Machine-Learning enhanced Quantum State Tomography: Covariance matrix approach

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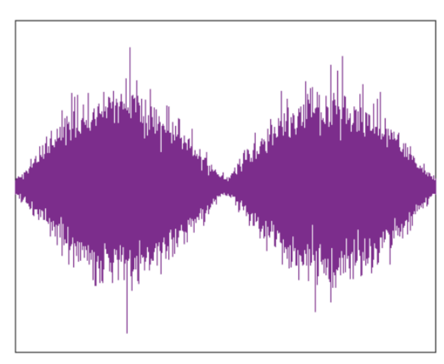
Introduction

- Quantum state reconstruction through homodyne measurements.
- Utilization of single-mode covariance matrices as output for quadrature data.
- QST has been tackled by MLE, density matrix reconstruction and direct parameter estimation.

Methodology

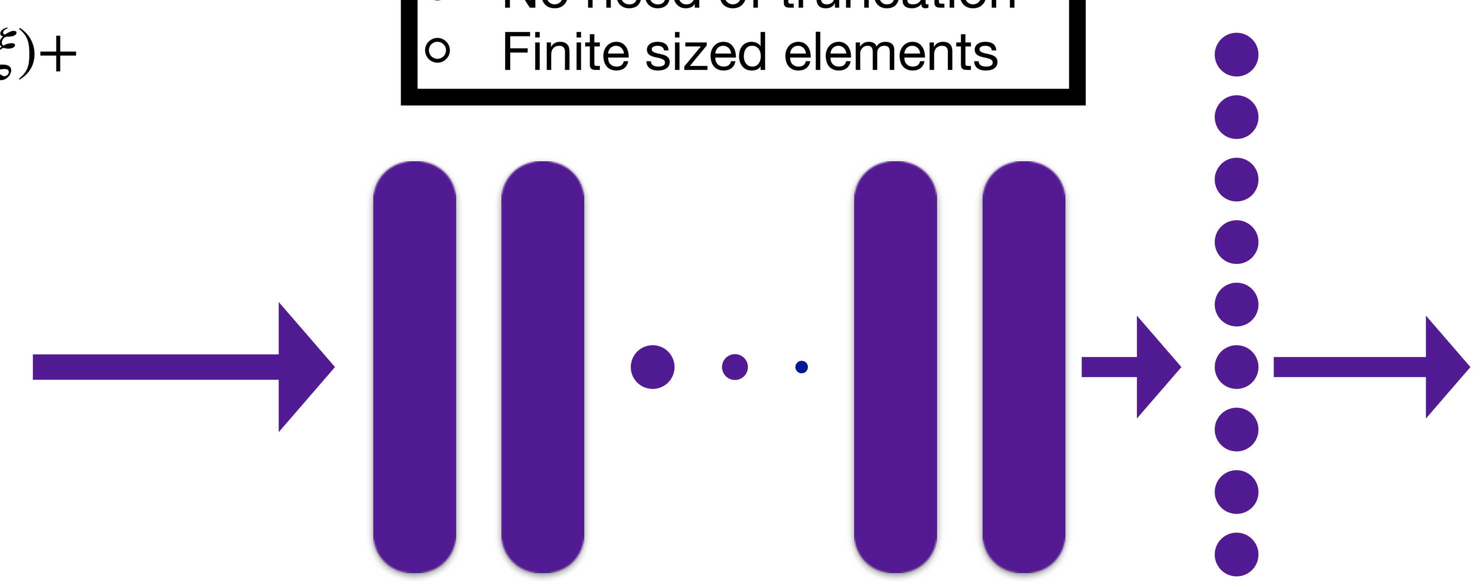
Training set
490.000 quadrature seq.
10.000 covariance matrix

$$\rho = \begin{matrix} c_1 S(\xi)\rho_{\text{vac}} S^\dagger(\xi) + \\ c_2 S(\xi)\rho_{\text{th}}(n_1) S^\dagger(\xi) + \\ c_3 \rho_{\text{th}}(n_2) \end{matrix}$$



Quadrature sequence

Output: Covariance matrix
Characteristics:
- Positive definite (P.D)
- No need of truncation
- Finite sized elements



1D-CNN kernel

Uncertainty principle

$$\sigma + i\Omega \geq 0$$

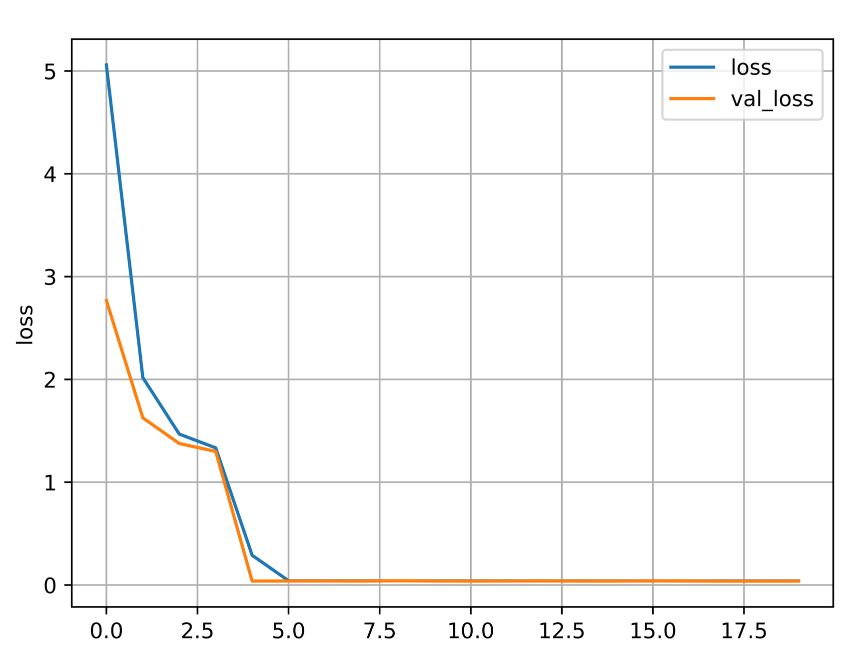
$$\det \sigma \geq 1$$

$$\begin{bmatrix} \sigma_X^2 & \sigma_{XP} \\ \sigma_{PX} & \sigma_P^2 \end{bmatrix}$$

Covariance Matrix

Reconstruction of states

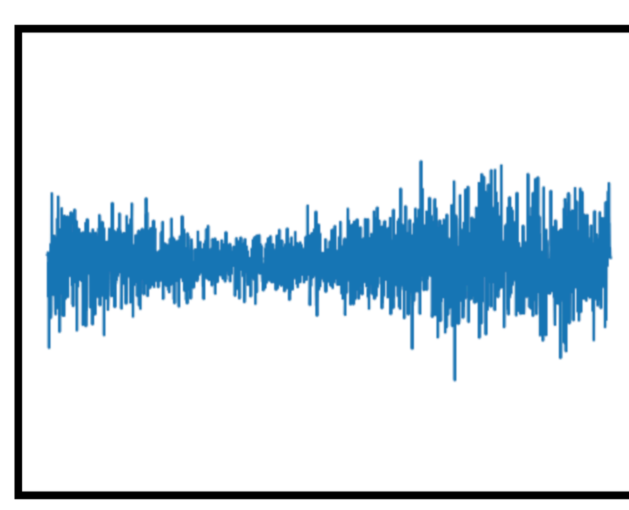
Learning curve



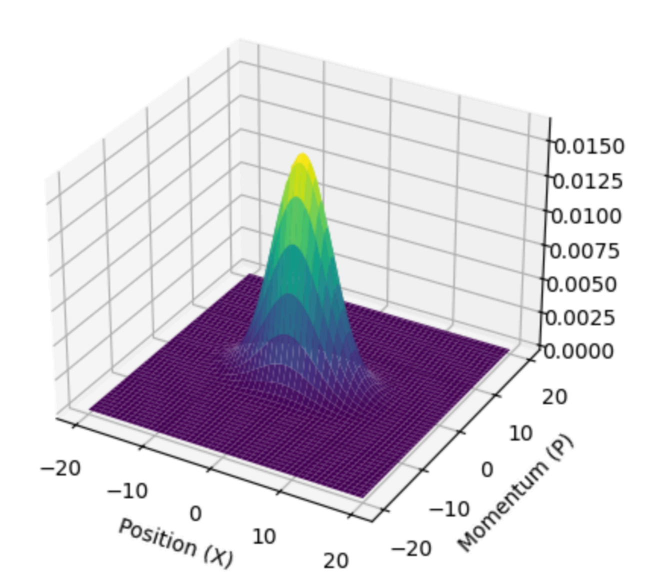
loss: 0.0402

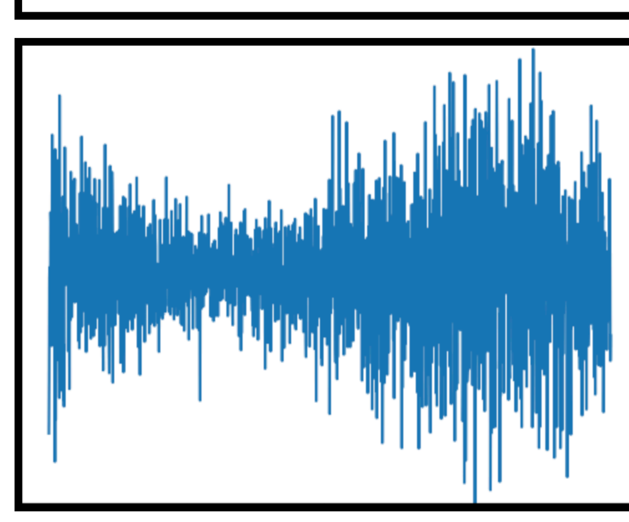
val_loss: 0.0381

σ_{ij}	$\sim$
σ_X^2	e^{-2r}
σ_P^2	e^{2r}
σ_{XP}	0
σ_{PX}	0

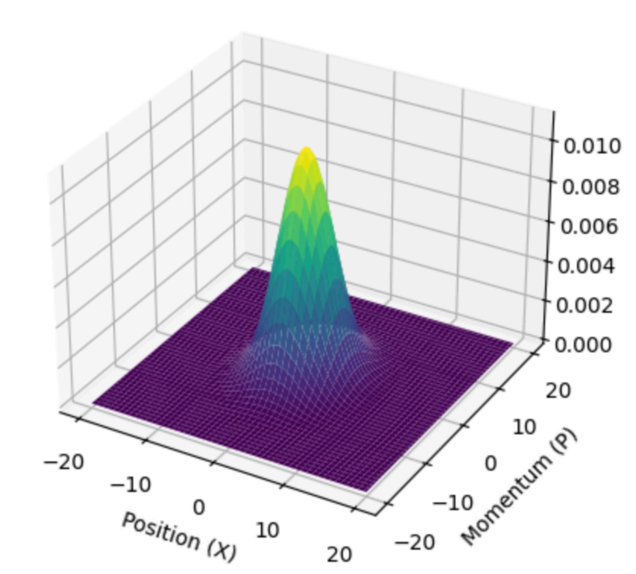


$$\begin{pmatrix} 2.9353 & 0 \\ 0 & 1.6593 \end{pmatrix}$$
$$\theta = 1.0812423$$





$$\begin{pmatrix} 2.4108 & 0 \\ 0 & 2.9542 \end{pmatrix}$$
$$\theta = 0.93159511$$



Prospects

- Two modes squeezing

$$\Sigma^{(2)} = \begin{bmatrix} \sigma_{X_1}^2 & \sigma_{X_1X_2} & \sigma_{X_1P_1} & \sigma_{X_1P_2} \\ \sigma_{X_2X_1} & \sigma_{X_2}^2 & \sigma_{X_2P_1} & \sigma_{X_2P_2} \\ \sigma_{P_1X_1} & \sigma_{P_1X_2} & \sigma_{P_1}^2 & \sigma_{P_1P_2} \\ \sigma_{P_2X_1} & \sigma_{P_2X_2} & \sigma_{P_2P_1} & \sigma_{P_2}^2 \end{bmatrix}$$

(entanglement!!!)

- Perform QST for experimental homodyne detection data.
- Analyze KAGRA data for gravitational wave detection using squeezed light technology with Covariance matrix QST.

Conclusions

- A single scan measurement effectively captures the quadrature sequence data, providing an accurate depiction of the quantum state.
- Covariance matrix approach can deal with large Hilbert spaces while preserving high-precision feature extraction.

References

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[2] H.-Y. Hsieh et al., "Direct parameter estimations from machine-learning enhanced quantum state tomography," Special Issue "Quantum Optimization & Machine Learning"; Symmetry 14, 874 (2022).
[3] Kumar, Chandan. "Estimation of the Wigner distribution of single-mode Gaussian states: A comparative study." *Physical Review A* 105.4 (2022).
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