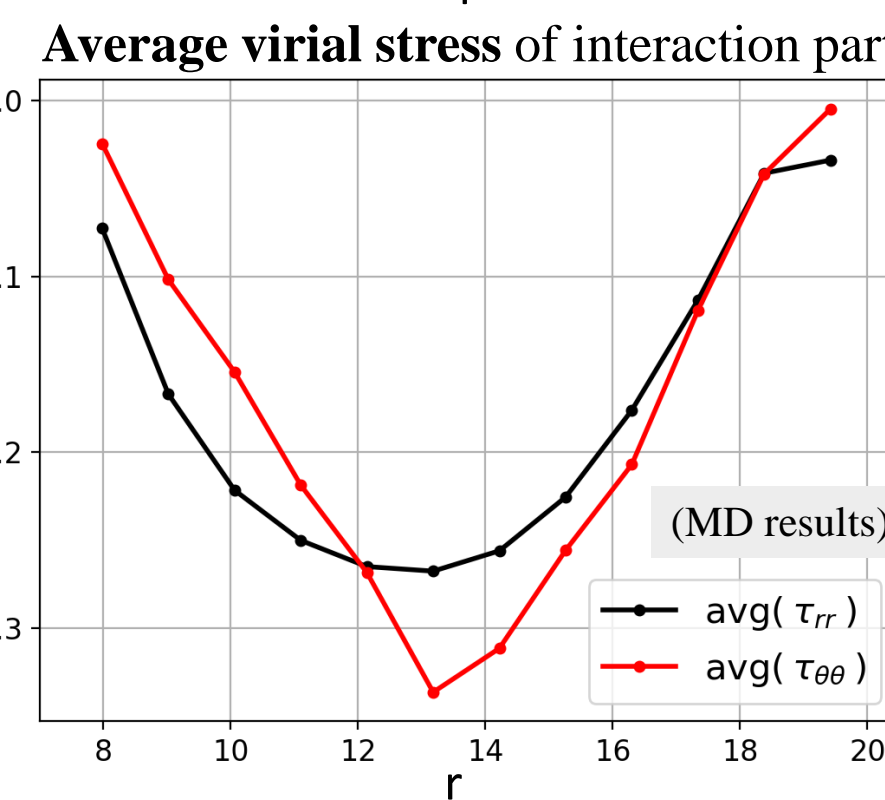
Illustration of orientation evolution according to the rule on the left.



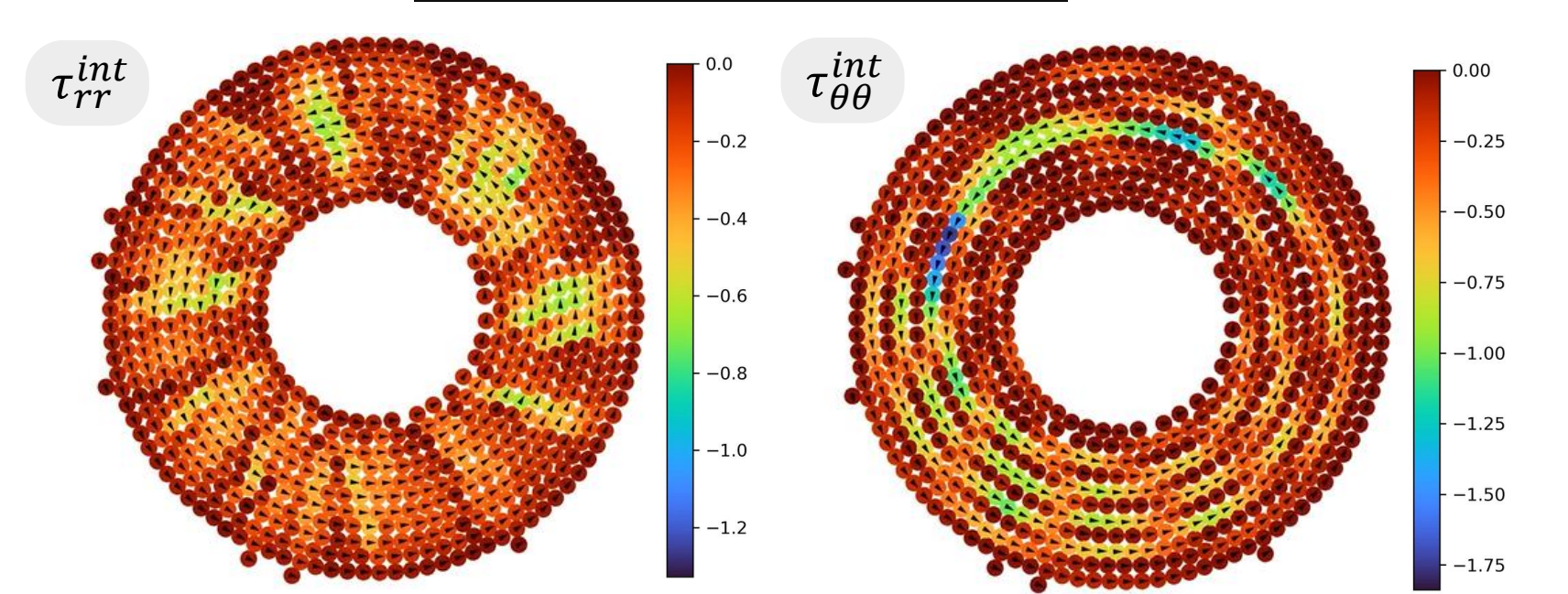
Tangential Velocity Profile



- By the difference between $\dot{\theta}$ (orientation) and ω (orbital), we know how much additional angular velocity $\dot{\theta}'$ will cause particles to deflect relative to the ring, and then allow their orientations to evolve at fixed positions.
- Although each particle is almost moving tangentially, on average, the tangential velocity on the inner ring is slightly less than v_0 , while on the outer ring it is slightly greater than v_0 , due to the shear motion in adjacent layers and the deflection of orientations.
- The compression in the radial direction also plays an important role for particles on the middle ring.
- It can be used to predict the transition phase boundary from milling to aster states.



Instantaneous virial stress



Conclusion & Evolutionary Implication

- A minimal agent-based model is proposed to rediscover milling structures observed in fish mills, bait ball, and reindeer cyclone, which are found to be an efficient defensive strategy from predators.
- We show that active particles self-organized into a milling structure as the vision angle and activity fall in a certain parameter space. According to psychological studies [3-4], when organisms are under threatening events and stress, their attention becomes more focused and narrowed (likely the change in activity and vision span in our case).
- For example, animals may be initially at rest, and after sensing the predator, they start moving (or increase their speed) and milling. Another scenario is that when animals are threatened, they get nervous, shrinking the vision angle, and changing the state from flocking to milling.
- The mixture of particles of two species with different velocities or different vision angles may provide additional evolutionary implications.

Reference

- [1] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen, and O. Sochet, “Novel type of phase transition in a system of self-driven particles”, *Phys. Rev. Lett.* **75**, 1226 (1995).
- [2] A. Costanzo and C. K. Hemelrijk, “Spontaneous emergence of milling (vortex state) in a Vicsek-like model”, *J. Phys. D: Appl. Phys.* **51**, 134004 (2018).
- [3] J. A. Easterbrook, “The effect of emotion on cue utilization and the organization of behavior”, *Psychol. Rev.* **66**, 183–201 (1959).
- [4] H. van Steenbergen, G. P. H. Band, and B. Hommel, “Threat but not arousal narrows attention: evidence from pupil dilation and saccade control”, *Front. Psychol.* **2**, 281 (2011).