Entanglement renormalization of non-Hermitian critical systems and emergent dS space

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Motivation

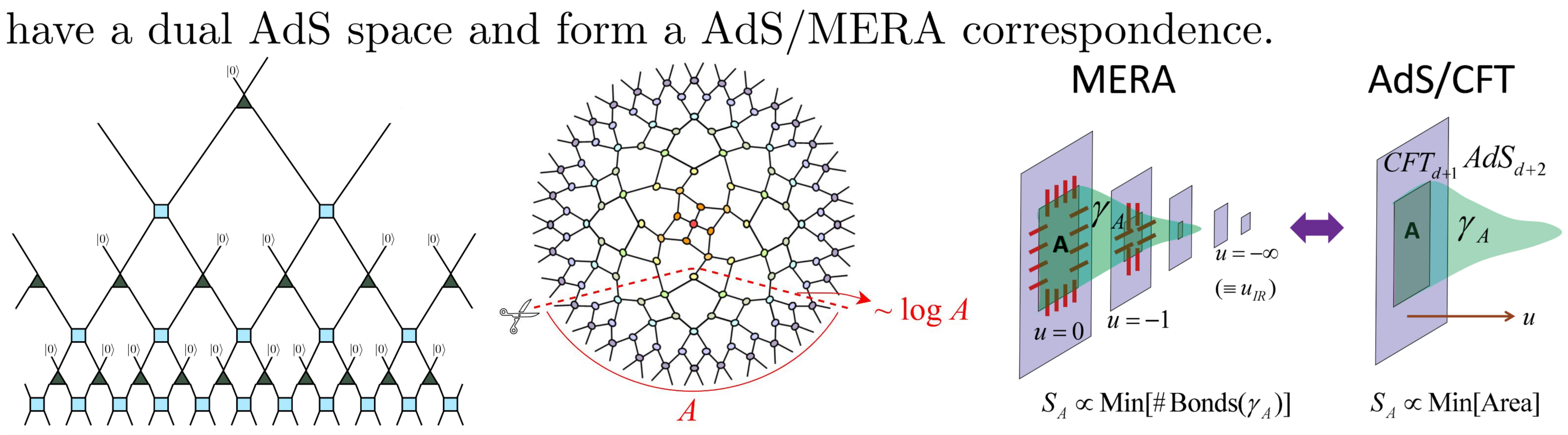
One of the most important development in AdS/CFT correspondence is the RT formula[1]. This formula states that entanglement entropy of boundary CFT will equal to bulk AdS minimum surface

$$S(V) = min[\frac{A(m)}{4G_N}]$$

Which gives a big convenience to calculate entanglement entropy in CFT. Later research[2] shows that this conjecture becomes obvious in the AdS/MERA. Since our universe is a dS space, so people have a big interest in dS/CFT. Research[3] shows that here the dual CFT will be a non-unitary one with negative/complex central charge. Recent research[4] shows that non-Hermitian system can have a negative central charge. Therefore, to push the understanding of RT formula in dS/CFT case and learn more about dS/CFT, we study the cMERA of non-Hermitian system.

MERA and AdS/MERA

MERA is a real space renormalization group (RG) which particularly useful for understanding systems with long-range entanglement due to its ability to capture entanglement correlations across different length scales. The tensor network of MERA has a layer structure, which each layer combined a disentangler and a coarse-grainer. At each layer, disentangler removes the entanglement and the coarse-grainer throw away the irrelevant part. This structure has been showed that have close connection with the holographic duality, especially on the RT formula. The MERA of hermitian gapless system has been argued to have a dual AdS space and form a AdS/MERA correspondence.



cMERA and emergent geometry

The continuous version of MERA (cMERA) was proposed to understand entanglement in quantum field theory within the MERA scheme. The formulation of cMERA is very convenience for making solid connection between it and AdS/CFT. In particular, with its continuous property, the emerget metric of extra holographic dimension can be exactly defined in cMERA.
The unitary transformation is

$$|\Psi_{UV}\rangle = \mathcal{P}\exp\left\{[-i\int_{-\infty}^{0}(K(u)+L)du]\right\}|\Psi_{IR}\rangle$$

where L is the generator of scale transformation, $K(u)$ is the disentangler. Interacting picture will be more useful, define $|\Psi^I(u)\rangle = e^{iuL}|\Psi^I(u)\rangle$

$$i\partial_u|\Psi^I(u)\rangle = \hat{K}(u)|\Psi^I(u)\rangle$$

The emergent quantum metric is the information metric

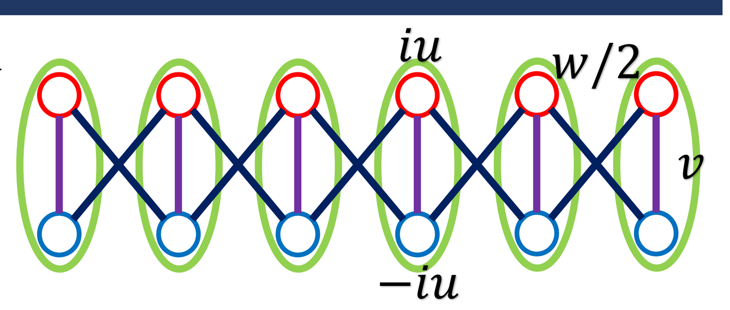
$$g_{uu}(u)du^2 = 1 - |\langle\Psi(u+du)|\Psi(u)\rangle|^2$$

full geometry will be

$$ds^2 = g_{uu}(u)du^2 + \Lambda^2 e^{2u}dx^2$$

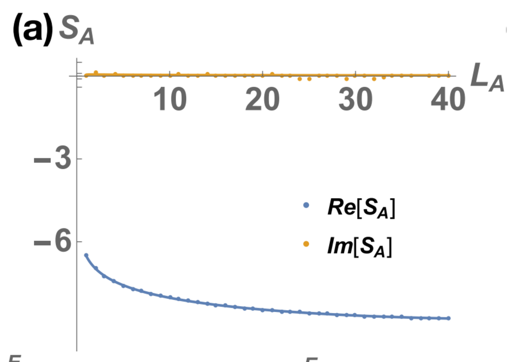
cMERA of nH-SSH

We study critical tog-legged nH-SSH which has been reported have central charge $c=-2$



$$\mathcal{H}_k = \begin{bmatrix} iu & w\cos k + v \\ w\cos k + v & -iu \end{bmatrix}$$

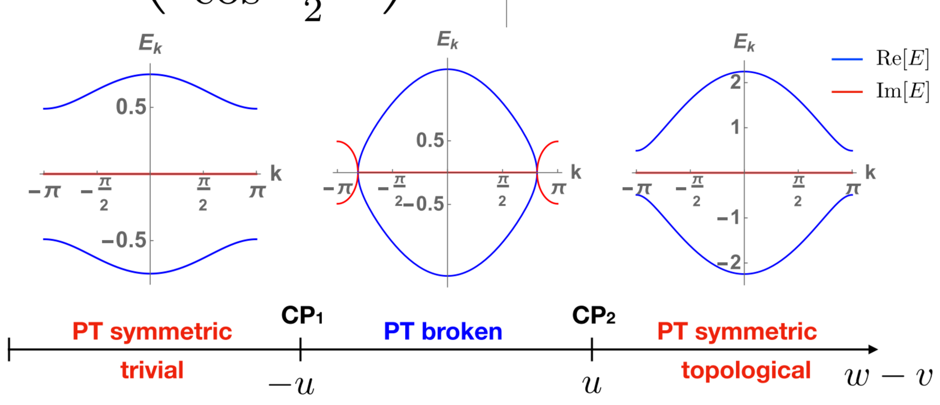
With eigenstae



$$|R(L)_{k,+}\rangle = \begin{pmatrix} \cos\frac{\phi_k^{(*)}}{2} \\ -\sin\frac{\phi_k^{(*)}}{2} \end{pmatrix}, |R(L)_{k,-}\rangle = \begin{pmatrix} \sin\frac{\phi_k^{(*)}}{2} \\ \cos\frac{\phi_k^{(*)}}{2} \end{pmatrix}$$

where $\phi_k = \tan^{-1}\frac{v_k}{iu}$, at $k=0$

$$\phi(k) = i\ln k + \frac{i}{2}\ln\frac{w}{4u} - \frac{\pi}{2} + \mathcal{O}(k^2)$$



IR state will be $\phi_k = \phi_{k=\pi}$ with entangler

$$\hat{\mathcal{K}}(k,u) = g(u)\sigma_y\Gamma(\frac{|k|e^{-u}}{\pi})$$

then we have

$$\begin{pmatrix} \psi_+(k,u) \\ \psi_-(k,u) \end{pmatrix} = \begin{bmatrix} \cos\theta_k(u) & -\sin\theta_k(u) \\ \sin\theta_k(u) & \cos\theta_k(u) \end{bmatrix} \begin{pmatrix} \psi_+(k) \\ \psi_-(k) \end{pmatrix}$$

The angle rotate under cMERA will be

$$\theta_k(u_{IR}) = \int_{\ln\frac{k}{\pi}}^{0} g(s)ds = \frac{1}{2}\phi(k) - \frac{1}{2}\phi(\pi)$$

One can solve $g(u)$

$$g(u) = -\frac{k}{2}\partial_k\phi(k)|_{k=\pi e^u} = -\frac{i}{2}$$

The emergent quantum metric for nH system

$$g_{uu}(u)du^2 = 1 - \langle\Psi_L(u+du)|\Psi_R(u)\rangle\langle\Psi_L(u)|\Psi_R(u+du)\rangle$$

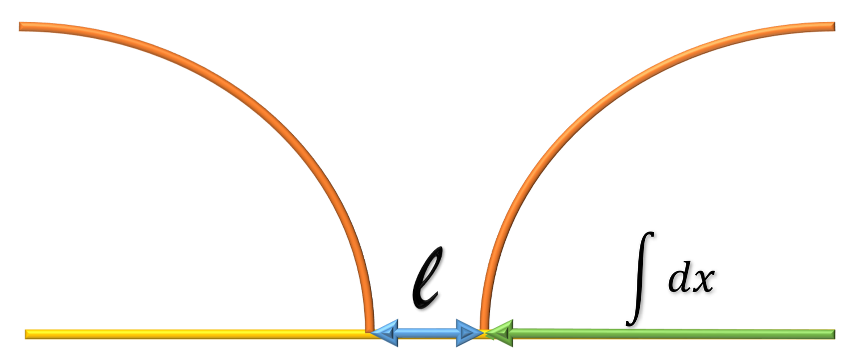
substitute the ansatz we can get the full metric will be

$$ds^2 = -\frac{1}{4}du^2 + \pi^2 e^{2u}dx^2$$

RT formula in dS/cMERA

Within the emergent dS space, we can calculate the geodesic, the geometry can be simplify and have gedesic as

$$ds^2 = -dt^2 + e^{2t}dx^2, \int ds = \int dtL$$



the Euler-Lagrange equation give

$$\dot{x}^2 = \frac{Q^2}{e^{4t} + Q^2e^{2t}}$$

where Q is integral constant. The end of geodesic should at the point $\dot{x}^2 \to \infty$, which means the end point should at $t \to -\infty$

$$\int ds = \int_0^{-\infty} dt(-1 + \frac{Q^2e^{2t}}{e^{4t}+Q^2e^{2t}})^{1/2} = -i\sinh^{-1}(Q^{-1})$$

and the total length will be

$$\int dx = \int_0^{-\infty} dt\frac{Q}{(e^{4t}+Q^2e^{2t})^{1/2}} = \infty - \frac{1}{Q}(1+Q^2)^{1/2}$$

one can see that the total length diverge, but the subsystem size should not diverge, this is because here the geodesic is going outward, the subsystem size should be defined as

$$l = 2\int dx(\infty) - 2\int dx(Q) = \frac{2}{Q}(1+Q^2)^{1/2} - 2$$

then for the long range limit $l >> 1$, we have

$$S \sim -i2\log l$$

Which is indeed a logarithmic scaling.

Reference

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