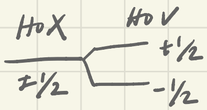
Kramer's Degeneracy (Zero field splitting)

- e^- ↑ 1/2, ↓ −1/2 → spin — an electron can be arranged either upward/downward direction

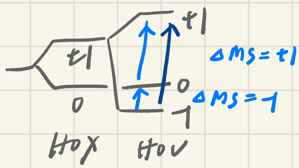


- by adding magnetic field, these energy levels will split; splitting is taking place only in the magnetic field

- If splitting take place in the absence of Ho also → Kramer's degeneracy; we can observe in metals which have more than 1 electron
- d^2 system (2 electrons)

  ↑↑ +1
  ↑↓ 0
  ↓↓ −1
  ] ±1

  can be arranged in 3 ways → 3 different spins; in absence of Ho, it has 2 energy level: ±1, 0



ΔMs = 0 → forbidden

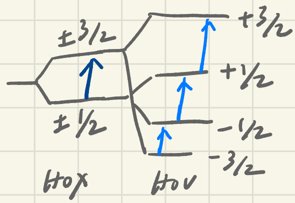
There are 2 selection rules:

ΔMi = 0 → nuclear transi.

ΔMs = ±1, ±2, ±3 … → spin transi (we discuss this)

- d^3 system

  ↑↑↑ +3/2
  ↑↓↑ +1/2
  ↓↑↓ −1/2
  ↓↓↓ −3/2



in EPM spectroscopy, we'll get 3 lines

- in Kramer's degeneracy, in absence of Ho, we can get 1 line

- d^5 system

  Spins: +5/2, +3/2, +1/2, −1/2, −3/2, −5/2 (6 lines) 同理

  energy levels: ±5/2, ±3/2, ±1/2 (3 lines)

- **Def.** (a quantum mechanical principle)

  for any system with ① time-symmetric Hamiltonian * ② odd num of half-integer spin particles (eg. e^-)

  all energy levels must be at least double degenerate

*: time reversal operator T satisfy $T^2 = -1$ → for an eigenstate ψ (psi), the time-reversed state Tψ is distinct, but has same energy.

Application I: band structure

T. Time-Reversal symmetry: $E_\uparrow(k) = E_\downarrow(k)$ — flip both momentum k and spin

- case 1: materials with "Inversion Symmetry" P
  flips "only k" but not spin: $E_\uparrow(k) = E_\uparrow(-k)$
  ⇒ combinding $T \rightarrow E_\uparrow(k) = E_\downarrow(k)$
  ⇒ Twofold spin degeneracy exist.
- case 2: materials without P
  band split: $E_\uparrow(k) \neq E_\downarrow(k)$
  but at TRIM points $(k=0)$, $E_\uparrow(TRIM) = E_\downarrow(TRIM)$

Application II: Anderson thm.

- superconductivity
  "cooper pairs" electrons pair up in time-reversed states $|k,\uparrow\rangle$ and $|-k,\downarrow\rangle$
- problems!: "Impurities"
  e^- ↗ scatter, would randomize k
  theoretically would destroy cooper pairs
- Thm: non-magnetic impurities do not break TRS
  since TRS preserved, kramer's dege guarentees that for any destroted $|\psi\rangle$, there exist an exact degenerate, TR partner $T|\psi\rangle$

  ⇒ non-magnetic impurities won't decrease T_c of superconductors.
  but if magnetic impurities, T broken → kramer's degen dissapear
  → superconductor will die.