Beyond Calabrese–Cardy Scaling: Exceptional-Point Sensitivity and its de Sitter RT Origin

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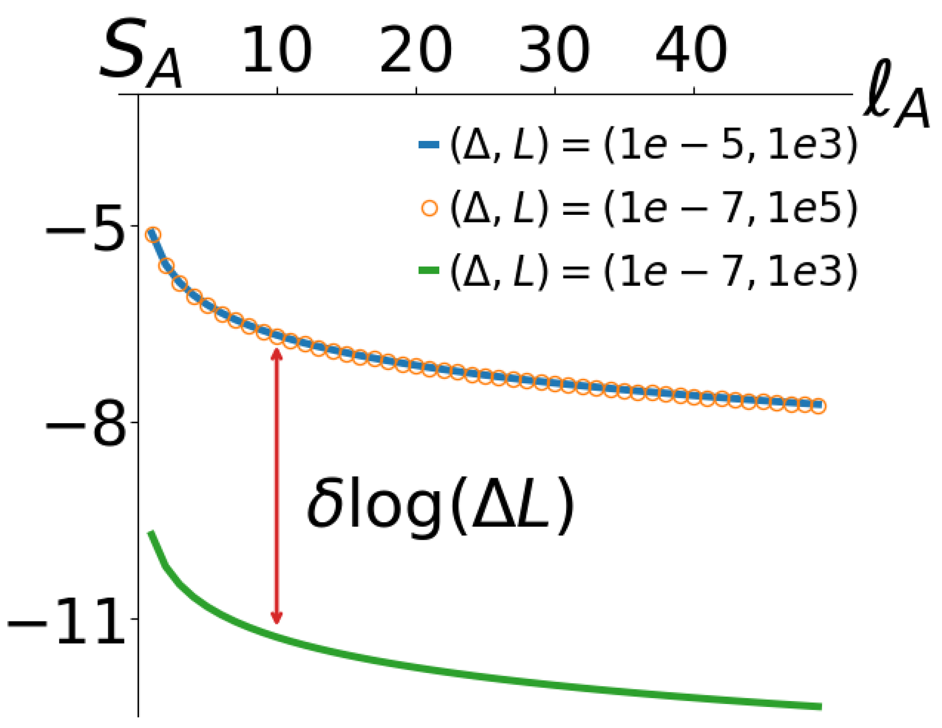
Motivation

Critical entanglement usually follows the **Calabrese–Cardy scaling** [1].

Here we identify a new **beyond-Cardy** contribution, $\log(\Delta L)$, in non-unitary critical chains near exceptional points.

It produces an **ℓ-independent offset** and reveals an unusual IR sensitivity **absent in Hermitian systems.**



$$S_A(\ell) = \underbrace{\frac{c}{3}\log\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)\right]}_{\textbf{Critical Scaling}} + \underbrace{\log(\Delta L)}_{\textbf{Residual Entropy}} + \cdots .$$

We trace the origin of this long-range entanglement through **holography** and **entanglement renormalization.**

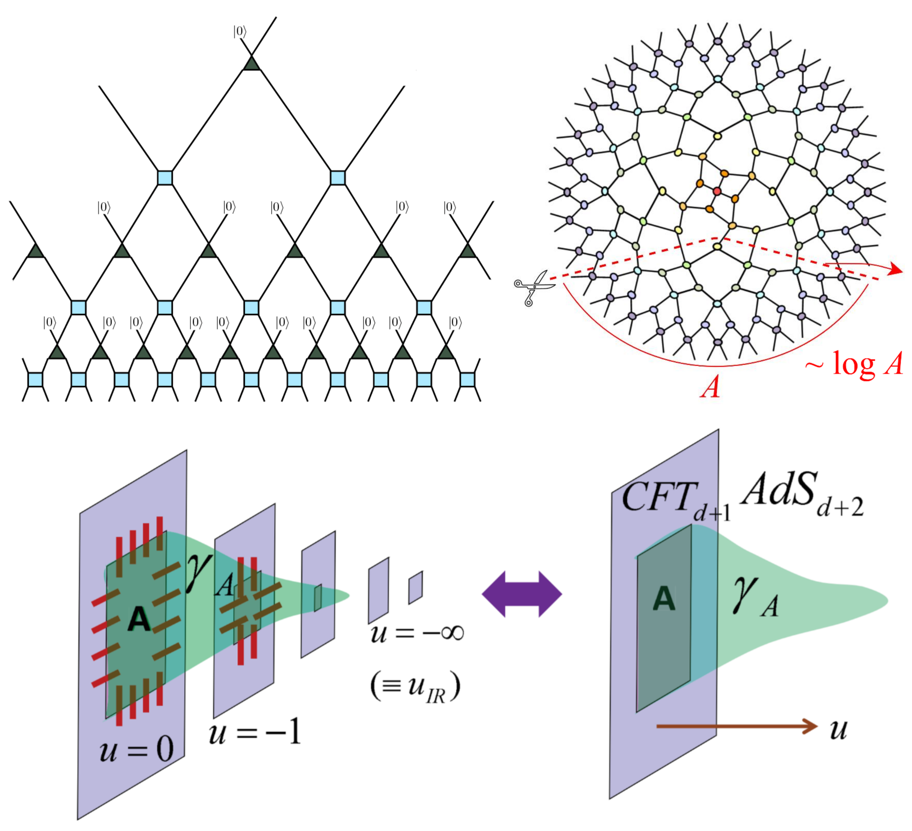
MERA and holography

MERA is a real-space entanglement renormalization scheme. [2]

Its entropy can be estimated by a **minimal bond cut**, reproducing the critical scaling.

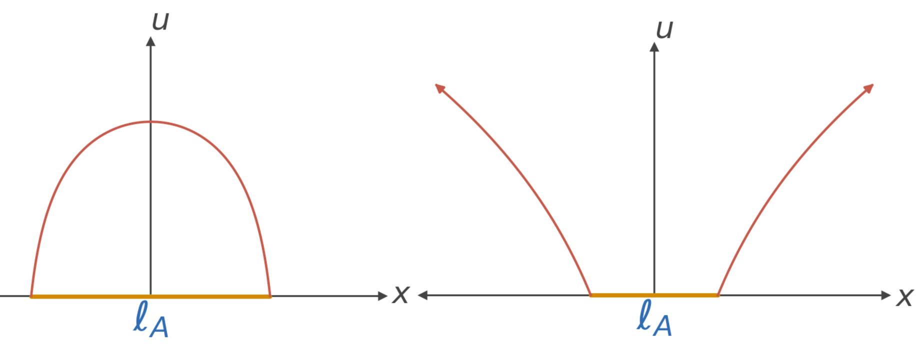
This directly parallels the **Ryu–Takayanagi formula** in holography [3]

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N},$$



which motivates the **AdS/MERA** correspondence [4,5].

For non-unitary/**non-Hermitian** critical systems, the dual geometry becomes **de Sitter**.



Although dS again admits an RT / minimal-cut picture, its extremal surface **opens outward toward the IR end**, rather than returning to the boundary [6].

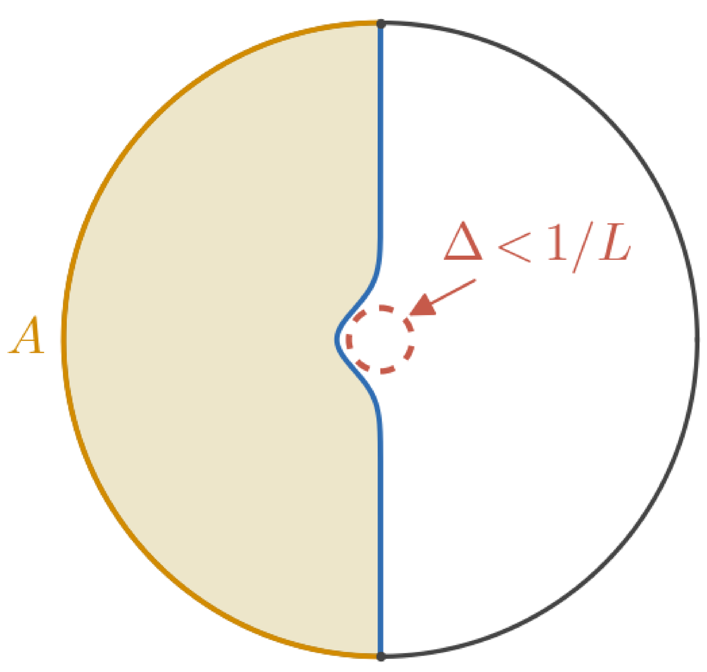
This qualitative difference underlies the **extra residual term.**

Unavoidable IR divergence

For a finite-size system, the real-space RG circuit has only a **finite depth $\sim \log L$.**

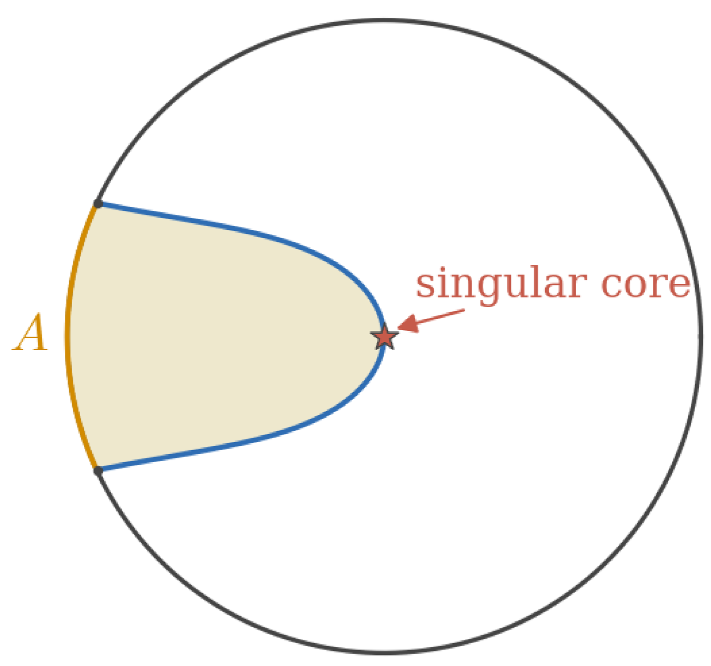
> A critical system cannot be fully disentangled by a finite-depth circuit.

Forcing a finite-depth circuit to end in a **fully disentangled IR** state therefore produces an **IR singularity.**



In **AdS**, the geodesic can **detour** this singularity. Thus, we **cannot resolve** a **gap below $1/L$** in Hermitian systems.

In **dS**, the geodesic is instead driven toward the IR endpoint, so this singularity becomes **unavoidable.**



To obtain a non-singular description, we instead retain a **partially disentangled IR state** $|\Psi_{IR}\rangle$ for non-Hermitian systems.

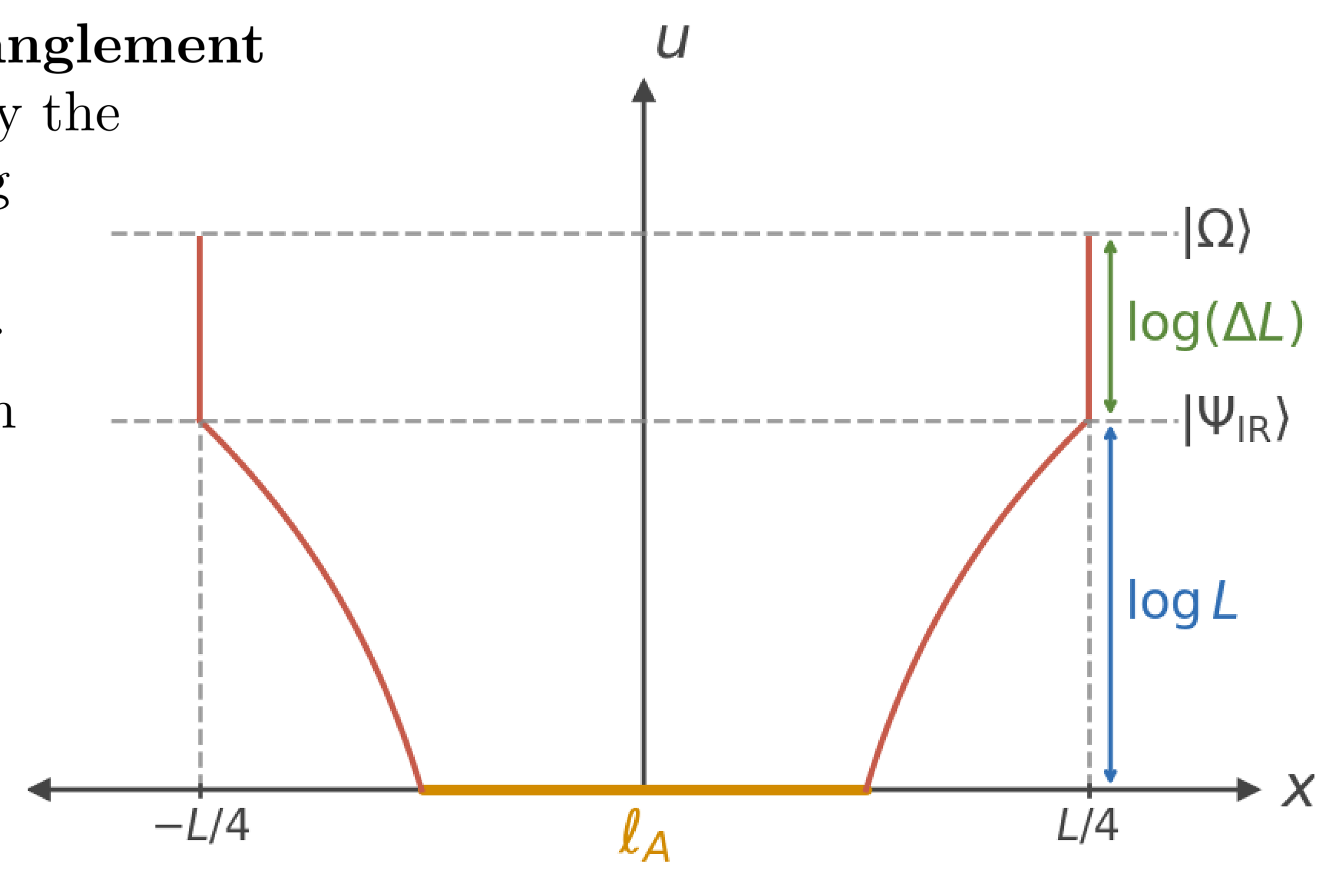
Squeezed geodesic in finite system

The **residual entanglement** can be estimated by the geodesic connecting $|\Psi_{IR}\rangle$ to the true reference state $|\Omega\rangle$.

The geodesic length corresponds to the **residual rotation**

$$2\int ds = \Delta\phi_{\text{res}}$$

yielding



$$\Delta\phi_{\text{res}} = \phi_{k=0} - \phi_{k=\pi/L} = i\ln\left(\frac{\Delta L}{\pi}\right).$$

Geometrically, finite size limits the outward extension of the geodesic, squeezing the excess part into an additional vertical segment.

> The geodesic toward past infinity is finite only because **spatial and temporal contributions cancel.** Finite size cuts off the spatial part and leaves an extra **vertical IR contribution $\sim\log(\Delta L)$.**

Residual entropy in IR state

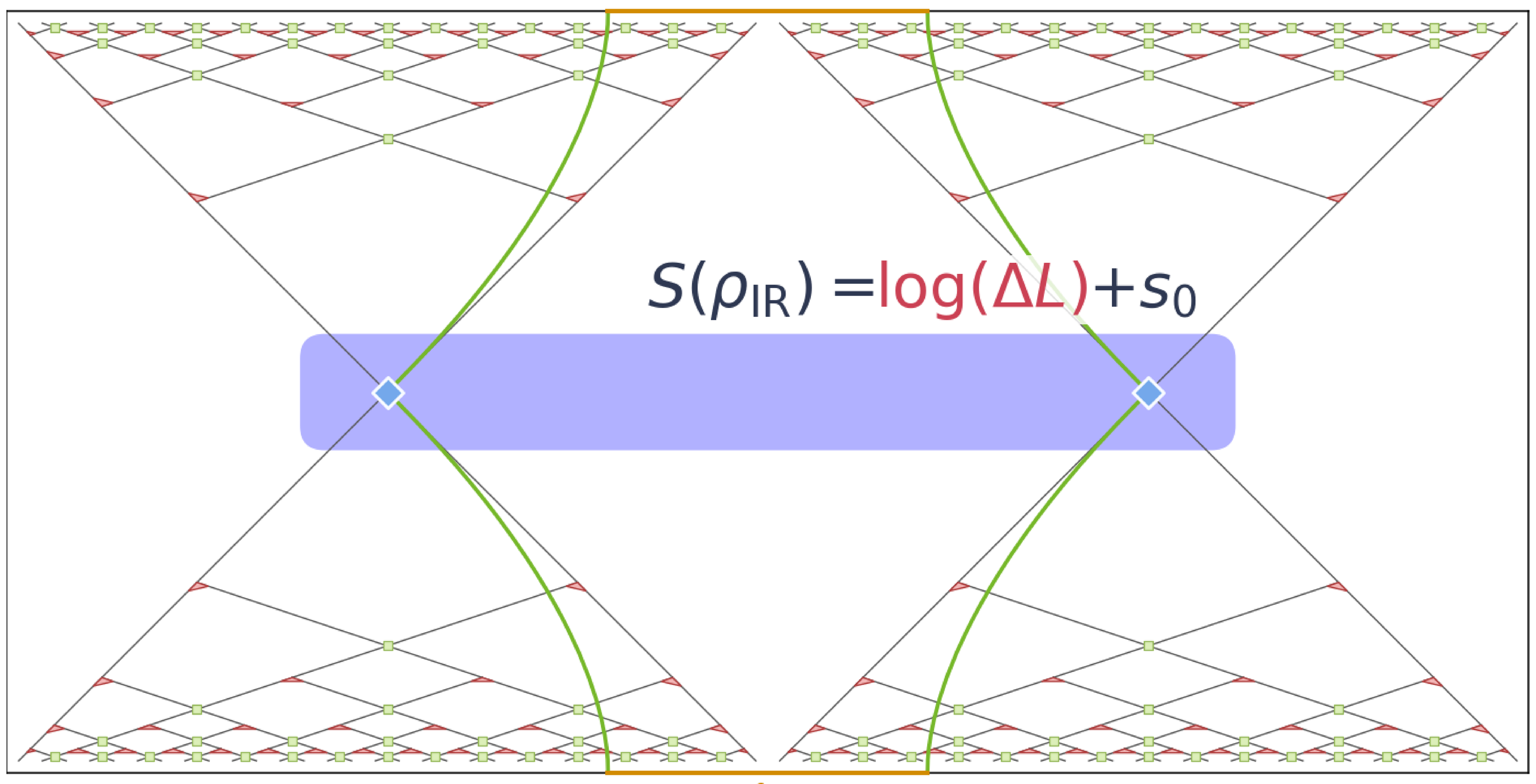
The **finite-depth IR state $|\Psi_{IR}\rangle$** is derived explicitly in cMERA as a two-particle state

$$|\Psi_{IR}\rangle = \psi_R'^{\dagger}(0)\,\psi_R'^{\dagger}(\pi)\,|0\rangle\,.$$

The corresponding phase are

$$\phi'(\pi) = \phi(\pi), \qquad \phi'(0) = \phi(0) - \Big[\phi_{k=\pi/L} - \phi_{k=\pi}\Big],$$

The entanglement carried by this **surviving IR pair** is then



Because outward RT curves for different boundary intervals all converge to the **same IR poles**, they intersect the same surviving pair. This makes the residual contribution **ℓ-independent.**

Reference

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