Raman Scattering Approach to the $\mathbb{Z}_2$ Gauge Theory in the Kitaev Model

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Abstract

The Kitaev honeycomb model can be viewed as a system consisting of Majorana fermions coupled to a $\mathbb{Z}_2$ gauge field, for which the accompanied $\mathbb{Z}_2$ vortex is called the *vison*. Recently, a study introduced a Green's function approach to analytically compute the energy cost of generating a vison in the Kitaev honeycomb model. On the other hand, some studies proposed that the Raman scattering method can be applied to investigate the Kitaev spin liquid. Based on These approaches, we can delve deeper into the system's characteristics when two visons recombine through the formation of a loop, which may reflect the information about the self statistics of visons in the Kitaev model. This study offers further insights into the $\mathbb{Z}_2$ gauge theory within the Kitaev spin liquid.

Introduction

►Kitaev honeycomb model [1]

$$\hat{H} = J \sum_{\langle ij \rangle} \sigma_i^{\alpha_{ij}} \sigma_j^{\alpha_{ij}}$$

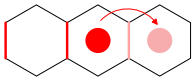
►$\sigma_i^{\mu} \to i b_i^{\mu} c_i, \quad \hat{u}_{ij} \equiv i b_i^{\alpha_{ij}} b_j^{\alpha_{ij}}$ ($\mathbb{Z}_2$ gauge field $\hat{u}_{ij}^2 = 1$)

$$\hat{H} = iJ \sum_{\langle ij \rangle} c_i \hat{u}_{ij} c_j$$

The eigenstate can be separated into two parts: $|\psi\rangle = |M\rangle |\{u_{ij}\}\rangle$

►**Vison**, i.e. the $\mathbb{Z}_2$ vortex : $\hat{W}_p \equiv \prod_{(ij)\in p} \hat{u}_{ij}$

►Vison movement: $\sigma_i^{\alpha_{ij}} \hat{u}_{ij} \sigma_i^{\alpha_{ij}} = -\hat{u}_{ij}$



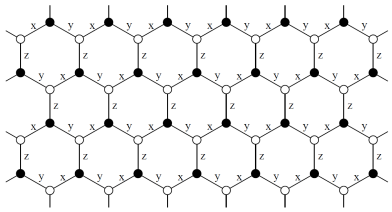
►Visons in the Kitaev model are **semions**. An additional phase π is accumulated while a vison goes around another vison.

Motivation

►**Anyonic** feature is crucial to identify quantum spin liquid phases. However, the **mutual** or **self statistics** of anyons is not easy to be measured directly. It is important to propose a experimental method to confirm the theory of the quantum spin liquid.

►**Raman scattering** is a candidate. The interaction between the electromagnetic wave and the spin system might reflect the information about the statistics of emerged anyons. Some previous works have theoretically proposed the methods using Raman scattering to investigate the feature of the Kitaev spin liquid. [2, 3]

►**The situations we are interested in:** the Raman scattering caused by the process in which a photon creates vison pairs, and then visons annihilate each other to scatter another photon.



Raman Scattering with Impurities

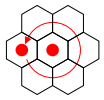
►External magnetic field creates visons through the following interacting Hamiltonian

$$\hat{H}_{\text{v-p}} = \sum_i \mathbf{H}(\mathbf{r}_i) \cdot \boldsymbol{\sigma}_i$$

Expressing the magnetic field in the second quantization form allows us to discuss the **photon-vison coupling**.

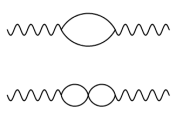
►We introduce an **impurity** making visons circle around one another

$$\hat{H}_h = J' \left(\frac{1 - \hat{W}_p}{2} \right) \prod_{i \in p} \sigma_i^{\alpha_i}$$



The term in the parenthesis ensures that the nearby visons circle around the plaquette only when there is a vison inside the plaquette.

There are two leading contribution to the amplitude of the Raman scattering. Here we applied the Green's function approach developed in Ref.[4]

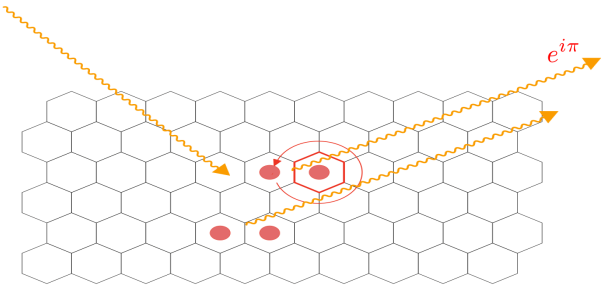


$$M^{(0)} = \langle M'; \{u\}^{(0)} | \hat{H}_{\text{v-p}} \hat{G}_0 \hat{H}_{\text{v-p}} | M_0; \{u\}^{(0)} \rangle$$

$$M^{(1)} = \langle M'; \{u\}^{(0)} | \hat{H}_{\text{v-p}} \hat{G}_0 \hat{H}_h \hat{G}_0 \hat{H}_{\text{v-p}} | M_0; \{u\}^{(0)} \rangle$$

Result

$$I_{z\bar{z}}(\Omega) = I_{z\bar{z}}^{(0)}(\Omega) \left| 1 + e^{i\pi} J' f(\omega_{\text{in}}) \right|^2$$



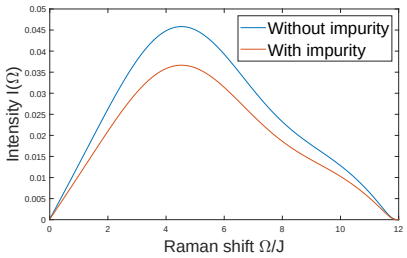


Figure 1: Schematic plot of the Raman spectrum with the impurity.

Conclusion

In our model, the additional phase will be carried by the scattered light, which will interfere with the original scattered light caused by the pure Kitaev term. For visons in the Kitaev model, the additional phase π leads to the destructive interference, thus the signal of Raman scattering will be weaker than the original material without impurities.

Further Questions

1. How to realize it?
2. Loudon-Fleury type Raman vertex? [2]
3. Is this approach general for all types of anyons in quantum spin liquids?

References

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[4] A. Panigrahi *et al. Phys. Rev. B* **108**, 045151 (2023)