

PAST, PRESENT AND THE FUTURE

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2023.09.26



- Quantum Walks and its applications

Overview

- Quantum Walks and its applications
- SSH Model and it's variations

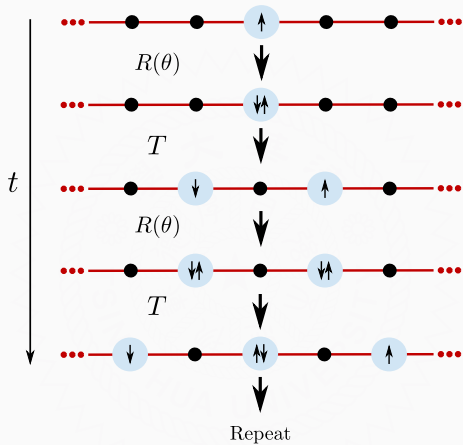
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- SSH Model and it's variations
- Quantum Entanglement

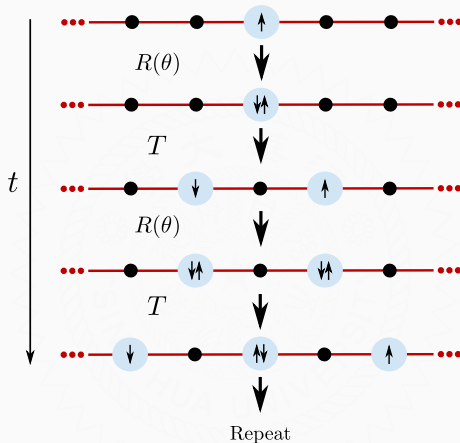
Overview

- Quantum Walks and its applications
- SSH Model and it's variations
- Quantum Entanglement
- A desire for more and more!!!

1D DTQW

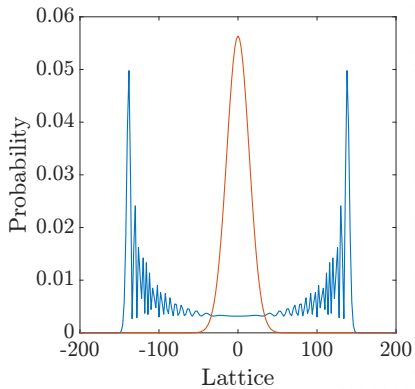


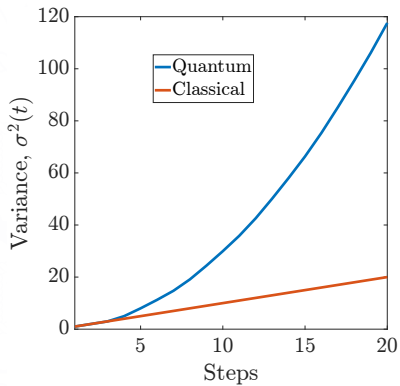
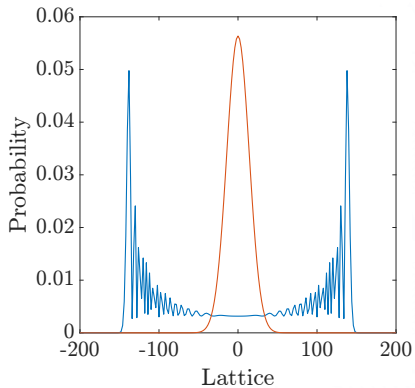
1D DTQW



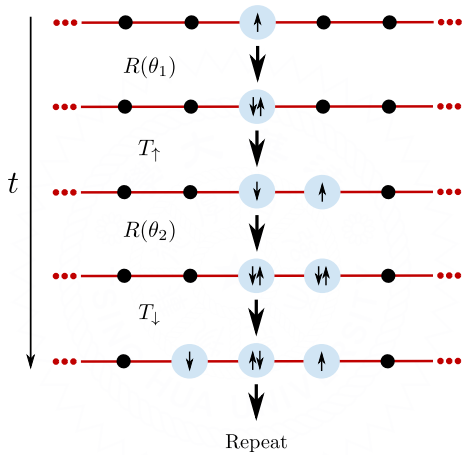
Time evolution operator for 1D DTQW

$$U(\theta) = TR(\theta)$$

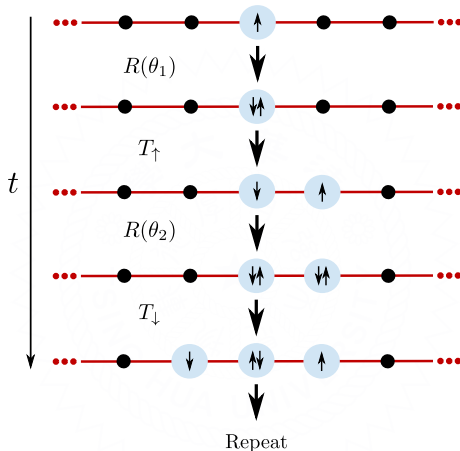




1D SSQW

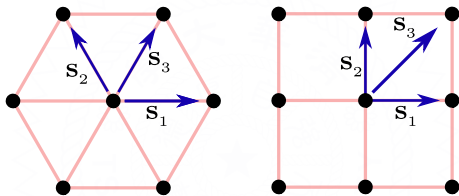


1D SSQW



Time evolution operator for 1D SSQW

$$U_{ss}(\theta_1, \theta_2) = T_{\downarrow} R(\theta_2) T_{\uparrow} R(\theta_1)$$



Time evolution operator for 1D SSQW

$$U_{2D}(\theta_1, \theta_2) = T_y R(\theta_1) T_y R(\theta_2) T_x R(\theta_1) T_x.$$

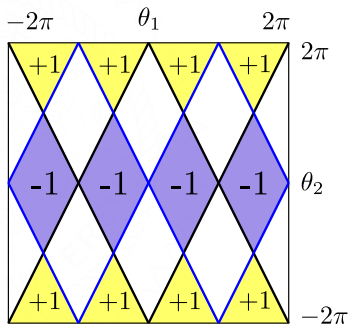
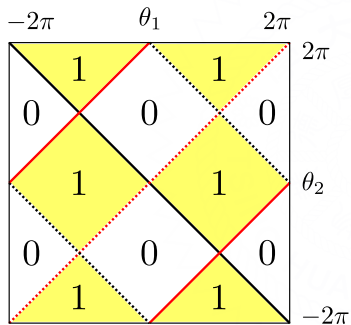
$$U_{\text{SS}} = e^{-iHt} \longrightarrow H_k(\theta_1, \theta_2) = \sum_k \underbrace{E_k(\theta_1, \theta_2) [\hat{\mathbf{n}}_k(\theta_1, \theta_2) \cdot \boldsymbol{\sigma}]}_{\mathcal{H}_k(\theta_1, \theta_2)} \otimes |k\rangle\langle k|.$$

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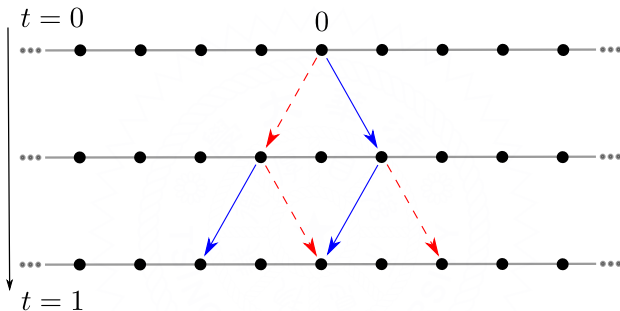
The **dispersion** relation is given by (Kitagawa, 2010)

$$\cos E_k(\theta_1, \theta_2) = \cos(\theta_1/2) \cos(\theta_2/2) \cos k - \sin(\theta_1/2) \sin(\theta_2/2).$$

Topological Phases in Quantum Walks

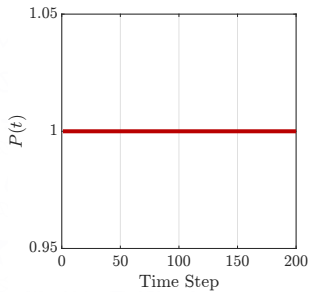
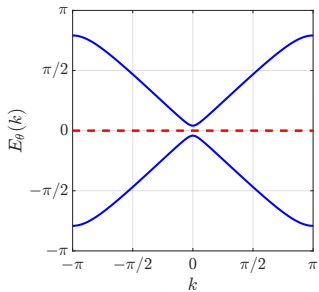


Non-Unitary/Non-Hermitian Quantum Walk

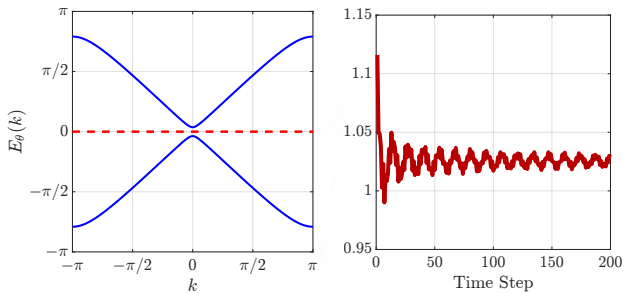


(Regensburger et al (2012))

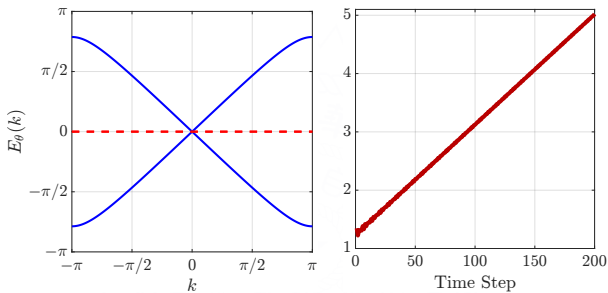
$$T_{\downarrow} R(\theta_2) T_{\uparrow} R(\theta_1) \xrightarrow{G_{\gamma} = e^{\gamma \sigma_z}} T_{\downarrow} G_{\gamma}^{-1} R(\theta_2) T_{\uparrow} G_{\gamma} R(\theta_1)$$



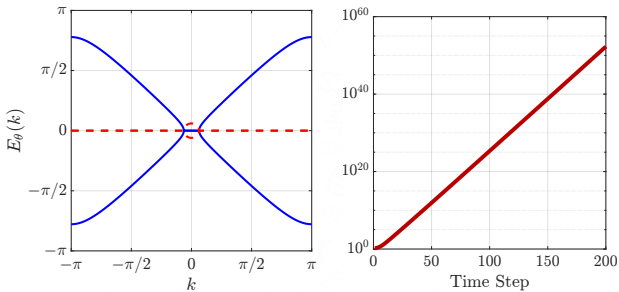
$$\cos E(k) = \cos(\theta_1/2) \cos(\theta_2/2) \cos k - \sin(\theta_1/2) \sin(\theta_2/2) \cosh 2\gamma$$



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In continuous systems, we have

Parity symmetry (Unitary)

$$\mathcal{P} : x \rightarrow -x \text{ and } p \rightarrow -p, \mathcal{P}^2 = \mathbb{1}$$

Time reversal symmetry (Anti-unitary)

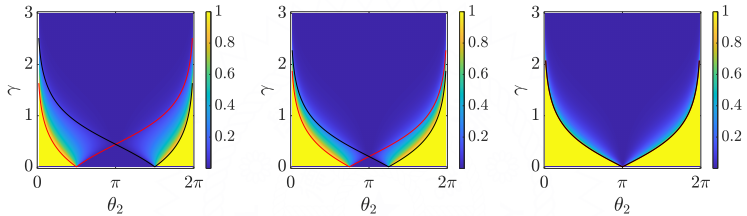
$$\mathcal{T} : t \rightarrow -t, p \rightarrow -p, x \rightarrow -x, \text{ and } i \rightarrow -i$$

such that

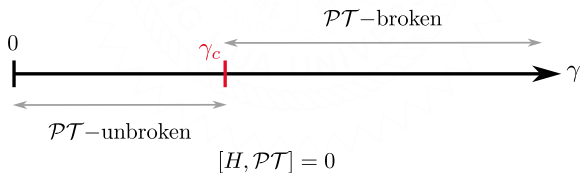
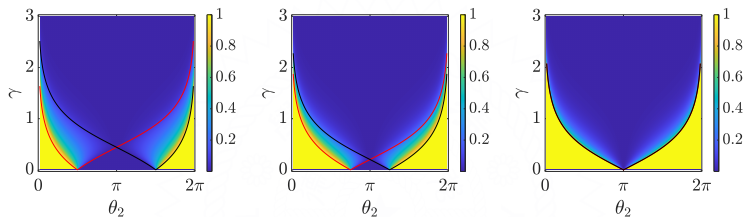
\mathcal{PT} -symmetric Hamiltonian

$$(\mathcal{PT})H(\mathcal{PT})^{-1} = H$$

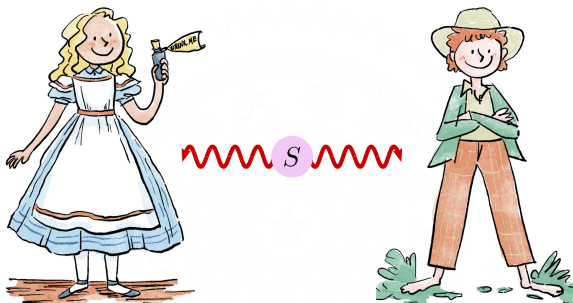
1D NH-SSQW



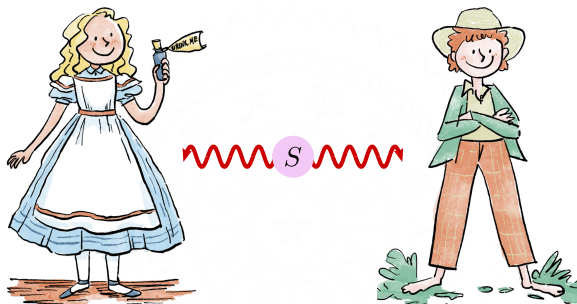
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Quantum Walk and Entanglement

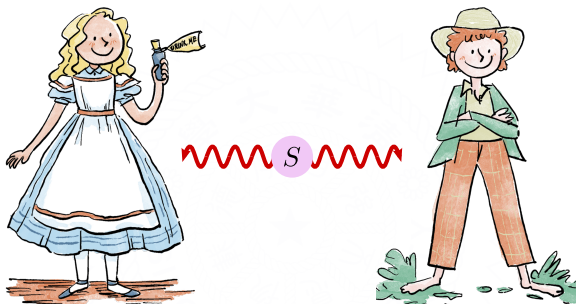


Quantum Walk and Entanglement



$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B)$$

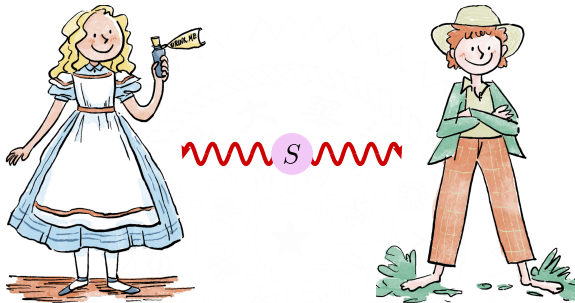
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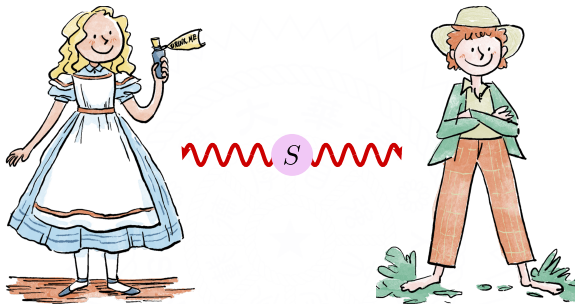
TRANSMISSION EFFICIENCY!!! DETECTION EFFICIENCY!!!

Quantum Walk and Entanglement



$$|\Psi\rangle_{AB} = S_{AB}(\beta) |0,0\rangle, \quad S_{AB}(\beta) = \exp\left[\beta(a_A^\dagger a_B^\dagger - a_A a_B)\right]$$

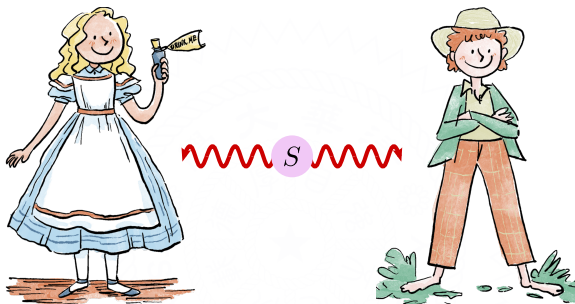
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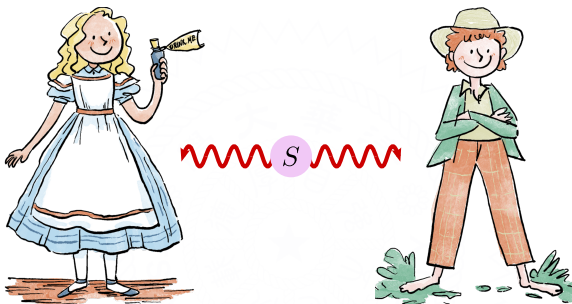
IDEAL HOMODYNE DETECTION AT TELECOM λ 's !!!

Quantum Walk and Entanglement



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Quantum Walk and Entanglement



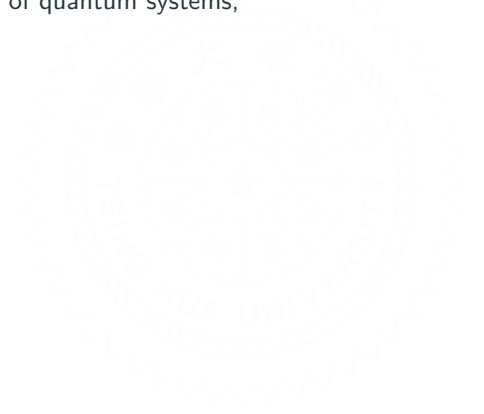
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Middle ground!! Useful Entanglement over 300 km

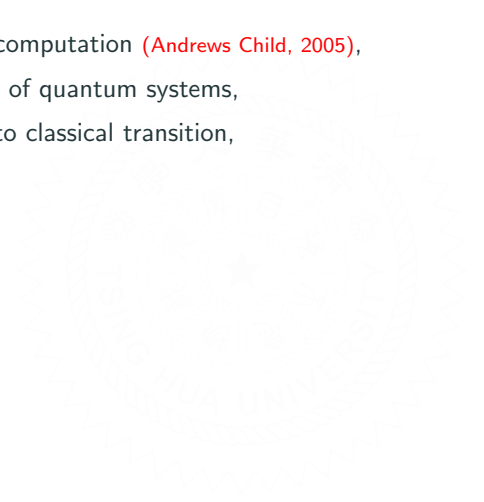
- quantum computation ([Andrews Child, 2005](#)),



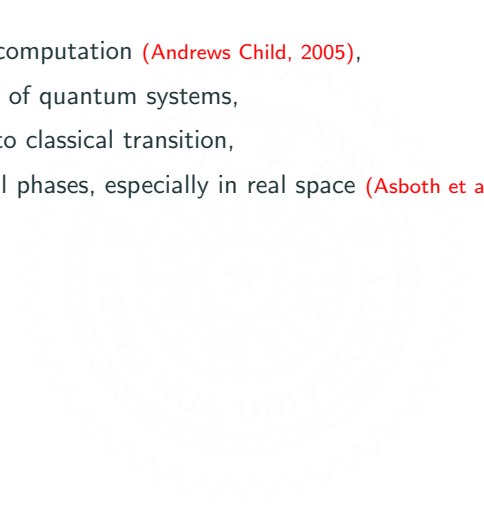
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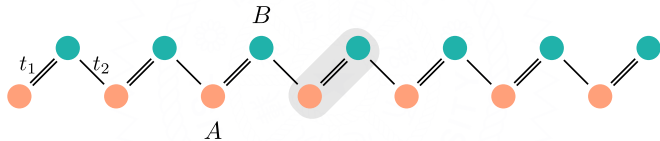
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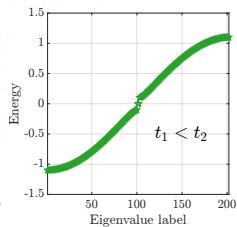
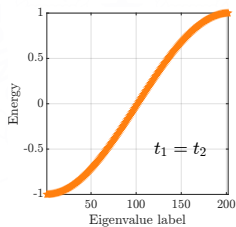
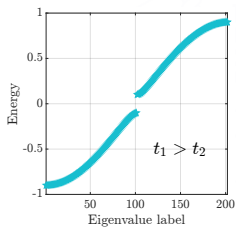
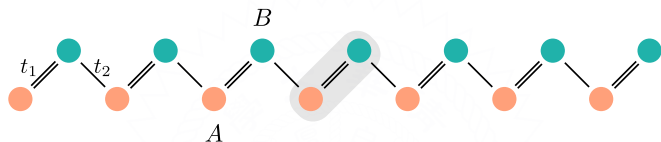
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- analogous to SSH model,

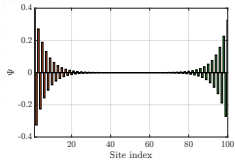
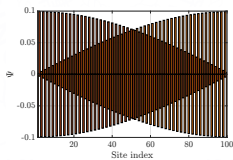
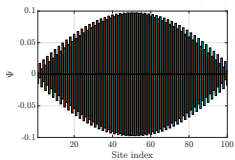
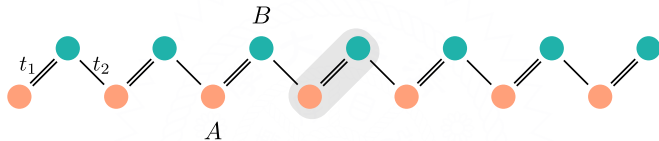
Our beloved SSH



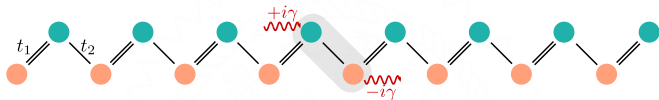
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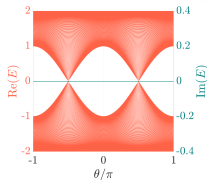
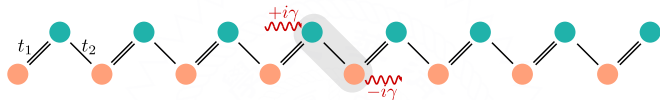
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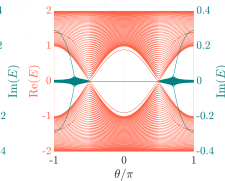
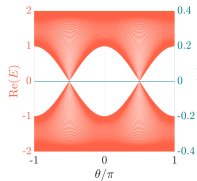
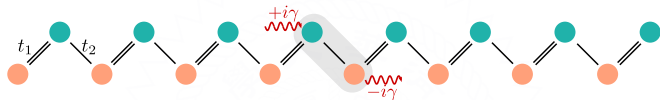
Siblings of SSH (Even)



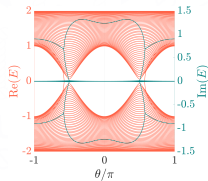
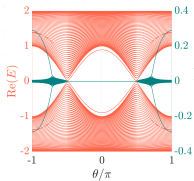
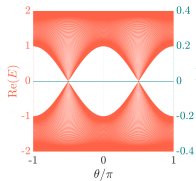
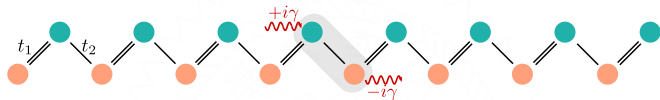
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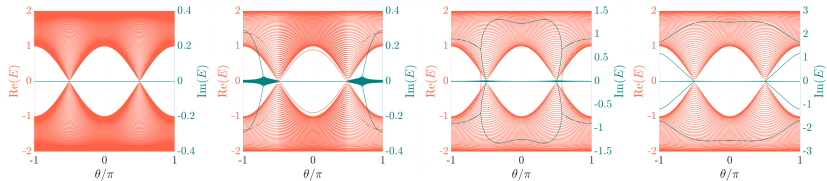
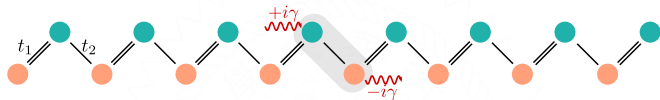
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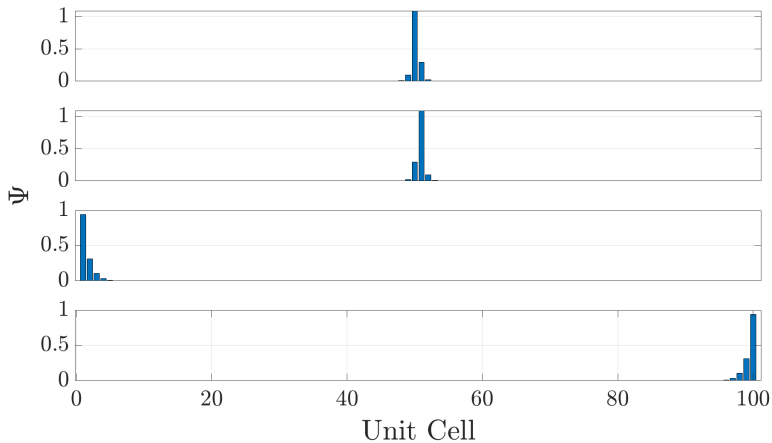


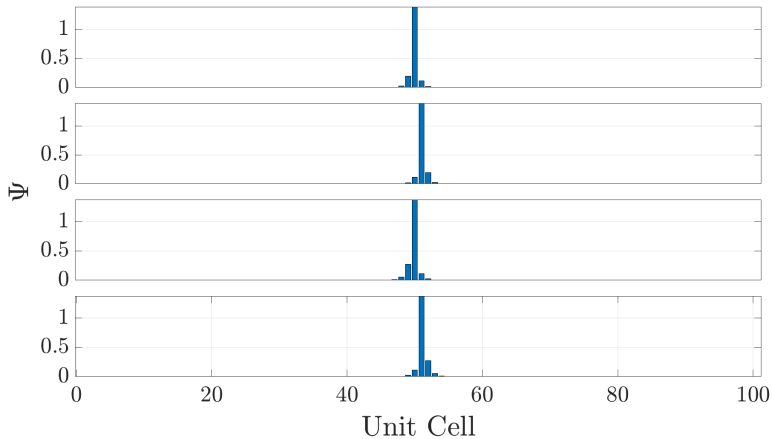
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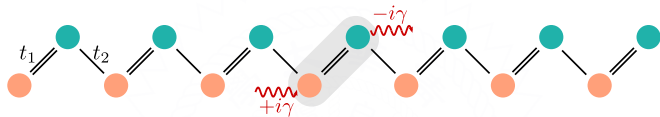
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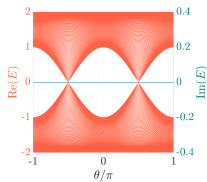
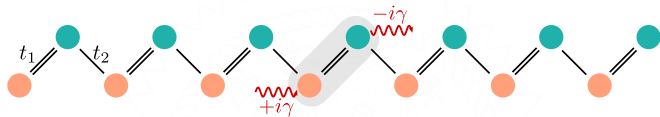




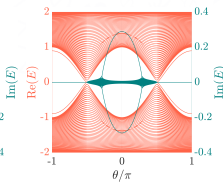
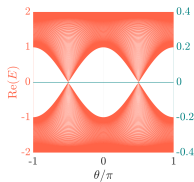
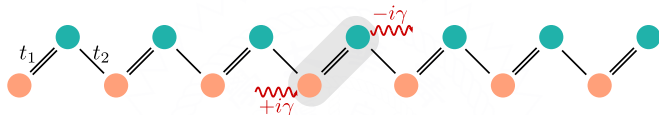
Siblings of SSH (Odd)



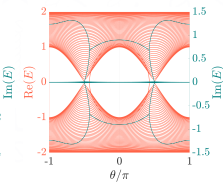
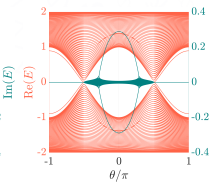
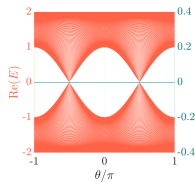
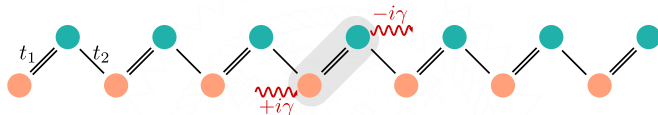
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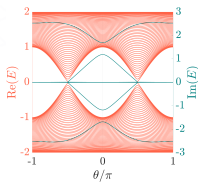
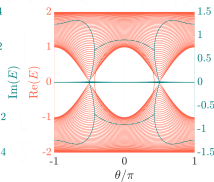
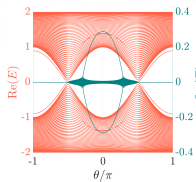
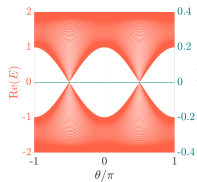
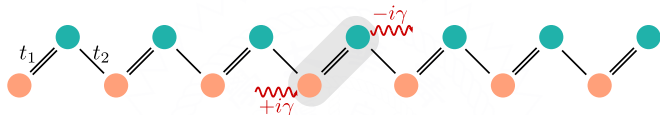
Siblings of SSH (Odd)

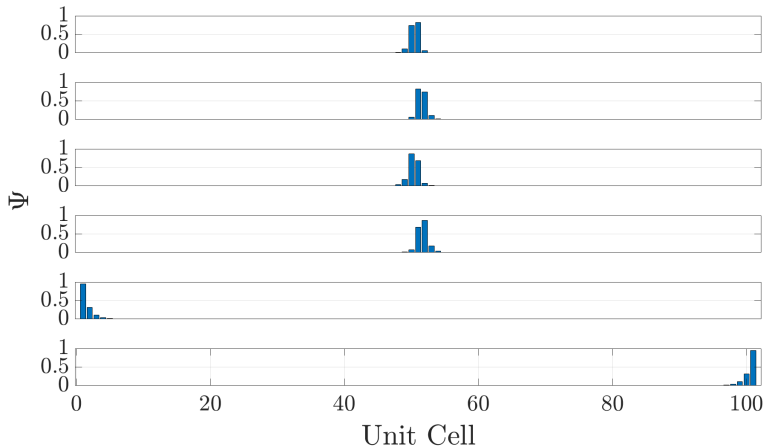


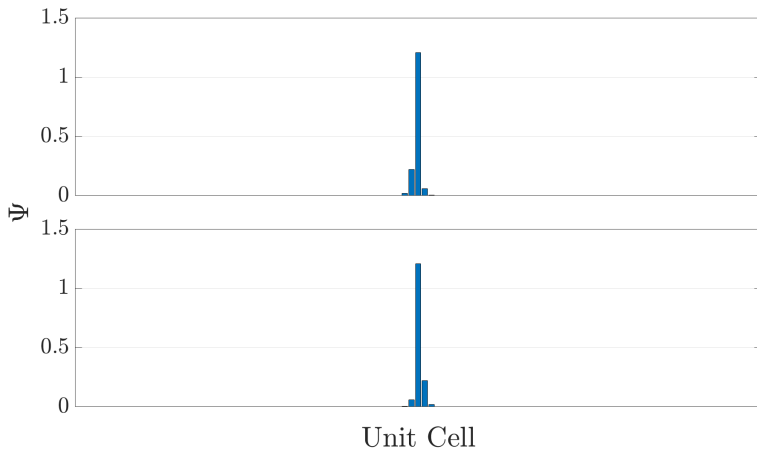
Siblings of SSH (Odd)



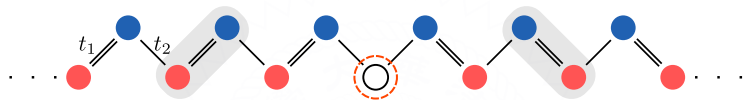
Siblings of SSH (Odd)



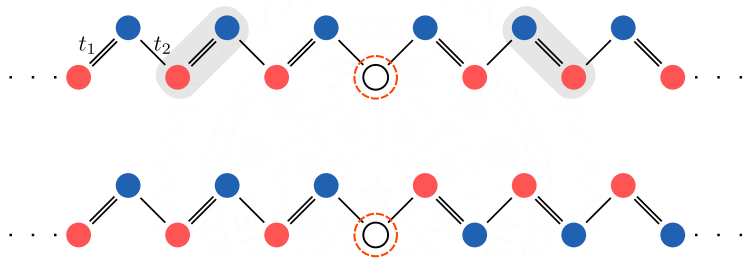




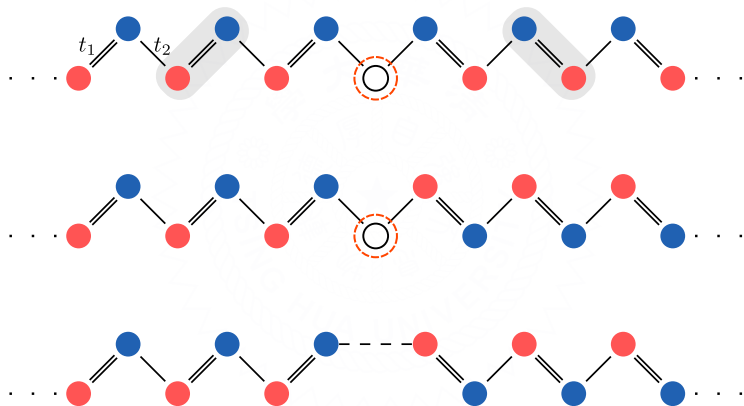
Other siblings of SSH Model



Other siblings of SSH Model



Other siblings of SSH Model





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PAPER

Non-Hermitian topological phases and dynamical quantum phase transitions: a generic connection

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Keywords: non-Hermitian physics, dynamical quantum phase transition, topological phases of matter

Abstract

The dynamical and topological properties of non-Hermitian systems have attracted great attention in recent years. In this work, we establish an intrinsic connection between two classes of intriguing phenomena—topological phases and dynamical quantum phase transitions (DQPTs)—in non-Hermitian systems. Focusing on one-dimensional models with chiral symmetry, we find DQPTs following the quench from a trivial to a non-Hermitian topological phase. Moreover, the critical momenta and critical time of the DQPTs are found to be directly related to the topological invariants of the non-Hermitian system. We further demonstrate our theory in three prototypical non-Hermitian lattice models, the lossy Kitaev chain (LKC), the LKC with next-nearest-neighbor hoppings, and the nonreciprocal Su–Schrieffer–Heeger model. Finally, we suggest a proposal to experimentally verify the found connection by a nitrogen-vacancy center in diamond.

Topological invariants in quantum walks

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Discrete-time quantum walks (DTQWs) provide a convenient platform for a realization of many topological phases in noninteracting systems. They often offer more possibilities than systems with a static Hamiltonian. Nevertheless, researchers are still looking for DTQW symmetries protecting topological phases and for definitions of appropriate topological invariants. Although the majority of DTQW studies on this topic focus on the so-called split-step quantum walk, two distinct topological phases can be observed in more basic models. Here we infer topological properties of the basic DTQWs directly from the mapping of the Brillouin zone to the Bloch Hamiltonian. We show that for translation-symmetric systems they can be characterized by a homotopy relative to special points. We also propose a topological invariant corresponding to this concept. This invariant indicates the number of edge states at the interface between two distinct phases.

DOI: [10.1103/PhysRevA.107.032201](https://doi.org/10.1103/PhysRevA.107.032201)


Speeding Up Entanglement Generation by Proximity to Higher-Order Exceptional Points

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Entanglement is a key resource for quantum information technologies ranging from quantum sensing to quantum computing. Conventionally, the entanglement between two coupled qubits is established at the timescale of the inverse of the coupling strength. In this Letter, we study two weakly coupled non-Hermitian qubits and observe entanglement generation at a significantly shorter timescale by proximity to a higher-order exceptional point. We establish a non-Hermitian perturbation theory based on constructing a biorthogonal complete basis and further identify the optimal condition to obtain the maximally entangled state. Our study of speeding up entanglement generation in non-Hermitian quantum systems opens new avenues for harnessing coherent nonunitary dissipation for quantum technologies.

DOI: [10.1103/PhysRevLett.131.100202](https://doi.org/10.1103/PhysRevLett.131.100202)

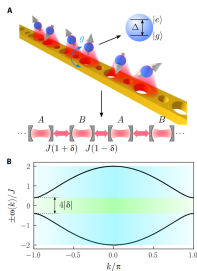
PHYSICS

Unconventional quantum optics in topological waveguide QED

M. Bello¹, G. Platero¹, J. I. Cirac², A. González-Tudela^{2,3*}


The discovery of topological materials has motivated recent developments to export topological concepts into photonics to make light behave in exotic ways. Here, we predict several unconventional quantum optical phenomena that occur when quantum emitters interact with a topological waveguide quantum electrodynamics bath, namely, the photonic analog of the Su-Schrieffer-Heeger model. When the emitters' frequency lies within the topological bandgap, a chiral bound state emerges, which is located on just one side (right or left) of the emitter. In the presence of several emitters, this bound state mediates topological, tunable interactions between them, which can give rise to exotic many-body phases such as double Néel ordered states. Furthermore, when the emitters' optical transition is resonant with the bands, we find unconventional scattering properties and different super/subradiant states depending on the band topology. Last, we propose several implementations where these phenomena can be observed with state-of-the-art technology.

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PHYSICAL REVIEW B **108**, 085126 (2023)

Quantum quench dynamics of Berry and Uhlmann phases in topological systems

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We study the time evolution of geometric phases of one-dimensional topological models under the quench dynamics. Taking the Creutz ladder model as an example, it is found that the Berry phase is fixed as the parameter is suddenly tuned across the topological phase boundary, given that the inversion symmetry of the model is preserved. At finite temperature, the Uhlmann phase displays abrupt jumps between the two quantized values, which indicates the topological transition at certain times after the quench. Both the Berry and Uhlmann phase will deviate from quantized values if the inversion symmetry of the model is broken.

DOI: [10.1103/PhysRevB.108.085126](https://doi.org/10.1103/PhysRevB.108.085126)

हज़ारों ख्वाहिशें ऐसी कि हर ख्वाहिश पे दम निकले
बहुत निकले मिरे अरमान लेकिन फिर भी कम निकले

मिर्ज़ा ग़ालिब

I have a thousand yearnings , each one afflicts me so
Many were fulfilled for sure, not enough although