Characterization of dynamical phases for periodic-driven systems on the Poincaré disk

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tl;dr:

In conformal invariant $(1+1)$-dimensional systems subjected to periodic driving, there are heating and non-heating phases characterized by linear growth and oscillation of the entanglement entropy respectively [arXiv preprint arXiv:1805.00031]. In this work, we explore different setups without conformal symmetry by employing Poincaré disk realizations for periodic driven systems with $SU(1,1)$ symmetry. We demonstrate these realizations by two examples: (a) Bose-Einstein condensates (BEC) quenching dynamics and (b) periodic-driven oscillators, both of which are experimentally accessible. For BEC quenching dynamics, the heating and non-heating phases can be determined by both excitations and entanglement entropy. On the other hand, for the driven coupled oscillators, the phase diagram is enriched. We observed there are distinct phases inside the heating phase which can only be captured by the entanglement measures.

Settings and Algebra [1]

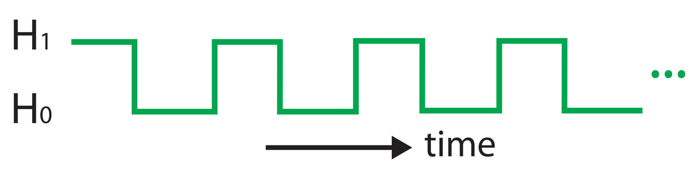
SU(1,1): Three generators K_0, K_1, K_2 with the rules

$$[K_0, K_1] = iK_2, \quad [K_1, K_2] = -iK_0, \quad [K_2, K_0] = iK_1$$

and can work out the unitary representation

$$K^2 |k,m\rangle = k(k-1)|k,m\rangle, \quad K_+|k,m\rangle = \sqrt{(m+1)(m+2k)}\,|k,m+1\rangle,$$
$$K_0|k,m\rangle = (m+k)|k,m\rangle, \quad K_-|k,m\rangle = \sqrt{m(m+2k-1)}\,|k,m-1\rangle,$$

Setting: Taking from [1], we take $|\psi(0)\rangle = |GS \text{ of } H_0\rangle$, and drive the system by the Hamiltonian



Take $H_0 \sim K_0$ and H_1 is some linear combination of all generators, the evolution operator reads

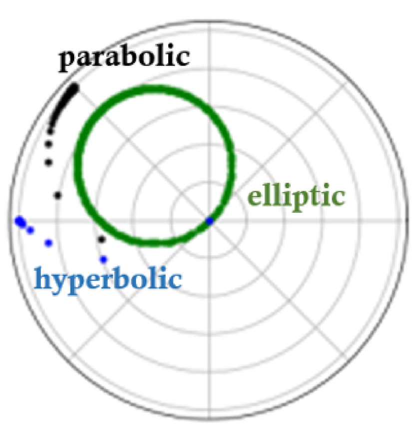
$$\hat{U} = e^{a_+\hat{K}_+ + a_-\hat{K}_- + a_0\hat{K}_0} = e^{A_+\hat{K}_+} e^{\ln(A_0)\hat{K}_0} e^{A_-\hat{K}_-} \sim (A_0)^k e^{A_+K_+}$$

The problem narrows down to track the *SU(1,1) coherent state (CS)* on the *Poincaré disk* $\mathcal{D} = \{z = A_+ | z \in \mathbb{C}, |z| \le 1\}$ and each SU(1,1) elements serve as *Möbius transformation (MT)* $\mathcal{M}$ on $\mathcal{D}$. This allows us to:

1. **Every cycle can be realized on PD**($U_1 = \mathcal{M}_1$, $U_0 = \mathcal{M}_0$, $U = \mathcal{M}_0\mathcal{M}_1$)
2. By property of MT:

$$\mathcal{M}\cdot z = \frac{\alpha z + \beta}{\beta^\star z + \alpha^\star}, \quad \gamma_\pm = \frac{\alpha - \alpha^\star \pm \sqrt{\Delta}}{2\beta^\star}$$

the *fixed point* can be different by the trace of MT $\Delta = \mathrm{Tr}(\mathcal{M})^2 - 4$, and MT has corresponding different behaviors [1].



$$\frac{z_n - \gamma_1}{z_n - \gamma_2} = \eta^n \cdot \frac{z - \gamma_1}{z - \gamma_2}, \quad \eta = \begin{cases} e^{i2\phi}, & \text{Elliptic,} \\ 1, & \text{Parabolic,} \\ e^{2\phi'}, & \text{Hyperbolic.} \end{cases}$$

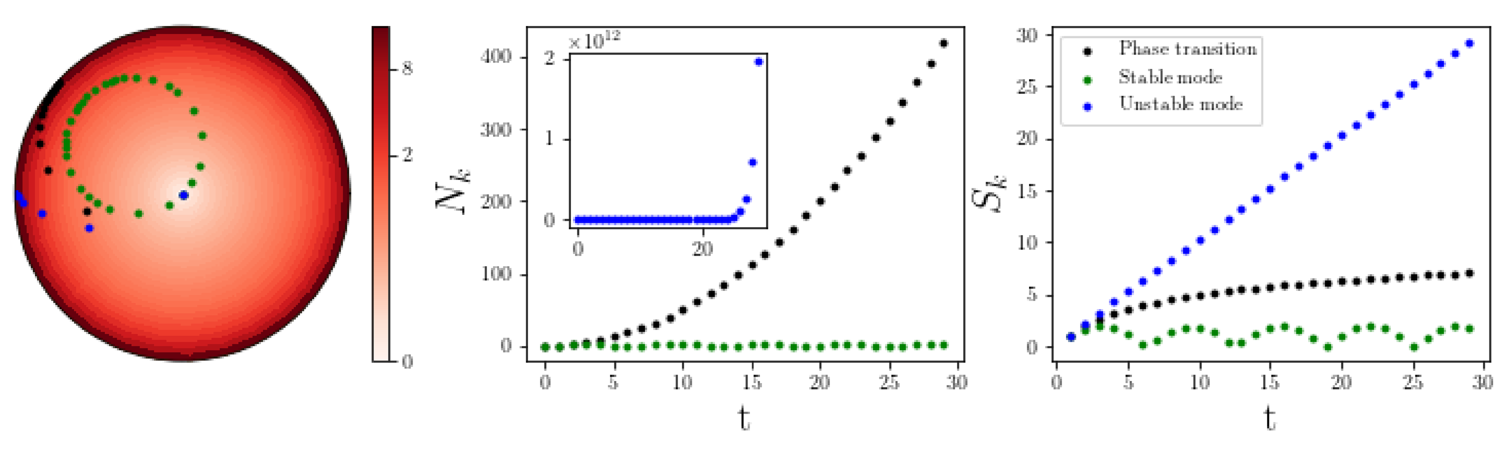
This implies that when **characterizing evolution, knowing one cycle = knowing n cycle**, and the **trace of MT** works as an important index.

BEC quenching dynamics [2,3,4]

Setting: Starting from interacting (controlled by Feshbach resonance) Bosonic Hamiltonian

$$\hat{H}(t) = \sum_{\boldsymbol{k}} E_{\boldsymbol{k}} \hat{c}^\dagger_{\boldsymbol{k}} \hat{c}_{\boldsymbol{k}} + \frac{\tilde{U}}{2V} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \hat{c}^\dagger_{\boldsymbol{k}+\boldsymbol{q}} \hat{c}^\dagger_{\boldsymbol{k}'-\boldsymbol{q}} \hat{c}_{\boldsymbol{k}'} \hat{c}_{\boldsymbol{k}}$$

1. **GS = BEC**. MF (pairing k to $-k$ excitations) gives $H \simeq \Sigma_k H(k)$
$$\hat{H}(\boldsymbol{k}) = \xi_0(\boldsymbol{k})\hat{K}_0 + \xi_1(\boldsymbol{k})\hat{K}_1 + \xi_2(\boldsymbol{k})\hat{K}_2$$
2. K_0 works as counting excitations, $K_\pm$ excites (resp. annihilates) the state.
3. Using the algebra, the trajectory of states on PD
$$z(t) = -i\frac{(\xi_1 - i\xi_2)\sin(\xi t/2)}{\xi\cos(\xi t/2) + i\xi_0\sin(\xi t/2)}$$
with $\mathrm{Tr}(\mathcal{M}) = 2\cos\left(\sqrt{\xi_0^2 - \xi_1^2 - \xi_2^2}\,t/2\right)$.
4. **The stability directly determines the growth behavior of excitations**: $S_{\boldsymbol{k}} = (n_{\boldsymbol{k}}+1)\ln(n_{\boldsymbol{k}}+1) - n_{\boldsymbol{k}}\ln n_{\boldsymbol{k}}$



Periodic driven oscillators (PDOs) [5]

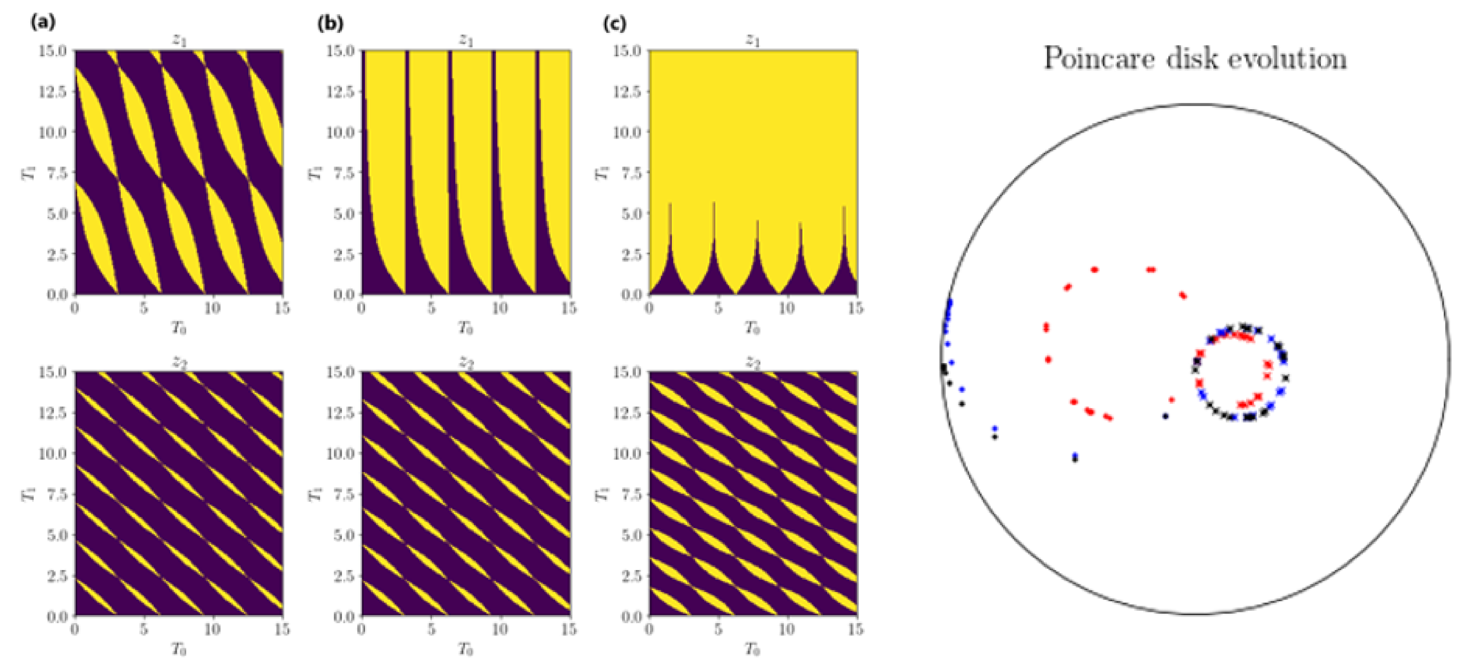
Settings: We coupled the system in the fashion of LHS

$$\hat{H} = \begin{cases} \hat{H}_1 = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2\hat{q}_1^2 + \frac{1}{2}m\omega^2\hat{q}_2^2 + C\hat{q}_1\hat{q}_2 \\ \hat{H}_0 = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2\hat{q}_1^2 + \frac{1}{2}m\omega^2\hat{q}_2^2 \end{cases}$$

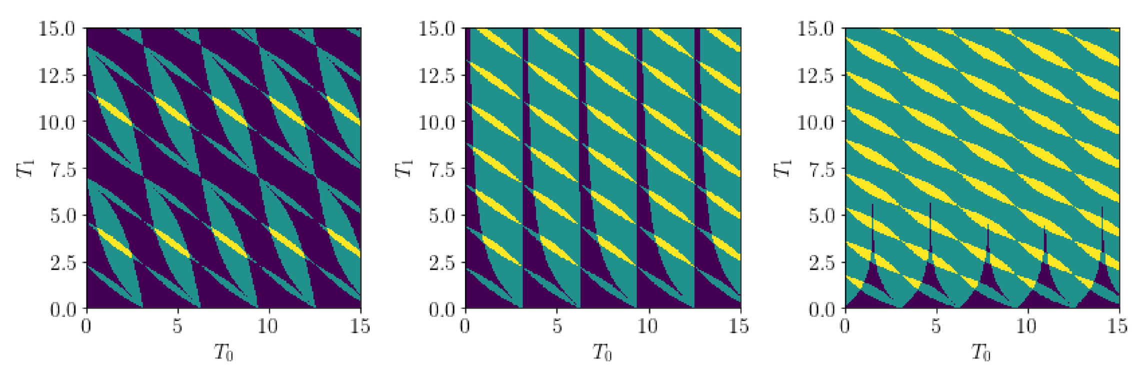
can be diagonalized just like CM (symm., anti-symm.)

$$\hat{H} = \begin{cases} \hat{H}_1 = \sum_{i=1}^2 \left(2(\omega + U_i)\hat{K}_{0,i} + 2U_i\hat{K}_{1,i}\right) \\ \hat{H}_0 = \sum_{i=1}^2 2\omega\hat{K}_{0,i} \end{cases}$$

1. **GS** $= |0\rangle_{1'}|0\rangle_{2'}$, K_0 works as counting excitations, $K_\pm$ excites (resp. annihilates) the state.
2. Unlike BEC, we have to separate two modes into two PDs, and each disk has their own evolution. (Each MTs have to be counted)



together determine the phase diagram:

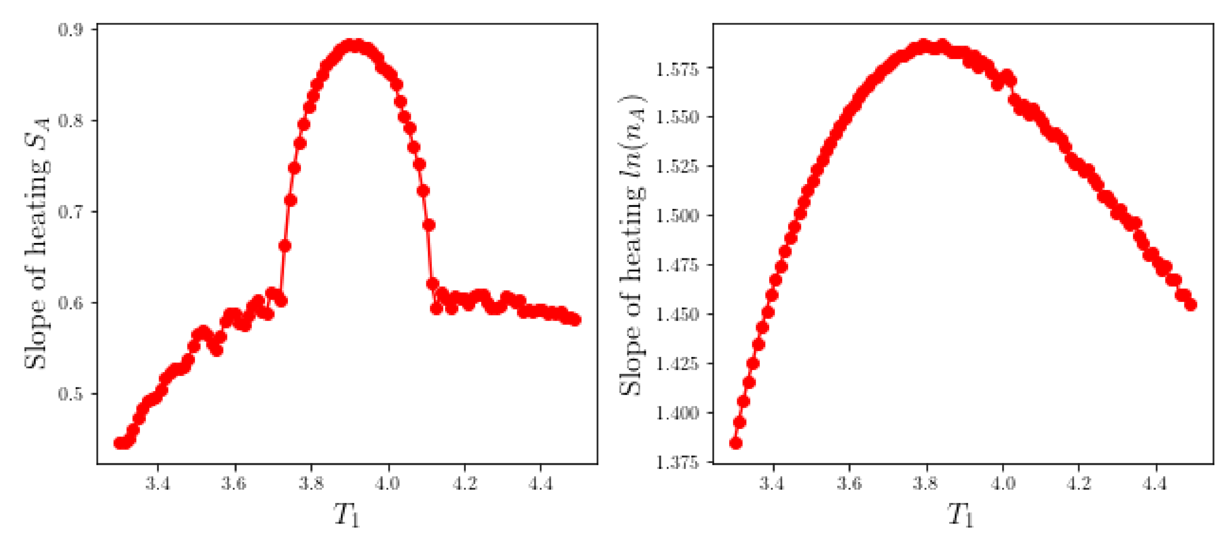


The stability *does not* directly determines the growth behavior of excitations here. and the entropy and excitations are no longer related in the easy way as BEC.

$\mathrm{Tr}(\mathcal{M}_2)$ \ $\mathrm{Tr}(\mathcal{M}_1)$	elliptic (<2)	parabolic ($=2$)	hyperbolic (>2)
elliptic (<2)	$\sin^2(c_1 t)$	$c_1 t^2 + k\sin^2(c_2 t)$	$\sinh^2(c_1 t) + k\sin^2(c_2 t)$
parabolic ($=2$)	$c_1 t^2 + k\sin^2(c_2 t)$	$c_1 t^2$	$\sinh^2(c_1 t) + k c_2 t^2$
hyperbolic (>2)	$\sinh^2(c_1 t) + k\sin^2(c_2 t)$	$\sinh^2(c_1 t) + k c_2 t^2$	$\sinh^2(c_1 t)$

$\mathrm{Tr}(\mathcal{M}_2)$ \ $\mathrm{Tr}(\mathcal{M}_1)$	elliptic (<2)	parabolic ($=2$)	hyperbolic (>2)
elliptic (<2)	$\ln(\alpha\cos(t) + \beta)$	$\ln(\alpha(T_0, T_1)t)$	$\alpha(T_0,T_1)t + \beta(T_0,T_1)$
parabolic ($=2$)	$\ln(\alpha(T_0, T_1)t)$	$\ln(\alpha(T_0, T_1)t)$	$\alpha(T_0,T_1)t + \beta(T_0,T_1)$
hyperbolic (>2)	$\alpha(T_0,T_1)t + \beta(T_0,T_1)$	$\alpha(T_0,T_1)t + \beta(T_0,T_1)$	$\alpha(T_0,T_1)t + \beta(T_0,T_1)$

Only entanglement captures the difference of the trace of MT.



What's the difference?

- Two cases have similar behavior of scaling (EE and excitations), but PDO has finer structure on phase diagram and is captured by entanglement.
- No conformal invariance assumed, only non-compactness of SU(1,1) (Different from but claimed in [1]).
- Unlike Floquet CFT, BEC quenching dynamics is experimentally realized in [2,3] and we expect PDO can also be experimentally done as well.

References

1. Wen, Xueda, and Jie-Qiang Wu. 'Floquet conformal field theory.'arXiv preprint arXiv:1805.00031 (2018).
2. Zhang, Jing, et al. 'Quantum dynamics of cold atomic gas with SU (1, 1) symmetry.'Physical Review A 106.1 (2022): 013314.
3. R. Yamazaki, S. Taie, S. Sugawa, and Y. Takahashi, Submicron spatial modulation of an interatomic interaction in a bose-einstein condensate, Phys. Rev. Lett. 105, 050405 (2010).
4. Lyu, Changyuan, Chenwei Lv, and Qi Zhou. 'Geometrizing quantum dynamics of a Bose-Einstein condensate.'Physical Review Letters 125.25 (2020): 253401.
5. Except figure one, all the figures are taken from our upcoming paper.