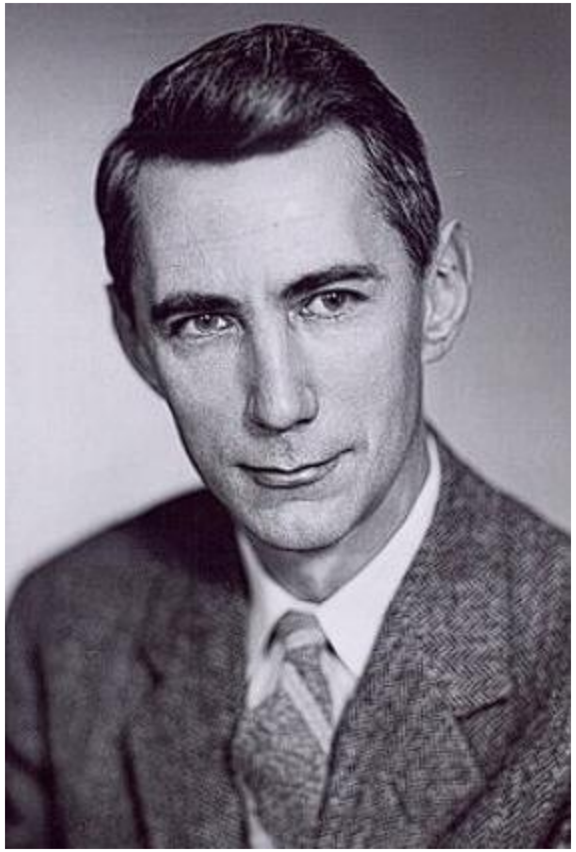
Information Entropy

Information Entropy Introduction

first introduced in 1948 by Claude Shannon:

$$H(x) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$



https://en.wikipedia.org/wiki/Claude_Shannon

Information Entropy vs. Classical Entropy

Information Entropy:
uncertainty or average amount of information

$$H(x) = -\sum_{i=1}^{n} p_i \log_2(p_i)$$

Classical Entropy:
degree of disorder or randomness within a system

$$S = k_B ln\Omega$$

Their Purity and Mixedness

Pure State:

the most complete knowledge of a system’s state

fully described by a single quantum state vector

“uncertainty” is minimal

Mixed State:

we do not have complete knowledge of a system’s state

described as a probabilistic mixture of multiple pure states

“uncertainty” is higher

Von Neumann entropy

To quantify the mixedness of a quantum system or its quantum information uncertainty

$$S(\rho) = -T_r(\rho \log_2 \rho)$$

diagonalize the density matrix

$$S(\rho) = -\sum_i \lambda_i \log_2 \lambda_i$$

https://en.wikipedia.org/wiki/John_von_Neumann

Von Neumann entropy Purity and Mixedness

Pure State
the most complete knowledge of its state

entropy of a pure state is 0

Mixed State

knowledge of state is incomplete

multiple non-zero eigenvalues(λ_i), and these satisfy $\sum_i \lambda_i = 1$

the von Neumann entropy is greater than 0

Why is it important

Quantifying the mixedness and uncertainty of quantum states

Information compression and quantum encoding

What do we know from "analyzing quantum systems"

State and evolution

Entanglement and correlations

Information storage, processing, and transmission

References

- Shannon, C. E. (1948). *A mathematical theory of communication*. Bell System Technical Journal, 27(3), 379–423.
- Cover, T. M., & Thomas, J. A. (2006). *Elements of information theory* (2nd ed.). Wiley-Interscience.
- Nielsen, M. A., & Chuang, I. L. (2010). *Quantum computation and quantum information* (10th anniversary ed.). Cambridge University Press.
- Von Neumann, J. (1955). *Mathematical foundations of quantum mechanics*. Princeton University Press.
- Boltzmann, L. (1877). *On the relation between the second law of thermodynamics and probability theory*.
- Pathria, R. K., & Beale, P. D. (2011). *Statistical mechanics* (3rd ed.). Elsevier.
- Breuer, H.-P., & Petruccione, F. (2002). *The theory of open quantum systems*. Oxford University Press.
- Horodecki, R., Horodecki, P., Horodecki, M., & Horodecki, K. (2009). Quantum entanglement. *Reviews of Modern Physics, 81*(2), 865–942.