Magnetic Point Group

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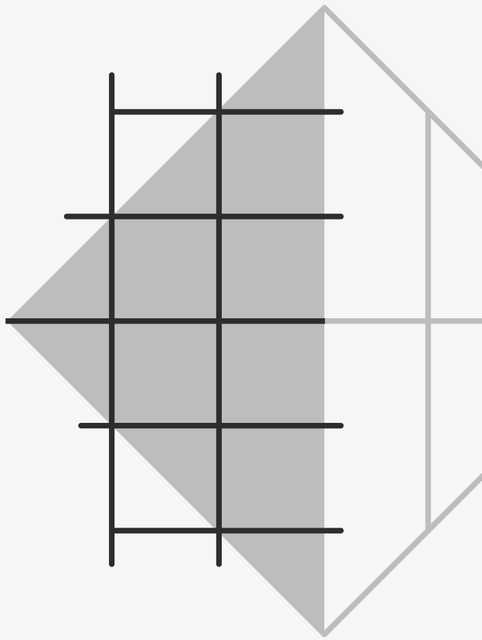
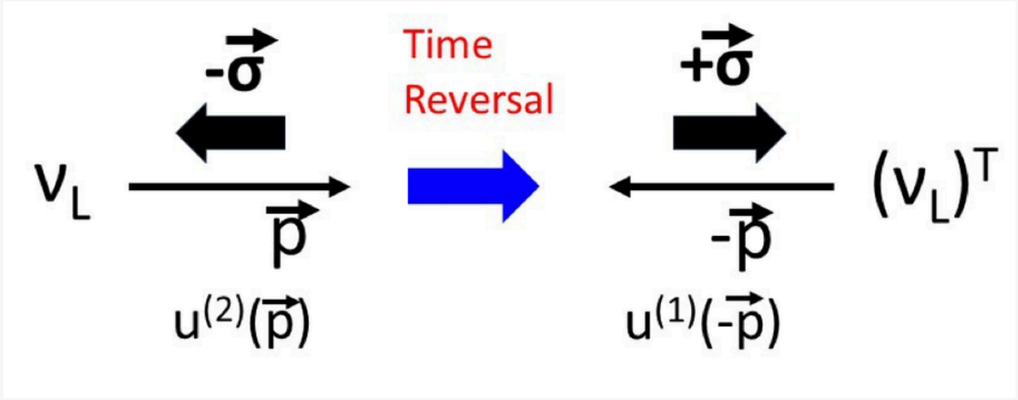
Overview

1. Time-Reversal Operator
2. Point Group
3. Applications



Time-Reversal Operator



- Reverses the direction of time, thereby reversing momentum (p→-p) and angular momentum / spin (S→-S).
- The reversal of currents and spins leads to the reversal of the macroscopic magnetization vector: TM=-M.
- Commutes with spatial rotation operations, [T,R] = 0.

Point Group

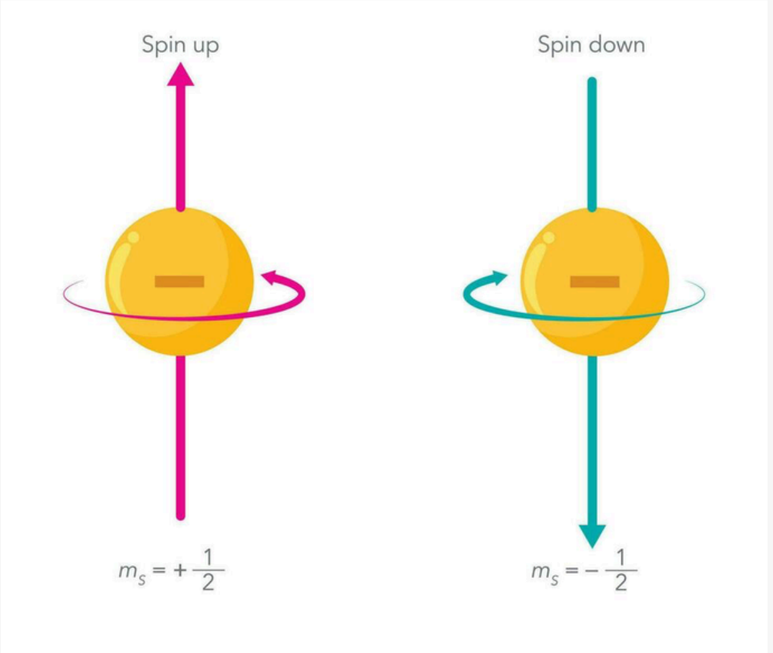
- Definition: A set of symmetry operations that leave at least one fixed point in space unchanged.
- Basic Building Blocks: Rotation, reflection, and inversion.
- Physical Constraints: Coupled with the restriction of lattice translation, only 32 standard crystallographic point groups can be formed in three-dimensional space.

Point Group

- Type I (Colorless Groups): The conventional point group G itself. It does not possess time-reversal symmetry. (32 groups)
- Type II (Gray Groups): The crystal perfectly possesses time-reversal symmetry T. (32 groups)

$$M_{II} = G \cup TG$$

These operations only deal with spatial coordinates (x, y, z), completely ignoring time and spin!



Type III (magnetic point group) ⇒ 58 groups

- To describe magnetic crystals

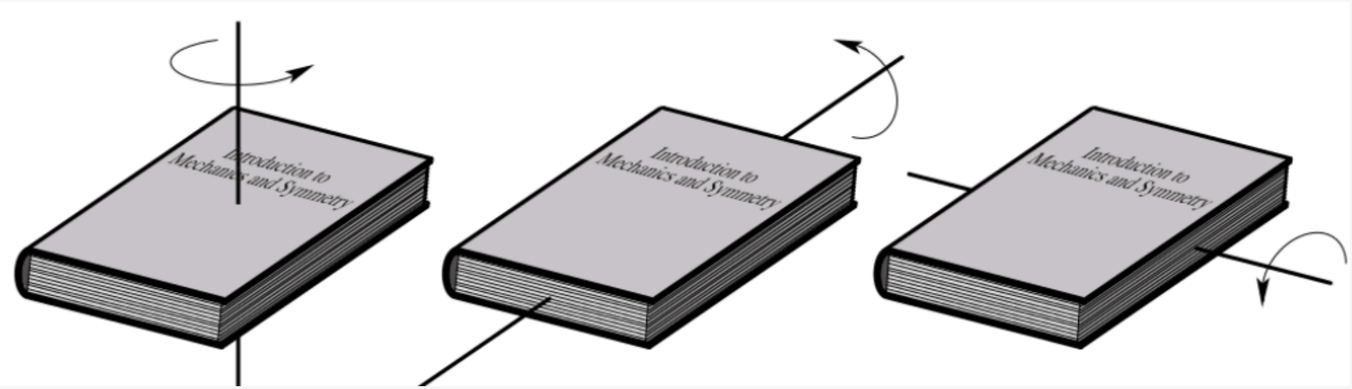
⇒ incorporate the time-reversal operator T

- This combined space-time extension gives rise to Shubnikov groups, also known as Magnetic Point Groups.
- Physical Significance: There is no pure T operation within the group; time-reversal T must appear coupled with specific spatial operations (such as translation or rotation).

$$G = H \cup aH$$

$$M_{III} = H \cup T(aH)$$

Example



Hermann–Mauguin notation:

- “1" represents "doing nothing" (Identity element)
- “2" represents "180-degree rotation".
- “m” represents "reflection" (mirror plane).
- ' (prime) represents time-reversal.

⇒ A rectangular cuboid has “222” (three 2-fold axes), “2/m” (2-fold axis + perpendicular mirror plane), and “mmm” (three mirror planes).

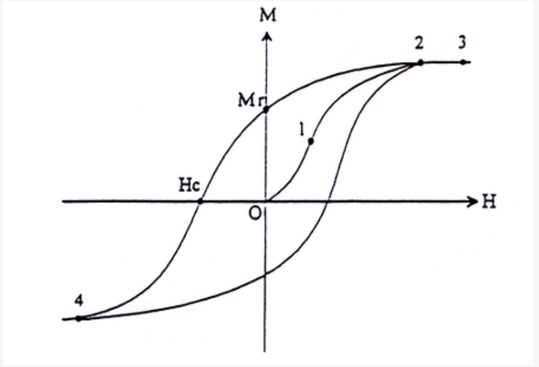
Applications

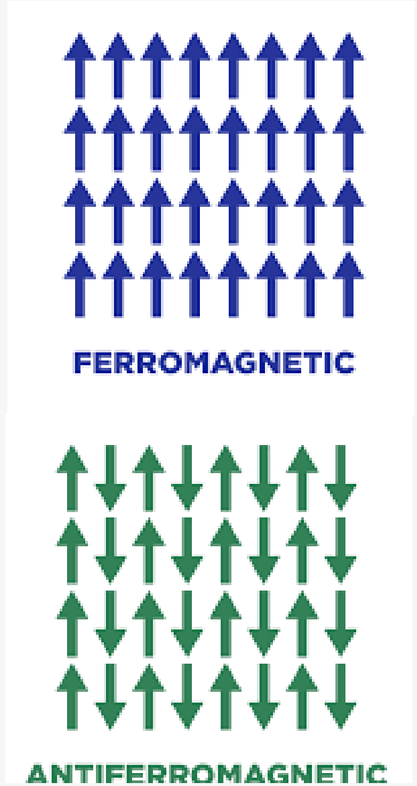
Neumann's Principle

Any macroscopic physical property of a crystal must obey "all the symmetries" of the point group to which the crystal belongs

$$R(P) = P$$

1. Ferromagnetic or Not?





1.If the crystal belongs to Type II (Gray Groups):

- Symmetry requirement: T(M)=M & Physical requirement: T(M)=-M
- Unique solution: M=-M ⇒ M=0. (Strictly forbids ferromagnetism!)

2.If the crystal belongs to Type III:

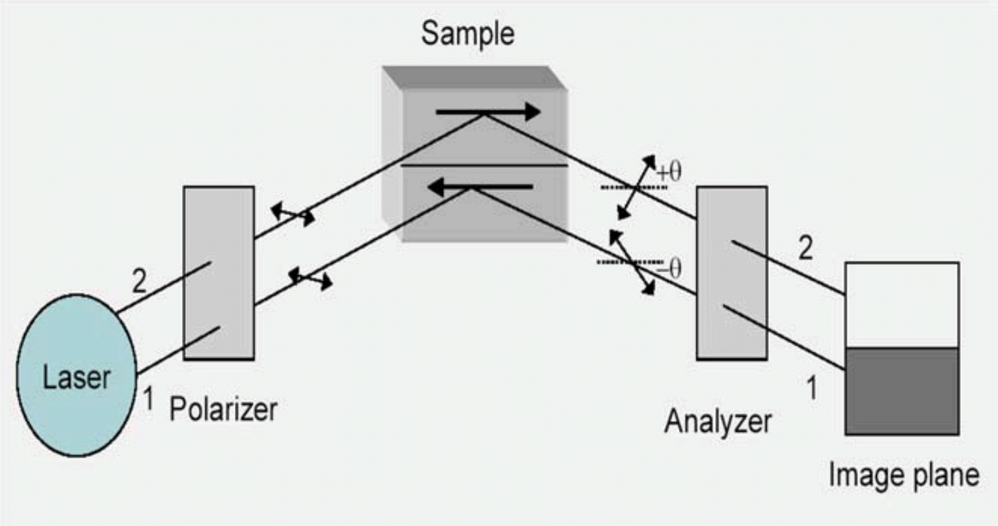
- Since there is no pure T operation within the group, M is not forced to be zero. As long as M remains invariant under a specific combined space-time symmetry T(aH), ferromagnetism is allowed to exist.

3.Example:

- Iron at high temperatures (non-magnetic) → $Im\bar{3}m1'$ ⇒ Type II
- Iron at room temperature (ferromagnetic) → 4/m**m'm'** ⇒ Type III

2. Magneto-optic Kerr effect (MOKE)

- When light reflects off the surface of a magnetic material, its polarization plane rotates, changing the direction of polarization.
- Determined by the off-diagonal elements of εij.
- For a Gray Groups, these off-diagonal elements must be zero making MOKE impossible to occur.



Magnetic moments in different directions cause the polarization of light to rotate by different angles
⇒ convert “magnetic moment directions" into "brightness".

Thanks for listening

Reference

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