Spectropolarization of Synchrotron Radiation in Astrophysics

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Abstract

The ratio of the polarized synchrotron emission to the total emission, i.e., the polarization degree, is known to be (p+1)/(p+7/3) or (α +1)/(α +5/3), for electrons with a power-law energy distribution of index p, where α = (p − 1)/2 is the spectral index. In this article, we first show the limitation of the formula, and then we propose a generalized version of this formula which could serve as a universally applicable formula for estimating the polarization degree.

Problem statement

The derivation of the polarization degree is based on an assumption that the energy distribution of the electrons spans from Lorentz factor γ = 0 to infinity. However, the value for the Lorentz factor cannot be smaller than one; in fact, it has to be much larger than one for the synchrotron radiative process to be valid. Furthermore, in a realistic astrophysical situation, electrons cannot be accelerated to infinite energy either. The unrealistic condition is applied for mathematical convenience. This raises a fundamental question: how well does this formula work under realistic conditions?

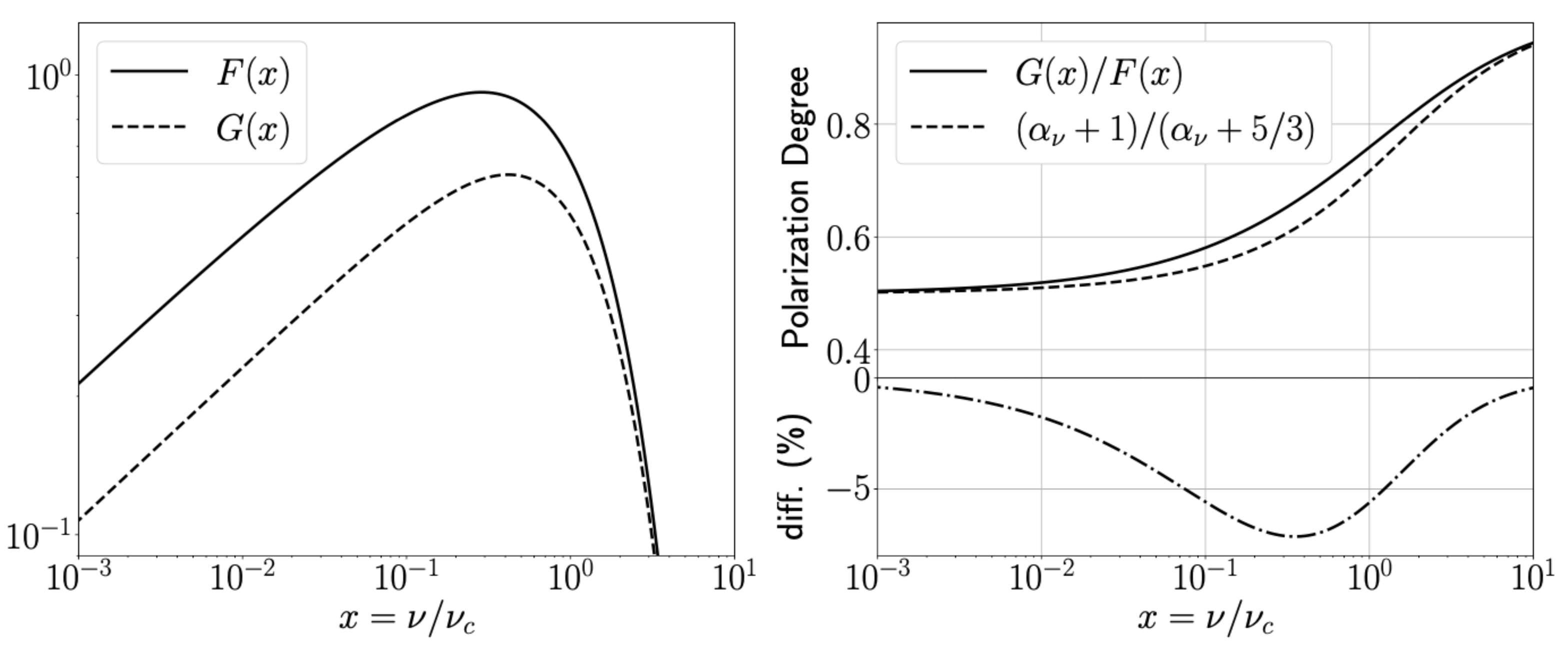
Results

We show

- ❖ The polarization degree of synchrotron radiation due to I) single electron II) power-law (PL) electrons.
- ❖ The percentage difference between the full formulation and the empirical formula (c.f. Fig 1,2) .
- ❖ The previous step is repeated by replacing the PL electron distribution with the commonly used energy distributions: Broken PL, double PL, PL with exponential cutoff and log-normal distribution formula (c.f. Fig 3,4).
- ❖ A generalization of the empirical formula, where the global spectral index α is replaced by the local spectral index αν of the radiation, i.e.

$$\Pi_{\alpha_\nu}(\nu) = \frac{\alpha_\nu + 1}{\alpha_\nu + 5/3}\ ; \qquad \alpha_\nu \equiv -\frac{\nu}{I_\nu}\frac{dI_\nu}{d\nu} = -\frac{d\log I_\nu}{d\log\nu} = \frac{d\log(\lambda^2 I_\lambda)}{d\log\lambda}$$

1



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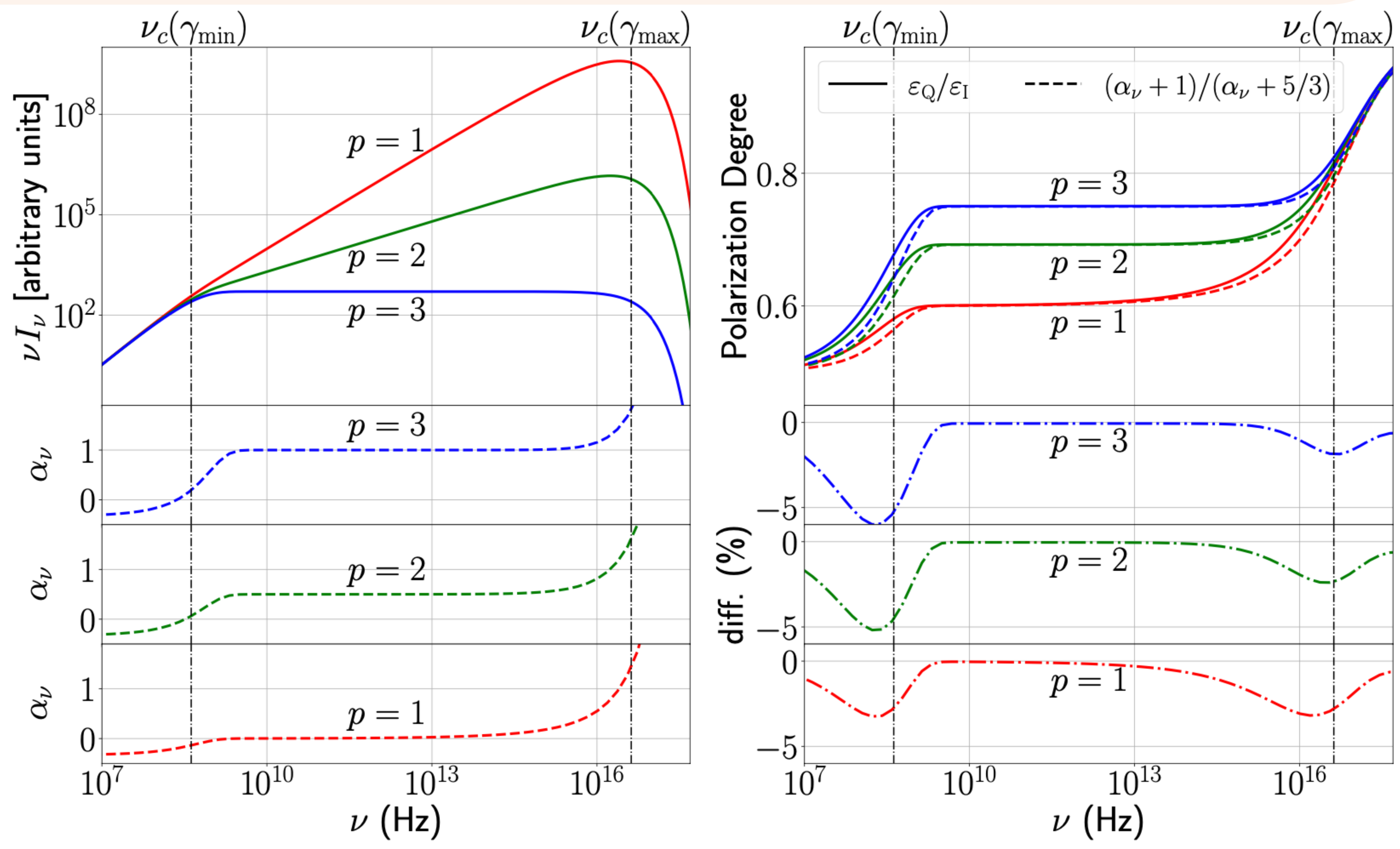

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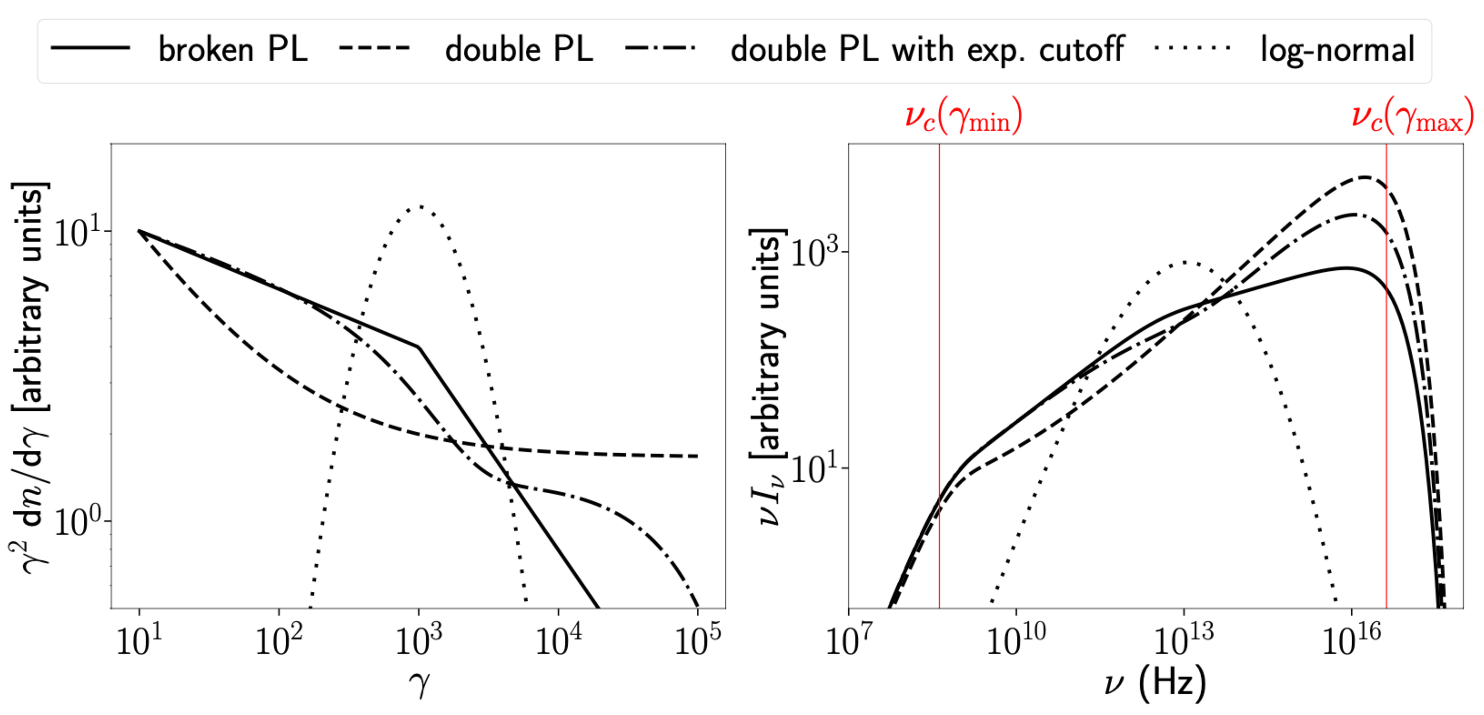

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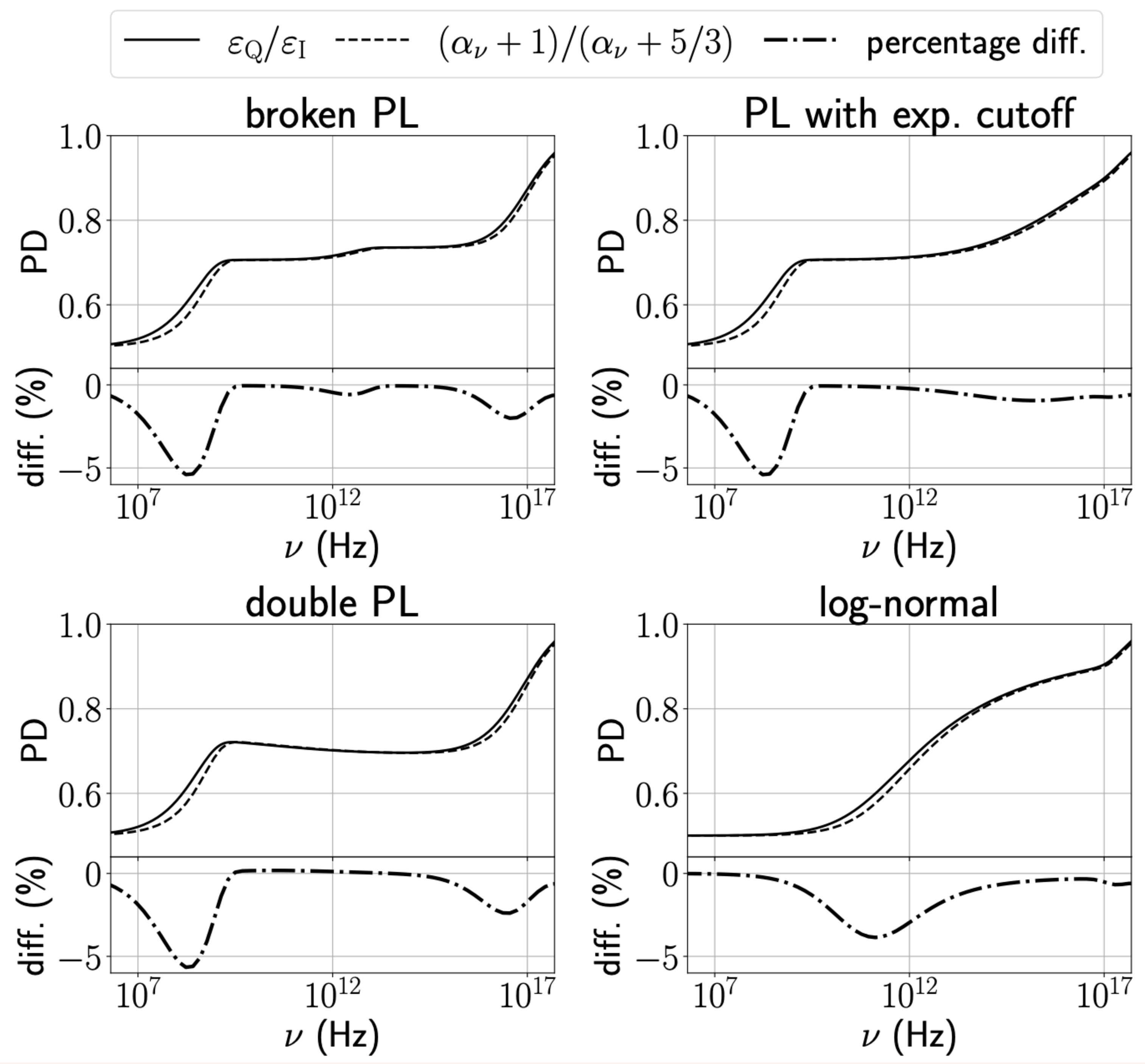

Extra information: Some key equations

1. Single electron emissivity perpendicular and parallel to the magnetic field when projected onto the sky plane.

$$\frac{dP_\perp(\nu)}{d\nu} = \frac{\pi\sqrt{3}\, e^2 \nu_G \sin\theta}{c}\left[F(x) + G(x)\right]$$

$$\frac{dP_\parallel(\nu)}{d\nu} = \frac{\pi\sqrt{3}\, e^2 \nu_G \sin\theta}{c}\left[F(x) - G(x)\right]$$

2. Total emissivity given any distribution of electrons.

$$\varepsilon_I(\nu,\theta) = \int d\Omega(\chi,\phi) \int d\gamma\, n(\gamma,\chi)\, \frac{d^2P}{d\nu d\Omega}\big(\nu,\gamma,\chi,\varphi(\theta,\chi,\phi)\big)$$

3. Polarization degree

$$\Pi(\nu) \equiv \frac{\sqrt{\varepsilon_Q{}^2 + \varepsilon_U{}^2}}{\varepsilon_I}$$

Summary

We showed that the formula Π = (p + 1)/(p + 7/3) breaks down at the low and high frequency limits when realistic energy cutoffs are present. In fact, the applicable frequency range for this formula is narrower than [$\nu_c(\gamma_{min})$, $\nu_c(\gamma_{max})$], and the range depends on the power-law index p. We then introduced a generalized formula of the polarization degree that only relies on the local spectral index. We showed numerically that it can robustly predict the polarization degree with only single digit percentage difference. The generalized formula would be particularly useful for relatively narrow-band observations in astronomy.

References. A. G. Pacholczyk, (Pergamon Press, 1980). K. C. Westfold, Astrophys. J. 130, 241 (1959). G. B. Rybicki and A. P. Lightman, (Wiley-VCH, 1979). 尤峻汉 (科学出版社, 1998).
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