a brief exploration into

Ergodic Theory

111022121 蔡翰宗

Recalling Virial Theorem

7.13 Virial Theorem (Optional)

Another important result of a statistical nature is worthy of mention. Consider a collection of particles whose position vectors $\mathbf{r}_\alpha$ and momenta $\mathbf{p}_\alpha$ are both bounded (i.e., remain finite for all values of the time). Define a quantity

$$S \equiv \sum_\alpha \mathbf{p}_\alpha \cdot \mathbf{r}_\alpha \qquad (7.199)$$

$$\frac{dS}{dt} = \sum_\alpha (\mathbf{p}_\alpha \cdot \dot{\mathbf{r}}_\alpha + \dot{\mathbf{p}}_\alpha \cdot \mathbf{r}_\alpha)$$

$$\left\langle \frac{dS}{dt} \right\rangle = \frac{1}{\tau}\int_0^\tau \frac{dS}{dt}\,dt = \frac{S(\tau) - S(0)}{\tau}$$

$$\Rightarrow \quad \langle T \rangle = -\frac{1}{2}\left\langle \sum_\alpha \mathbf{F}_\alpha \cdot \mathbf{r}_\alpha \right\rangle$$

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→

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Recalling Virial Theorem

given enough time, the "average" works even in non-periodic system as long as everything is bounded

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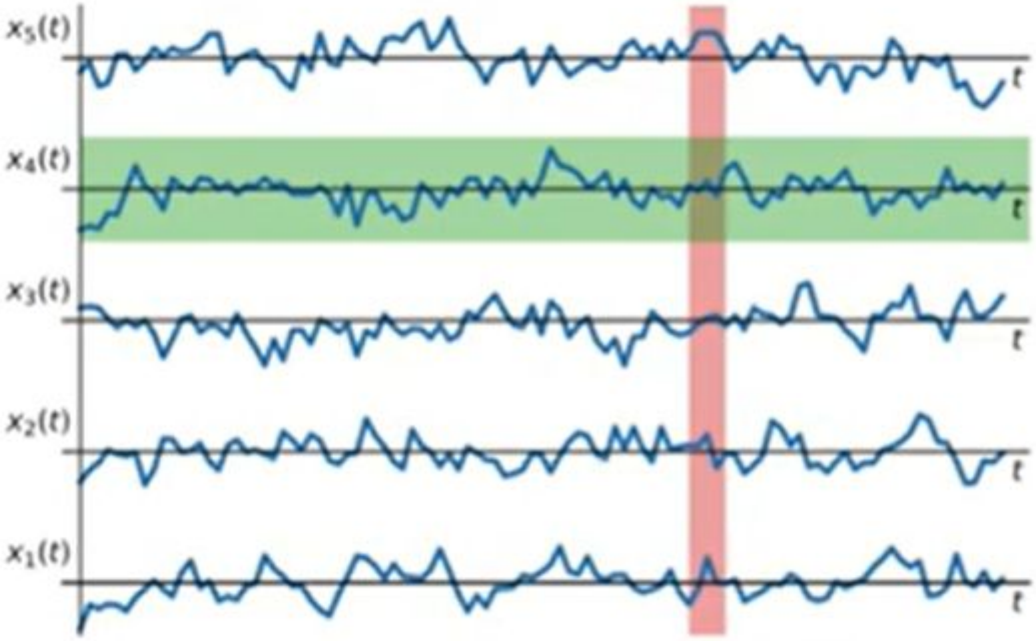
Average?

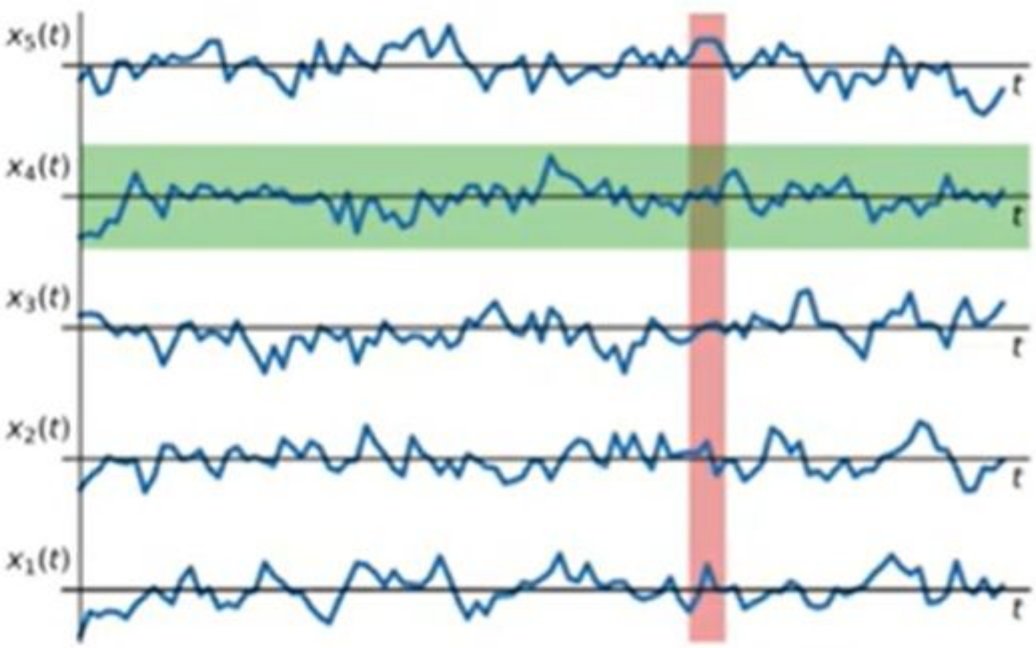


Time average

Phase average

Ensemble average

Average?



Time average

⬍

Phase average

Ensemble average

Leading to Birkhoff’s ergodic THM

measure space

almost every

ergodic?

Leading to Birkhoff's ergodic THM

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almost every

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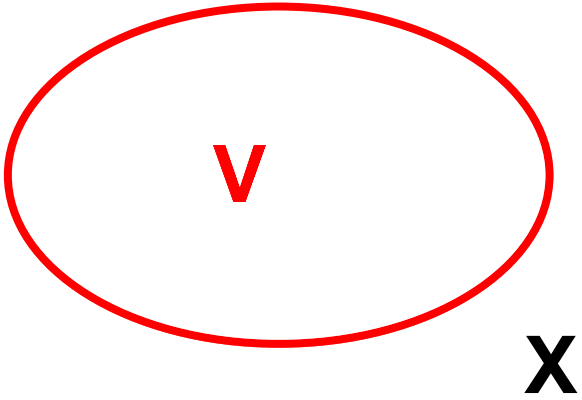


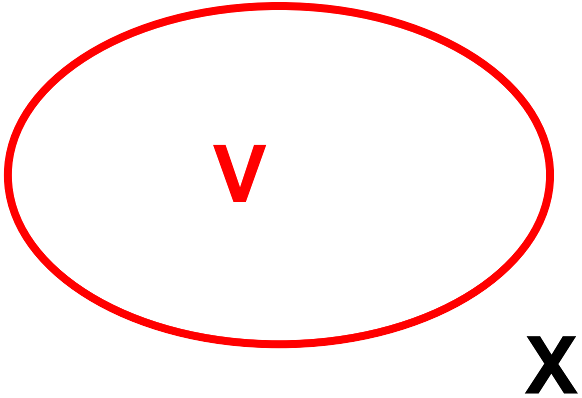
$$(X, V, \mu)$$

Leading to Birkhoff's ergodic THM

measure space

almost every

ergodic?



$$(X, V, \mu)$$

for probability measure, $\mu(X) = 1$

Leading to Birkhoff's ergodic THM

measure space

almost every

ergodic?

Let $f: X \to X$ be μ-measure preserving

$\Rightarrow \forall V \in X$, we got

$\mu(\{x \in V: \exists N \geq 1, s.t. f^{n}(x) \in V \forall n \in N\})$

Let $f: X \to X$ be μ-measure preserving

, $\mu(X) < \infty$

$\Rightarrow$ f is ergodic if $\forall V \in X, s.t. V\ is\ invariant$

$\Rightarrow \mu(V) = 0\ or\ \mu(V) = \mu(X)$

Leading to Birkhoff's ergodic THM

measure space

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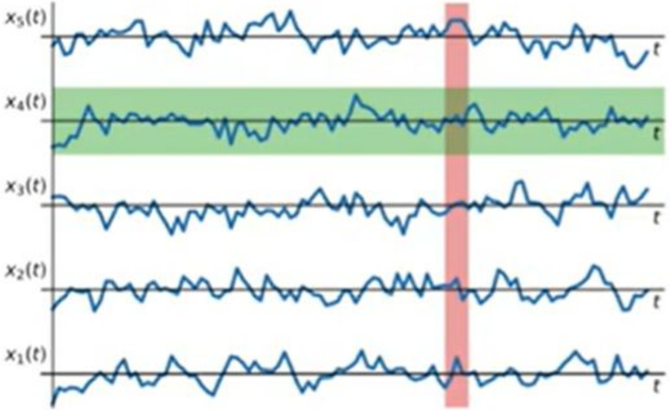
Birkhoff's ergodic THM



Let $f\colon X \to X$, system ergodic w.r.t. μ

$\Rightarrow \ \forall\, g\colon X \to R\,,\ s.t.\ \int_X |g|\,dx < \infty$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{1}{n}\Sigma g(f^k(x)) = \frac{1}{\mu(X)}\int_X g\, d\mu$$

(for almost every $X \in X$)

slide waiting to be added:(sorry..)

so what makes a system ergodic?
two star system?
three star system?
ideal gas
relations between chaotic and ergodic?
recalling experiment on duffing oscillator
equation of life? (where ergodicity don’t work)